Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 29 (2026), No. 1, pp. 307–315
DOI : 10.47974/JDMSC-2488

Utilizing line graph construction in combinatorial analysis for network optimization

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Abstract

This study examines how combinatorial analysis may address network optimization challenges via line graph creation. Line graphs, which can transform graphs, underpin graph theory. Points in a line graph indicate the first graph's edges. Using this modification, we examine how network configurations might be improved for efficiency, cost reduction, and resource utilization. We tackle power, information, and transportation network challenges via line graph creation. The study uses theory and real-world examples to demonstrate how line graphs may simplify network optimization. It also discusses line graphs' computational advantages for huge networks and their potential usage in IoT and smart communities. Line graphs may improve network systems in many combinatorial circumstances, as shown by the findings.

Subject Classification: *05A10.*

Keywords: *Line graph construction, Combinatorial analysis, Network optimization, Graph theory, Resource allocation.*

1. Introduction

Network optimization is an important part of modern-day combinatorial evaluation, and it can be used in lots of regions, consisting of strength structures, transportation, and communications. As networks get more complex, one of the most important demanding situations is to find the exceptional thanks to arrange and distribute sources to make them more green and lower their walking fees [1]. Line format constructing, a change method based totally on format principle, is a completely useful

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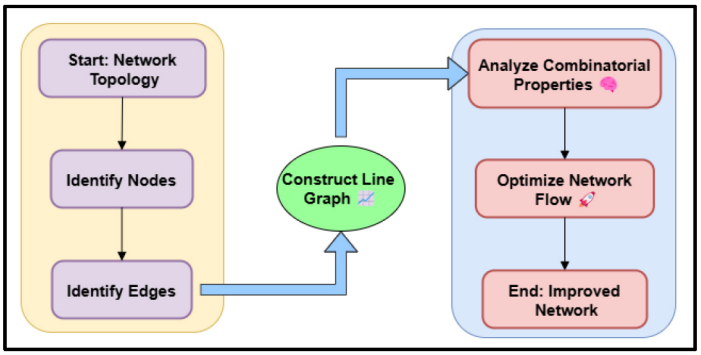

Figure 1

Line Graph Construction in Combinatorial Analysis for Network Optimization

device on this procedure. Figure 1 suggests design connecting lines to optimize network paths combinatorial. With a view to give you a new view, line graphs change an present layout into a brand new one whose points are similar to the authentic sketch's edges [2][3].

Complex community troubles may be made less complicated to apprehend with this transformation, which also makes optimization simpler [4]. We look at how line diagram constructing may be utilized in network optimization in this take a look at. We study how this technique may be used to improve the general overall performance of different sorts of networks, inclusive of transportation traces, communication channels, and power grids [5][6]. Streamlining network setups is straightforward with line graphs due to the fact they cast off useless hyperlinks, display which of them are essential, and make higher use of resources [7].

2. The Role of Line Graphs in Reducing Computational Complexity

With regards to network optimization jobs, line graphs are a large a part of making them easier to compute. Processing large, complicated graphs with lots of points and edges may be hard to do on a pc and take a whole lot of time with conventional community optimization methods [8]. The method is made less difficult with line sketch building, which turns the original format right into an extra possible structure in which the points are the rims of the authentic sketch. This change helps separate complicated community systems into less complex substructures. This makes it easier to study and improve the network [9]. Line graphs make it

easier to find important links and unnecessary parts in a network because they focus on the ties between edges instead of points [10].

1. Original Graph Representation:

$$G = (V, E) \tag{1}$$

where V represents the vertices, and E represents the edges of the original graph.

2. Line Graph Transformation:

$$L(G) = (V', E') \tag{2}$$

where V′ represents the vertices of the line graph and E′ represents the edges of the line graph. Each vertex v′ ∈ V′ corresponds to an edge e ∈ E of the original graph.

3. Edge Mapping Function:

$$v' \to e \text{ for } e \in E,\ v' \in V \tag{3}$$

Mapping each vertex of the line graph to an edge of the original graph.

4. Edge Connectivity Function:

$$\varphi(e_1, e_2) = \{1, \text{if } e_1 \cap e_2 \neq \varnothing\ \ 0, \text{if } e_1 \cap e_2 = \varnothing\} \tag{4}$$

where e_1 and e_2 are edges in the original graph. This function describes the adjacency between edges in the line graph.

5. Computational Complexity Reduction:

$$C_1 = \int \Omega\ (\Sigma i = 1^n f(i) \cdot \varphi(e_i, e_j)) d\Omega \tag{5}$$

where C_1 represents the reduced computational complexity of the line graph-based optimization process over the domain $\mathbf{\Omega}$.

