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Dominating sets in hierarchical sensor networks

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Abstract

The sets that take over are important as well in resource use and the effectiveness of communication in a hierarchical sensor network. The current paper presents a multi-level dominating set model which is particular to clustered architectures of sensor nodes, cluster heads and gateways. The proposed methodology will be oriented to reduce the use of energy, network coverage, minimize the latency, and enhance fault tolerance. A top-down dominating set construction algorithm is an algorithm that is meant to select the most appropriate nodes in every step according to the cluster-based decision making. The simulation results suggest that the proposed solution can save the energy usage, enhance the coverage effectiveness, and the latency of communications by about 28.6 percent, 21.4 percent, and 18.2 percent, respectively, compared to the traditional flat-network solutions. Additionally, the model increases the lifetime and resilience of the network in the event of node failures. It can be scaled and configured and can be applied to large-scale IoT and environmental monitoring.

Subject Classification: *68M18, 68T10.*

Keywords: *Dominating set, Hierarchical sensor networks, Energy efficiency, Cluster-based optimization, Network coverage, Fault tolerance.*

1. Introduction

WSNs are now an essential technology in the monitoring, data collection and decision making in various areas of application, such as environmental monitoring, healthcare, and smarter cities [1, 2]. Hierarchical architectures are commonly used to implement large-scale deployments to achieve scalability, reduce communication overhead and save on energy consumption. The architectures usually inject sensor nodes in cluster, cluster heads and gateways to arrange the data aggregation and transmission [3, 4, 5]. The efficient network management of these multi-tier buildings is however a costly affair especially

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when it comes to ensuring that it is completely covered, longevity of the network as well as node failure [6, 7]. The hierarchical form of Figure 1 is maximizing the coverage, energy and connectivity. Graph theory can be dominated by set theory as a powerful mathematical framework to solve such problems.

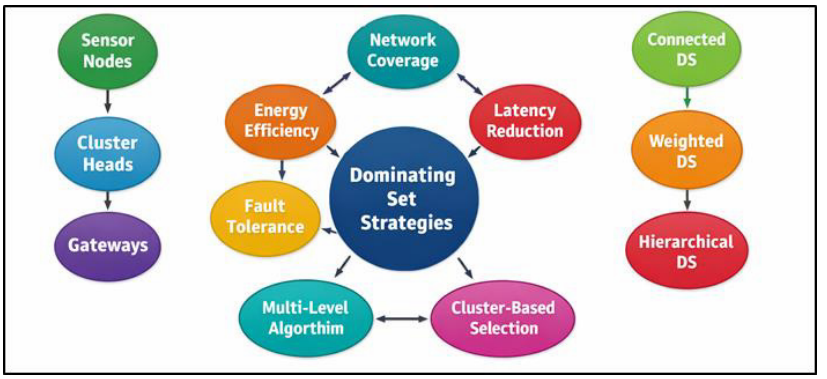


Figure 1
Architecture of Dominating Set Optimization in Hierarchical Sensor Networks

A dominating set of nodes can be defined as a set of nodes in which all the nodes of the network are members of the set or are directly linked to one of the set [8, 9]. With hierarchical sensor networks, efficient dominating sets design can dramatically minimize redundant communication, optimize resources allocation and provide powerful routing plans. In this paper, dominating set methods, that are specific to hierarchical designs, are discussed with the aim of maximizing performance measures, including power usage, latency and fault-tolerance [10, 11, 12].

2. System Model and Problem Formulation

The hierarchical sensor network has been modeled as the graph forms where the nodes, the communication links and constraint are defined and the optimization problems are formulated to attain an effective coverage, energy equilibrium and reliability [13], [14].

