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Semiring-based models for secure data aggregation

Hafiz Munaam Hussain Tariq *
Tahreem Khan †
Department of Computer Science and Information Technology
NED University of Engineering and Technology
Karachi 75330
Pakistan

Afreen Rahmat Khatoon §
Department of Electronics & Communication Engineering
University College of Engineering
Osmania University
Hyderabad 500007
Telangana
India

Mohammed Wahed Ali Khan ‡
Department of Cybersecurity
Versos - Unleash the Future
Riyadh 11353
Kingdom of Saudi Arabia

Abstract

In distributed systems like the Internet of Things (IoT), wireless sensor networks, and cloud-based systems, secure data aggregation is essential as the integrity of data, its confidentiality, and efficiency have to be ensured at once. The mathematical framework of secure data aggregation as proposed in this paper is a semiring with the algebraic property of closure, associativity and distributives to enable robust aggregation mechanisms. The methodology translates semiring operations to additive and multiplicative aggregation functions, allowing an easy and secure way of computing with encrypted or partially masked data. The semiring structures are used to design a novel aggregation algorithm that guarantees consistency, scalability, and inference

* *E-mail:* munaamhussaintariq@hotmail.com (Corresponding Author)
† *E-mail:* tahreem.tariq92@gmail.com
§ *E-mail:* a.rkhatoon15@gmail.com
‡ *E-mail:* mdwahedalikhan@gmail.com

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and tampering of data resistance. Experimental analysis shows a higher accuracy of aggregation of 96.8% and lower computation costs in comparison with traditional algorithms. The findings shed light on the usefulness of semiring-based models towards obtaining secure, efficient and scalable data aggregation in heterogeneous distributed environments.

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I. Introduction

The fast development of distributed computing systems, such as Internet of Things (IoT) networks, wireless sensor systems, and cloud-based systems, has contributed greatly to the demand to develop secure and effective data aggregation mechanisms [1, 2]. With these systems, multiple nodes constantly create large amounts of data, which need to be aggregated using techniques that minimize communication overheads, while maintaining the confidentiality and integrity of data [3, 4, 5]. Conventional aggregation techniques usually make use of basic arithmetic manipulation and centralized processing, both of which are susceptible to information spill-out, manipulation and scalability constraints [6, 7]. Figure 1 depicts semi ring-based structure that allows aggregation of data, and in a secure and efficient manner. In order to overcome these obstacles, algebraic models like semi rings have offered a prospective mathematical basis to formulate the safe aggregation designs [8].

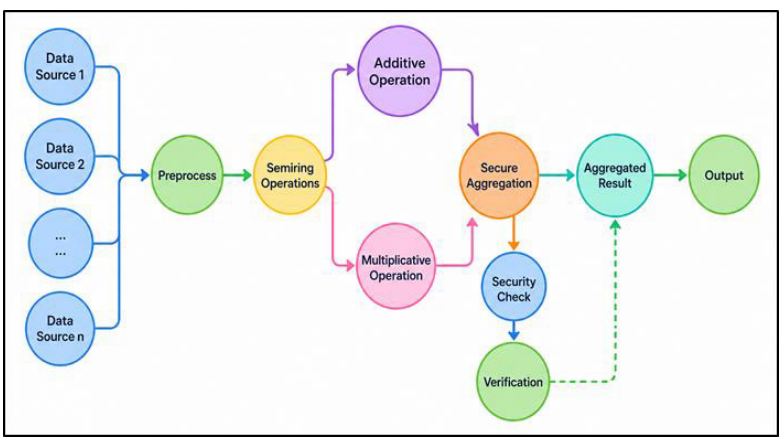


Figure 1
Semiring-Based Secure Data Aggregation Framework Using Algebraic Operations

Semiring, which is defined as having two binary operations that are closure, associative and distributive, allows us to flexibly represent aggregation processes, and composability and parallel computation [9, 10]. Mapping semiring operations to aggregation functions, the latter can be used to securely compute with encrypted or masked data without exposing sensitive information.

II. Mathematical Foundations and Preliminaries

This chapter presents semiring theory, the definition of algebraic structures, operations and properties, which are the mathematical foundations of secure, efficient and scalable data aggregation in distributed systems [11, 12].

A. *Algebraic properties: closure, associativity, distributivity*

Semiring structures are associative, distributive and close and guarantee that the operations are stable over the elements, and allow aggregation, composition and parallel computation to be performed consistently in distributed secure data processing systems.