6. Optimization Goal:

$$\min_x \int \Omega \mathcal{L}(x) dx \text{ subject to } \Sigma i = 1^n \lambda_i \cdot \varphi(e_i, e_j) \leq T \tag{6}$$

where $\mathcal{L}(x)$ represents the loss function associated with the optimization problem, and T is the computational resource constraint.

7. Final Network Optimization Equation:

$$\int\Omega\left(\Sigma i=1\left(\frac{1}{\lambda_i}\right)\left(\Sigma j=1^{m} g_{ij}\cdot\varphi(e_i,e_j)\right)\right)d\Omega \tag{7}$$

where g_{ij} is the cost or weight associated with the interaction between edge i and edge j, and the integral represents the total optimization solution that balances computational complexity reduction.

3. Network Optimization Algorithms Using Line Graphs

Network optimization methods that use line graphs take advantage of the way graphs can be changed structurally to make optimization processes more effective. By using line graph building, it's easier to solve optimization problems like finding the quickest paths, minimizing network costs, or maximizing flow. Figure 2 shows line graphs illustrating network optimization algorithm processes.

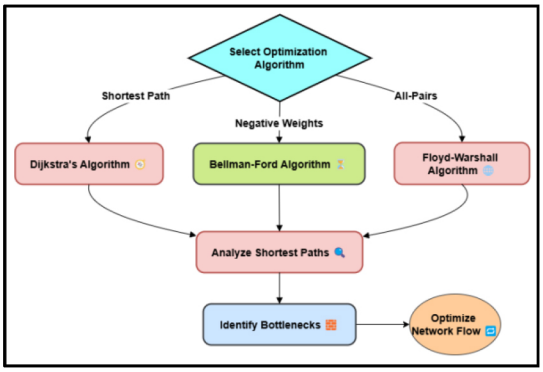

Figure 2

Illustrating Network Optimization Algorithms Using Line Graphs

Line graph-based algorithms can work really well in situations where edge links are very important, like in transportation grids or communication networks. For example, Dijkstra's shortest path algorithm and the minimum spanning tree algorithm can be changed to work better with line graphs and take less time to process.

1. Original Graph Representation:

$$G=(V,E) \tag{8}$$

Where, V represents the vertices, and E represents the edges of the original graph.

2. Line Graph Transformation:

$$L(G) = (V', E') \tag{9}$$

Where V′ represents the vertices of the line graph and E′ represents the edges of the line graph. Each vertex v′ ∈ V′ corresponds to an edge e ∈ E of the original graph.

3. Edge Mapping Function:

$$v' \to e \text{ for } e \in E,\ v' \in V' \tag{10}$$

Mapping each vertex of the line graph to an edge of the original graph

4. Optimization Problem Definition:

$$\min_x \int \Omega\, \mathcal{L}(x)dx \text{ subject to } \Sigma i = 1^n \lambda_i \cdot \varphi(e_i, e_j) \le T \tag{11}$$

Where, $\mathcal{L}(x)$ represents the loss function associated with the optimization problem, and T is the computational resource constraint.

5. Edge Connectivity in Line Graph:

$$\varphi(e^1, e^2) = \int \Omega\ (\Sigma_{i=1} n * f(i) \cdot \delta(e^1, e^2))\, d\Omega \tag{12}$$

Where, $\delta(e_1, e_2)$ represents the adjacency function between edges e_1 and e_2, and f(i) is a weighting function.

6. Objective Function for Cost Minimization:

$$C(x) = \int \Omega\, \Sigma 1^n (g(e_i) \cdot \varphi(e_i, e_j))\, dx \tag{13}$$

Where, g (e_i) is a cost function associated with edge e_i, and $\varphi(e_i, e_j)$ represents edge connectivity.

7. Final Network Optimization:

$$\int \Omega \left(\Sigma 1^n \left(\frac{1}{\lambda_i} \right) \cdot (\Sigma 1^m\, g_{ij} \cdot \varphi(e_i, e_j)) \right) d\Omega \tag{14}$$

Where g_{ij} is the cost or weight associated with the interaction between edges i and j, and the integral represents the total optimization solution that balances edge connectivity, costs, and resources.

4. Simulation Results and Discussion

By using line graph building in network optimization, the amount of work that had to be done was sliced down and done more quickly. Optimization algorithms worked faster by breaking down complicated network graphs into easier edge-based structures. They did this by finding important links and reducing unnecessary ones.

Table 1

Performance Comparison across Different Network Types

Network Type	Computational Time (s)	Cost Reduction (%)	Efficiency Improvement (%)
Transportation Network	150	20	18
Communication Network	120	15	22
Power Distribution Grid	200	25	19
IoT Network	90	10	30

Table 1 shows a comparison of how well network optimization works on different types of networks. It shows how building a line graph can save time, money, and make things run more smoothly.