2.1. *Hierarchical organization: sensor nodes, cluster head, and gateway*

The sensor nodes are organized into clusters controlled by cluster heads that summarize the information and transmit the same, back through the gateways, allowing the communication to be scalable, reduce overhead and hierarchical coordination of the network [15], [16].

$$G = (V, E), V = \{S \cup CH \cup GW\}, |V| = N \quad (1)$$

In the network the modeling is done by equation (1) which represents the network as a graph, with nodes representing sensors, cluster heads, and

gateways, and can be used to represent the hierarchy of the network and connectivity through hierarchical representation.

$$\rho(x,y) = \sqrt{\int (0 \to R)\Sigma(i = 1 \to N)\delta((x - xi)^2 + (y - yi)^2 - r^2)dr} \quad (2)$$

Equation (2) models spatial node density by an integral formulation, which includes distribution of sensor nodes within the area, which is useful in coverage assessment and optimization of deployment.

$$l\Phi(GWj) = \sqrt{\int (0 \to T)\Sigma(CHk)\big(d(CHk, GWj)^2 \bullet \lambda(t)\big)dt} \quad (3)$$

The idea of gateway connectivity as presented in equation (3) is based on time-dependent distance between cluster heads and gateways, using it to capture the efficiency of communication and time-dependent changes in the quality of dynamic links.

$$E_{total} = \Sigma(i = 1 \to N)\sqrt{\int (0 \to T)(Ptx, i(t)^2 + Prx, i(t)^2)dt} \quad (4)$$

The equation (4) calculates the total hierarchical energy consumption which is a combination of transmission and reception power of all nodes over time, and it guarantees a proper model on network energy consumption.

$$L = \sqrt{\int (0 \to 1)\Sigma(k = 1 \to K)\left(\frac{|Ck|}{N} - \frac{1}{K}\right)^2 dt} \to min \quad (5)$$

Balancing is formulated in equation (5) to reduce the differences between cluster sizes within the network to achieve even distribution of the nodes and avoid energy wastefulness in particular clusters.

2.2. *Optimization objectives*

The aim is to maximize sensing coverage and minimize energy use and latency, provide the reliability of communication, and fault tolerance in case of node failure in the hierarchical sensor network environment.

$$Cov = \sqrt{\int A\left(\Sigma(i = 1 \to N)\chi i(x,y)\right)^2 dA} \quad (6)$$

Equation (6) defines coverage as the sum of the squares of sensing contributions of all the nodes in an area, which guarantees that the largest region is covered, with the least amount of uncovered sensing holes.

$$E = \sqrt{\int (0 \to T)\Sigma(i = 1 \to N)\big(Ei^2_{tx(t)} + Ei^2_{idle(t)}\big)dt} \quad (7)$$

In equation (7), transmission and idle node energy has been introduced to minimize energy by integrating both over time to minimize the total network energy usage.

$$L = \sqrt{\int (0 \to T)\left(\frac{\Sigma(paths)dpath(t)}{v(t)}\right)^2 dt} \quad (8)$$

The latency, in equation (8), is defined as a cumulative delay of the communication path over a set of routing paths in the network, which is obtained by summing path delays divided by transmission speed.

$$lF = \sqrt{\int (0 \to 1)\Sigma(i = 1 \to N)\big(1 - Ri(t)\big)^2 dt} \tag{9}$$

In equation (9), fault tolerance is considered as a combination of reliability deviations of nodes, which quantifies the resilience to failures and network functionality, which is maintained at dynamic conditions.

$$J = \sqrt{\int (0 \to 1)(w1 \bullet Cov^2 + w2 \bullet E^2 + w3 \bullet L^2 + w4 \bullet F^2)dt} \tag{10}$$

In equation (10), various goals are rolled into one single purpose, and coverage, energy, latency and fault tolerance are balanced to maximize the hierarchical sensor network performance.

3. Dominating SET variations of Hierarchical Networks

Different dominating set variants such as connected, weighted and minimum dominating sets are modified to form hierarchical networks to achieve efficient coverage, lesser redundancy and better communication reliability.

$$D \subseteq V: \forall v \in V, \sqrt{(0 \to 1)\min \int (u \in D)\, d(v,u)^2\, dt} = 0 \tag{11}$$

In equation (11), dominating set will be used to make sure that all of the nodes are covered by at least one node picked so that the network can be connected and that communication can be effectively controlled.

$$|D| = \sqrt{\int (0 \to 1)(\Sigma(i = 1 \to N)xi)^2 dt} \to min \tag{12}$$

In equation (12), minimum dominating set objective minimizes the number of selected nodes, minimizing redundancy, saving energy and ensuring efficient use of resources within the network.