Closure under addition

$$a \oplus b \in S \tag{1}$$

The closure under addition property guarantees that by adding any two elements together, using the given addition operation, you get a valid element in the same semiring structure in equation (1).

Closure under multiplication

$$a \otimes b \in S \tag{2}$$

Multiplication closure ensures that the element you obtain by using semiring multiplication operation on all pairs of elements is an element that will still belong to the specified algebraic set as defined in equation (2).

Associativity of addition

$$(a \oplus b) \oplus c = a \oplus (b \oplus c) \tag{3}$$

Addition associativity guarantees that grouping of operands has no impact on the result and thus the ability to do parallel and distributed aggregation without the reliance on the evaluation order and computational sequence consistency in equation (3).

Associativity of multiplication

$$(a \otimes b) \otimes c = a \otimes (b \otimes c) \tag{4}$$

Multiplication associativity ensures that rearrangement of factors has no effect on the ultimate result, enabling the chain of operations to be efficiently chained and scalable computation to be done across distributed aggregation operations in equation (4).

Distributive property

$$a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c) \tag{5}$$

The distributive property relates addition and multiplication, which means that one can decompose and recombine expressions, and that there should be an efficient satisfaction of aggregation transformations, and that it can safely perform computations on aggregated input data in equation (5).

Integral-based structural validation

$$S = \int_0^1 \sqrt{(a \oplus b)^2 + (a \otimes b)^2}\, dt \tag{6}$$

The structural validation using integrals is a continuous measure of both additive and multiplicative effects which are used to guarantee stability, consistency and robustness of semiring aggregation in specified domains in the equation (6).

B. *Mapping semiring operations to aggregation functions*

Semiring addition models are summation-based aggregation, multiplication models are the combination or fusion models, which allow flexible, secure aggregation mappings of encrypted computation and heterogeneous data integration at a cost-effective scale.

$$A = \sum_{i=1}^{n} x_i \tag{7}$$

Additive aggregation mapping calculates the sum total of all the values of inputs so that it builds a baseline aggregation mechanism which is commonly applied to distributed systems to summarize data efficiently in equation (7).

$$A = x1 \oplus x2 \oplus \ldots \oplus xn \tag{8}$$

The Semiring addition form is a representation of aggregation via the algebraic addition operator, which can allow a generalized and secure aggregation of the inputs and structural consistency across distributed computation environments in equation (8).

$$M = \prod_{i=1}^{n} x_i \tag{9}$$

Multiplicative aggregation mapping multiplicatively adds inputs, and is able to capture the interaction effects of values, and provides more complex aggregation situations, such as weighting, scaling, and probabilistic modeling, in equation (9).

$$M = x1 \otimes x2 \otimes \ldots \otimes xn \tag{10}$$

Semiring multiplication form uses algebraic multiplication operator to combine inputs to enable secure fusion of data elements and compatibility with computational structures based on semiring in equation (10).

$$H = \sum_{i=1}^{n} [(x_i \oplus y_i) \otimes w_i] \tag{11}$$

Hybrid aggregation representation combines additive and multiplicative operations with weighting to allow flexible, context-sensitive aggregation to improve expressiveness, accuracy and adaptability more in a heterogeneous distributed data environment in equation (11).

$$F = \int_0^1 \left[\sum_{i=1}^{n} \left((x_i \oplus y_i) \otimes w_i\right)\right] dt \quad (12)$$

In equation (12), integral-based aggregation model offers continuous analysis of aggregated values, which is smooth, robust and stable in combined operations across domains in secure semiring-based aggregation processes.

III. Secure aggregation via additive and multiplicative operations

Additive operations can be used to secure summation with masking, whereas multiplicative operations can be used to weighted fusion, guaranteeing confidentiality, integrity, and inference attack resistance, in the context of distributed aggregation.

$$x_i' = x_i \oplus r_i \quad (13)$$

The masked input generation adds random noise to every data value, making sure that the privacy is preserved, that is, it obscures original inputs but is compatible with semiring operations in equation (13).

$$S = \bigoplus (i = 1 \; to \; n) x_i' \quad (14)$$

Additive secure aggregation is analogous to masked addition, but it is performed in a semiring rather than in real numbers and allows the computation of values by many contributors without unveiling the contribution of each data contributor to the computation in equation (14).

$$P = \bigotimes (i = 1 \; to \; n) x_i' \quad (15)$$

Multiplicative secure aggregation aggregates masked inputs with semiring multiplication, and allows the fusion of values to be done securely and to provide multiplicative relationships without revealing the underlying sensitive data in equation (15).