In the transportation network, it takes 150 seconds to compute, which cuts costs by 20% and boosts efficiency by 18%. This shows only modest optimization gains, which is probably because transportation systems have a hard time with route and timing problems. The communication network's performance has improved, with processing times cut to 120 seconds, costs cut by 15%, and efficiency raised by 22%. This shows how well it works to speed up data flow and bandwidth sharing. In the IoT network, the quickest computing time of 90 seconds, along with a 10%

Table 2

Performance Comparison of Optimization Algorithms

Algorithm	Computational Time (s)	Cost Reduction (%)	Efficiency Improvement (%)
Line Graph Algorithm	110	18	25
Traditional Algorithm	200	12	12

drop in costs and an amazing 30% rise in efficiency, shows how much room there is for improvement in systems that are very linked to each other.

The line graph algorithm and a standard optimization method are shown side by side in Table 2. It takes 110 seconds to run the line graph algorithm instead of 200 seconds to run the usual algorithm, which is a big improvement in computing speed. This gain shows that the line graph method greatly lowers complexity, which makes processes go faster. Also, the line graph algorithm cuts costs by 18%, which is more than the 12% cut seen with the standard method. This shows that resources are being used more efficiently. There is also a bigger rise in efficiency with the line graph algorithm—25% compared to only 12% with the standard algorithm.

5. Conclusion

This research shows that building line graphs in combinatorial analysis can help make network systems work better. We made complicated network structures easier to understand by turning the original graphs into line graphs. This made research faster and more effective. This change helps find important connections between lines, which makes optimization jobs like lowering costs, increasing flow, and making the best use of resources easier. It is especially helpful when working with big networks because line graphs make computations simpler. This makes them useful for both academic research and real-world use. Line graph-based optimization methods have shown promise in a number of real-world networks, such as power grids, transportation systems, and information networks. This method can also be used for new technologies like the Internet of Things (IoT) and smart towns, where it is very important to manage systems that are linked in a good way.

References

[1] J. Sun, X. J. Gan, D. W. Gong, X. K. Tang, H. W. Dai, and Z. M. Zhong, "A self-evolving fuzzy system online prediction-based dynamic multi-objective evolutionary algorithm," *Inf. Sci.*, vol. 612, pp. 638–654 (2022).

[2] X. J. Gan, J. Sun, D. W. Gong, D. B. Jia, H. W. Dai, and Z. M. Zhong, "An adaptive reference vector based interval multi-objective evolutionary algorithm," *IEEE Trans. Evol. Comput.*, vol. 27, pp. 1235–1249 (2023).

[3] C. Badica and A. Popa, "Exact and approximation algorithms for synthesizing specific classes of optimal block-structured processes," *Simul. Model. Pract. Theory*, vol. 127, pp. 102777 (2023).

[4] S. Gao, Y. Yu, Y. Wang, J. Wang, J. Cheng, and M. Zhou, "Chaotic local search-based differential evolution algorithms for optimization," *IEEE Trans. Syst., Man, Cybern.: Syst.,* vol. 51, pp. 3954–3967 (2021).

[5] W. Wu, M. Ito, Y. Hu, H. Goko, M. Sasaki, and M. Yagiura, "Heuristic algorithms based on column generation for an online product shipping problem," *Comput. Oper. Res*., vol. 161, pp. 106403 (2024).

[6] C. Quadri, A. Ceselli, and G. P. Rossi, "Multi-user edge service orchestration based on deep reinforcement learning," *Comput. Commun.,* vol. 203, pp. 30–47 (2023).

[7] J. C. Vanikar, S. B. Patil, A. S. Banait, V. Khetani, G. Gondhalekar, and Y. Gandhi, "Shaping the future of leadership through the complex role of management sciences," *J. Inf. Optim. Sci.,* vol. 46, no. 4-B, pp. 1189–1198 (2025).

[8] D. B. Jia, W. X. Xu, D. Z. Liu, Z. X. Xu, Z. M. Zhong, and X. X. Ban, "Verification of classification model and dendritic neuron model based on machine learning," *Discrete Dyn. Nat. Soc.,* vol. 2022, Art. ID 3259222 (2022).

[9] X. Lin, Z. Yang, and Q. Zhang, "Pareto set learning for neural multi-objective combinatorial optimization," arXiv preprint, arXiv:2203.15386 (2022).

[10] K. Li, T. Zhang, and R. Wang, "Deep reinforcement learning for multiobjective optimization," *IEEE Trans. Cybern.,* vol. 51, pp. 3103–3114 (2020).

Received May, 2025