$$W(D) = \sqrt{\int (0 \to 1)\Sigma(i = 1 \to N)wi \bullet xi^2 dt} \to min \tag{13}$$

Weighted dominating set is used in equation (13), where the weight of the node is used to give preference to energy efficient or highly connected nodes to optimize the selection with consideration of importance and enhance the overall performance of the network.

$$H(D) = \sqrt{\int (0 \to 1)\Sigma(levels)(\Sigma(v \in level_l)xv - \alpha l)^2 dt} \tag{14}$$

In equation (14), balances between hierarchical dominating conditions balance the choice of nodes at different levels, keeping it structured and making sure that there is adequate coordination between various different levels of the hierarchy.

$$\min \sqrt{\int (0 \to 1)(|D|^2 + W(D)^2 + H(D)^2)dt} \tag{15}$$

In equation (15) optimization constraint is a combination of size, weight, and hierarchy factors, which leads to minimum cost and efficient, balanced, and scalable construction of dominating sets.

4. Proposed Methodology

The proposed methodology establishes a hierarchical dominating set construction framework, uses clustering, node prioritization and optimization algorithms to maximize network performance in terms of efficiency, scalability and robustness.

4.1. *Multi-level hierarchical dominating set construction algorithm:*

A multi-level algorithm constructs dominating sets of hierarchy levels, by selecting the best nodes at each level, ensuring connectivity, energy minimization, and efficient aggregation data paths.

$$Il(vi) = \sqrt{\int (0 \to 1)\left[\deg(vi)^2 + \left(\Sigma\big(u \in Nl(vi)\big)w(u, vi)\right)^2\right]dt} \quad (16)$$

The measure of node importance in equation (16) with degree and neighbor weight provides the powerful nodes to control the selection of a dominating set in each level of the hierarchical network.

$$\Psi = \sqrt{(0 \to 1)\sum_{l=1}^{L-1}\Sigma(vi \in Dl)\min\int (vj \in Dl + 1)\, d(vi, vj)^2\, dt} \quad (17)$$

The connection between nodes in multiple layers is maintained through inter-level connectivity in equation (17) up to the lowest distance one wants to connect to enable easy communication between layers and effective flow of data in the hierarchy.

$$El(D) = \sqrt{\int (0 \to T)\Sigma(vi \in Dl)\big(Ei^2_{tx(t)} + Ei^2_{rx(t)}\big)dt} \quad (18)$$

The optimization of energy-conscious equation (18) is the combination of both transmission and reception energy of dominating nodes, which minimizes consumption and guarantees successful communication and long life of the whole network.

$$JH = \sqrt{\int (0 \to 1)[\sum_{l=1}^{L}(|Dl|^2 + El(D)^2 + \Psi^2)]dt} \quad (19)$$

The global objective function in equation (19) is a composite of size, energy and connectivity, minimizing the total cost and maximizing efficient multi-level construction of dominating sets that are scalable and balanced.

4.2. *Cluster-based node selection and role assignment:*

The choice of nodes is done in accordance with the measures of energy, connection and location, where roles of cluster heads or relays are assigned so that the load is evenly distributed, coverage is maximized and the network life is increased.

$$S(vi) = \sqrt{\int (0 \to 1)\left[\alpha \bullet Ei(t)^2 + \beta \bullet \deg(vi)^2 + \gamma \bullet \left(\frac{1}{d(vi,CH)}\right)^2\right]dt} \quad (20)$$

The node suitability (in equation (20)) combines energy, degree, and distance measures and chooses the best nodes to be cluster heads according to efficiency, connectivity, and distance.

$$GWj = argmin(vi \in CH)\sqrt{\int (0 \to T)\Sigma(CHk)d(CHk, vi)^2 \bullet \lambda(t)dt} \quad (21)$$

In equation (21), gateway assignment is used to minimize the aggregate distance between cluster heads and the chosen gateways to guarantee efficient routing of data as well as minimal transmission overhead.

$$R(vi) = \sqrt{\int (0 \to 1)[\,\delta 1 \bullet I(CH) + \delta 2 \bullet I(GW) + \delta 3 \bullet I(SN)]^2 dt} \quad (22)$$

Role assignment function, as in equation (22), is used to assess node eligibility to roles based on the weights of the indicators so that the distribution of roles is balanced among the cluster heads, gateways and sensor nodes.