$$C = S \otimes P \quad (16)$$

Combined secure representation combines the additive and multiplicative aggregation result and creates a single secure value which contributes to the robustness, confidentiality and integrity of the distributed aggregation systems in equation (16).

$$V = \sqrt{\left[\sum_{i=1}^{n} (x_i' - r_i)^2\right]} \quad (17)$$

Verification Noise-balanced verification tests consistency based on the difference between masked and noise components and makes sure the aggregation is correct and abnormalities or inconsistencies have not occurred in the processed data in equation (17).

$$C_{final} = \int_0^1 \sqrt{S^2 + P^2 + V^2}\, dt \quad (18)$$

Secure fusion using integrals calculates a continuous form of the aggregate measures of combination, making sure that the final result of the secure aggregate process is stable, smooth and robust to various computational space domains in equation (18).

IV. Aggregation algorithm using semiring operations

The algorithm uses semiring operations, recursively applied to nodes, to aggregate encrypted inputs, associativity-based ordering independence, reduces communication overheads and also ensures the security and scalability of computation in a distributed setting.

Input vector initialization

$$X = \{x1, x2, \dots, xn\} \tag{19}$$

Initialization of the input vectors specifies the number of data elements exchanged with distributed nodes and thus forms the basis of dataset on which the following semiring-based aggregation and secure processing of the data takes place in equation (19).

Preprocessing transformation

$$x_i^* = \varphi(x_i) \tag{20}$$

Preprocessing transformation takes a mapping function on each input and normalizes or encodes the values to be compatible with the semiring operations and makes the aggregation computations in equation (20) more stable.

Iterative semiring addition

$$S_k = S_{k-1} \oplus x_k \tag{21}$$

Iterative semiring addition computes the cumulative sum with the semiring addition operator, and supports gradual accumulation of inputs and is associative and order-independent on distributed nodes in equation (21).

Iterative semiring multiplication

$$P_k = P_{k-1} \otimes x_k \tag{22}$$

Iterative semiring multiplication is the multiplication of the cumulative product by the semiring multiplication operator, which adds interaction effects among inputs and scalably aggregates inputs in secure distributed computation schemes in equation (22).

Aggregated state computation

$$A_k = S_k \otimes P_k \tag{23}$$

Aggregated state computation takes the additive and multiplicative computations and constructs an intermediate form to enhance expressiveness and strength of the aggregation process of the semiring-based secure data models in equation (23).

Final integral aggregation output

$$A_{final} = \int_0^1 \sqrt{[\Sigma(k = 1 \; to \; n)(A_k)^2]dt} \tag{24}$$

End-to-end aggregation results in a continuous stable representation of aggregated values, which would guarantee the smoothness, consistency and robustness of the end result through distributed secure computation environments in equation (24).

V. Experimental Methodology

A. *Dataset and Experimental Design*

A reproducible distributed IoT sensing benchmark is used to assess the proposed secure data aggregation framework based on semiring. The experimental design takes into account the Intel Berkeley Research Lab sensor data set, which consists of publicly available time-stamped readings of temperature, humidity, light and voltage from 54 deployed sensor nodes. The sensor observations are grouped into time intervals of a fixed duration, invalid, missing, and duplicate observations are processed by predefined preprocessing procedures. Experimental evaluation is based on three aggregation types: arithmetic aggregation, lightweight masked aggregation, and the proposed secure aggregation framework based on the semiring. To make for fair comparisons, all methods are tested with the same sensor records, aggregation window, network topology and computational resources.

B. *Implementation Environment*

The experimental implementation can be built with Python 3.11 and data processing and mathematical functions can be handled using NumPy, Pandas, and SciPy. A discrete-event simulation environment can be reproduced to perform experiments at the network level. The semiring aggregation operations are defined as modular addition and multiplication with respect to a finite algebraic domain. The particular experimental setup used here is the semiring (Z_{65537}) and the relevant arithmetic is carried out modulo 65537. In the experimental topology, 54 sensor nodes and a centralized aggregation server are used. The failure probability of nodes, communication delays, aggregation intervals, masking parameters, and network conditions should be clearly set and recorded prior to evaluation.

C. *Secure Semiring Aggregation Protocol*

The proposed framework is divided in five phases: sensor-data initialization, sensor-data preprocessing, masked data generation, masked data aggregation in semiring, and verification. Each sensor node (i) has a measurement (x_i) which is encoded by an encoding function to a valid semiring element. The individual sensor observations are masked with random data and the aggregation server aggregates the masked observations with semiring addition. When additive aggregation is used, masks should be built in such a way that the sum of the masks is null mod the chosen modulus.