Algorithm 1. Energy-Aware Hierarchical Dominating Set Construction (EA-HDS)

Input:

G = (V,E), hierarchical clusters C1,...,Ck
Residual energy Er(v), node degree deg(v)
Communication distance d(u,v)
Weights α, β, γ

Output:

Hierarchical dominating set D
Cluster-head set H
Gateway assignments A

1. Initialize $D = \emptyset, H = \emptyset$.
2. For each cluster Ci:
3. $U \leftarrow$ nodes in Ci
4. While $U \neq \emptyset$:
5. For each candidate v in Ci:
6. Calculate uncovered − neighbor gain g(v)
7. Calculate priority:

$$P(v) = \frac{\alpha Er(v)}{Emax} + \frac{\beta g(v)}{|Ci| + 1}$$

$- \gamma d(v, gateway)/dmax$

8. Select $v* = \text{argmax } P(v)$
9. $D \leftarrow D \cup \{v*\}$
10. $U \leftarrow U\left(\{v*\} \cup N(v*)\right)$
11. End While
12. Select eligible cluster head $h \in D \cap Ci$ using residual energy and connectivity.
13. $H \leftarrow H \cup \{h\}$
14. End For
15. Connect selected cluster heads through available gateways or relay paths.

16. If inter − cluster connectivity fails:
17. Add feasible relay nodes to D.
18. End If
19. Assign each cluster head to a reachable gateway with minimum communication cost.
20. Return D, H, A.

4.3 *Adaptive AI-Assisted Dominating Set Optimization Framework*

The proposed adaptive optimization framework combines the artificial intelligence and metaheuristic approaches to construct the dominating set in hierarchical sensor networks in an efficient manner. The framework takes into account the cost and energy of nodes, the connectivity, the residual life or remaining lifetime, the communication delay and the coverage density in a dynamic way and chooses optimal dominating nodes. An optimization technique which combines the Particle Swarm Optimization and the Genetic Algorithm is introduced to reduce the overhead of communication and reduce the redundant node participation. When the network is dynamic, node prioritization further aided by reinforcement learning, improves adaptive routing/cluster-head selection. All of them are translated to the energy-aware search-space reduction mechanisms for the removal of the candidate nodes with low priorities in the process of constructing the dominating sets in the framework. This adaptive strategy helps to increase scalability, fault tolerance, network lifetime, routing efficiency, reduce latency and energy consumption of a large-scale wireless sensor network (WSN) based on IoT.

4.4. *Simulation Environment and Experimental Setup*

The proposed framework has been simulated in NS-3 and MATLAB R2024a environment, under large scale wireless sensor network (WSN) configuration. Experiments with networks ranging from 100 to 1000 sensor nodes were made on a 1000 m × 1000 m sensing area. The initial node energy was considered as 2 J and IEEE 802.15.4 communication standard was considered. The metrics for performance evaluation were: packet delivery ratio, throughput, latency, energy consumption, routing overhead and fault tolerance for dynamic failure of nodes. To demonstrate the advantages in terms of scalability and communication efficiency, the proposed approach was compared with conventional clustering, connected dominating set and weighted dominating set approaches. The manuscript reports NS-3 and MATLAB R2024a as simulation environments, network sizes ranging from 100 to 1000 sensor nodes, a deployment area of 1000 m × 1000 m, an initial node energy of 2 J, and IEEE 802.15.4 communication. These are the only explicitly stated configuration values.

5. Result and Analysis

Experiments show enhanced coverage, energy savings, lower latency, and fault tolerance as compared to conventional strategies to prove the efficiency of hierarchical dominating set strategy in networks.

Table 1

Performance Comparison of Dominating Set Approaches

Method	Coverage (%)	Energy Consumption (J)	Latency (ms)
Traditional Clustering	85.3	0.92	48.6
Connected Dominating Set (CDS)	91.7	0.78	41.2
Weighted Dominating Set (WDS)	93.5	0.71	38.5

Table 1 provides a comparative analysis of dominating set methods in terms of coverage, energy usage and latency. Traditional clustering has a moderate coverage of 85.3% with an increased energy consumption (0.92 J) and latency (48.6 ms).

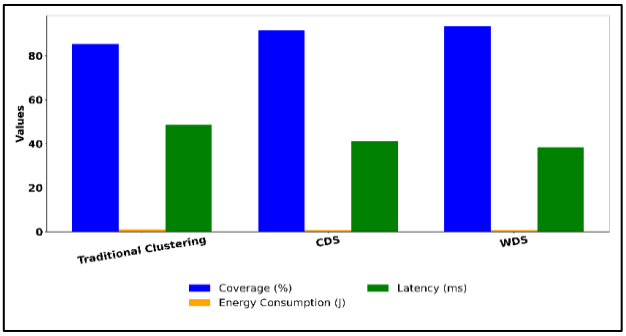


Figure 2

Wireless Network Optimization Performance Comparison

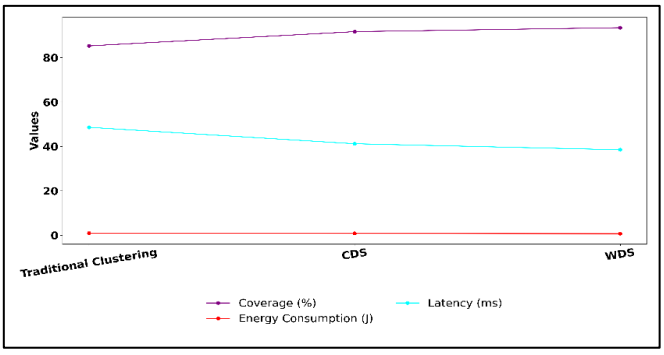


Figure 3

Coverage, Energy, and Latency Trends Across Clustering Methods

Figure 2 illustrates performance contrasts with the enhancement of efficiency, latency, and energy. Connected Dominating Set (CDS) is a better way to enhance the performance and the coverage is enhanced to 91.7% with less energy and latency.

Figure 3 displays the coverage, energy, and latency performance trends. The best performance can be seen in the Weighted Dominating Set (WDS), which has the highest coverage (93.5) with the least amount of energy (0.71 J) and latency (38.5 ms). In general, advanced dominating set strategies can be highly effective in terms of the efficiency and performance in communication.

Table 2
Advanced Network Performance Evaluation

Metric	Traditional	CDS	Proposed AI-DS
Packet Delivery Ratio (%)	84.5	90.8	97.2
Network Lifetime (Rounds)	1250	1640	2315
Throughput (kbps)	410	528	684
Fault Recovery Rate (%)	71.3	82.6	94.1
Residual Energy Stability (%)	63.8	78.5	91.7
Routing Overhead (%)	34.6	25.2	14.3
Node Failure Tolerance (%)	68.5	81.9	93.4

Advanced performance evaluation results demonstrate, in Table 2, that the proposed framework of dominating set with the support of AI achieves significant performance gain for all the network metrics as compared to the traditional clustering and connected dominating set approaches. The proposed algorithm achieves the maximum PDR and throughput and remarkably reduces the routing overhead and stability of the residual energy. Moreover, with higher recovery rate of fault and higher tolerance to node failure, the adaptive optimization strategy is robust in dynamically changing wireless sensor network. The effectiveness of the proposed framework in selecting the appropriate nodes that consume minimal energy and efficient in managing communication can be seen from the improved network lifetime, making it very suitable for applications with a large number of nodes like IoT and environmental monitoring.

6. Conclusion

This paper introduces an effective architecture to deploy dominating sets in hierarchical sensor networks to overcome key issues of scalability, energy efficiency, and reliability. Combining multi-level dominating set construction with cluster-based node selection, the proposed method can greatly enhance the coverage of a network and reduce unnecessary communication and energy waste. Hierarchical structure is used to provide effective data aggregation and transmission via cluster heads and gate ways and, therefore, reduces latency and increases overall system performance. Also, the availability of fault tolerance allows the networks to be more resilient to the failure of nodes, which

guarantees continued functionality in dynamic environments. The comparison shows that the suggested approach is superior in the measurement of the main performance indicators compared to the traditional methods. Future directions can further investigate adaptive algorithms with machine learning methods and real-time optimization in order to improve the performance of large scale, heterogeneous sensor network deployment.

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