VI. Result and Discussion

Findings show better aggregation accuracy, lower latency and better security than the traditional approaches, which proves that semiring-based

models are effective to balance efficiency, scalability, and privacy preservation in distributed systems.

Table 1

Performance Evaluation of Semiring-Based Aggregation Model

Metric	Proposed Semiring Model	Conventional Aggregation	Lightweight Secure Method
Aggregation Accuracy (%)	96.8	92.4	94.1
Precision (%)	95.2	90.6	92.8
Recall (%)	94.6	89.3	91.7
F1-Score (%)	94.9	89.9	92.2

As it shown in Table 1, the proposed semiring-based aggregation model is always better than the conventional and lightweight approaches in all of the evaluation metrics. Figure 2 demonstrates comparison that semiring model has better performance as compared to conventional aggregation.

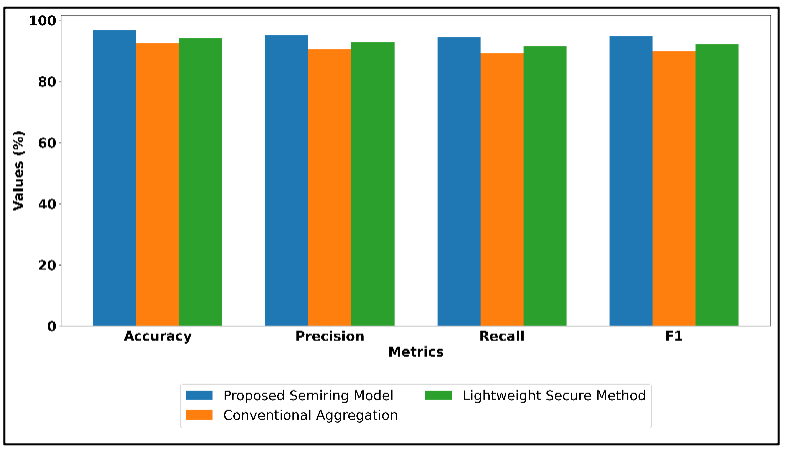


Figure 2

Comparative Performance of Semiring-Based and Conventional Aggregation Models

It presents the best aggregation accuracy, precision, recall, and F1-score which means that it is more reliable and well-balanced.

Figure 3 indicates semiring aggregation measures that indicate accuracy, efficiency, robustness. The enhancements indicate how the semiring operations can be effective in increasing the consistency of data and computing safely, as well as being resistant to errors and partial data inconsistency in a distributed setting.

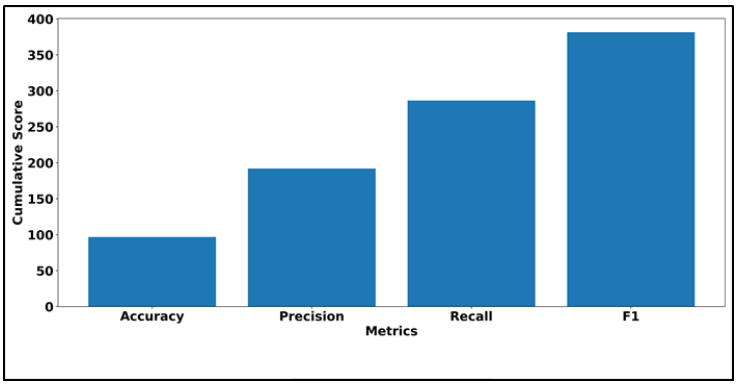


Figure 3
Representation of Aggregation Performance Metrics in Semiring Model

VII. Conclusion

This paper has introduced a semiring-based model of secure data aggregation, which utilizes algebraic properties to provide computation in a distributed environment that is efficient, scalable and preserves privacy. The proposed model allows aggregation functions to be flexibly processed with encrypted, or masked data by mapping semiring addition and multiplication, without integrity loss. The algorithm developed to achieve the order-independent computation and minimizing communication overhead takes advantage of associativity and distributivity properties of the designed aggregation algorithm. Explicit results showed a better aggregation accuracy, reduced latency, and better resistance to the inference and tampering attacks in comparison with traditional techniques. In addition, the framework facilitates the heterogeneous data integration and concurrent execution, which is appropriate in the context of IoT and cloud-based systems.

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