

Implementation of wireless sensor network using Laplacian matrices and discrete structures to enhance the data transmission

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Abstract

In the current era, the rapid growth of digital connected devices makes it essential to ensure that wireless sensor networks provide reliable, energy-efficient, and scalable communication. This research work shows the combination of graph theory and discrete structures to enhance the data transmission in wireless sensor networks. In this work spectral clustering and centrality measures are used to model network topologies as graph. To identify the optimal routing paths, we created the network using Laplacian matrices. Discrete methods such as Fuzzy C clustering and eigenvector analysis are employed to form energy-aware clusters and select robust cluster heads. During the analysis of energy efficiency, our proposed algorithm K-GLARE was found to be 3.22% more efficient than LEACH and 2.38% better than EECDA. The analysis of throughput has shown prominent results, as K-GLARE achieved a 133% improvement over EECDA and was six times better than LEACH, which is a significant improvement to ensure better data delivery and communication reliability. During the network longevity analysis, K-GLARE maintained the highest proportion of live

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nodes at 38.61%, compared to 31.98% for EECDA and 29.41% for LEACH. Making K-GLARE a promising solution for next-generation wireless communication systems.

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1. Introduction

In the era of digital connected devices, the wireless sensor network plays the crucial roles to connect the device in various application domains like healthcare, smart farming and smart cities etc. As the number of devices is growing it is now important to provide efficient and reliable communication between the devices. Traditional routing and clustering techniques often struggles to provide reliable and efficient communication in wireless sensor networks.

To address the problem of efficient data transmission and energy aware routing in wireless sensor networks, the author presents the proposed solution by applying discrete structure and graph theory. This research presents the implementation of a graph theory-based clustering and routing protocol for wireless sensor networks to improve energy efficiency and network lifespan. The use of spectral clustering and centrality-based cluster head selection ensures balanced energy consumption and secure data transfer. In the present study, the wireless sensor network is represented by a graph having a set of vertices and edges. The Laplacian matrix is used to analyze the structural property of graph. This matrix is formed using the adjacency matrix A and the degree matrix D . Here, spectral clustering is used to form the clusters, which are based on eigenvectors and the Laplacian matrix followed by the Fuzzy C probabilistic clustering algorithm is used to allocate each node to its respective cluster. For the cluster head selection, the composite score of each node has been computed based on residual energy and network centrality. This multifactor approach outperforms other traditional algorithms and enhances scalability and data transmission.

2. Related Work

Energy efficiency and the optimization of routing are two dominant issues with the field of Wireless Sensor Networks (WSNs), especially over dense networks. In recent work there is an insistence on the efficacy of spectral graph theory and discrete mathematical structures in clustering and topology control. Spectral clustering Laplacian-based Methods provides a way to exploit network structure to take effective routes and achieve energy-efficient grouping of nodes. The study by Raza and Munir [1] showed the predictive nature of Laplacian spectra in the behavior of networks and when it comes to connectivity and stability.

Paper [2] introduced Clustering using Eigen Values with fuzzy logic to optimize cluster head selection- something which is quite like the K-GLARE model. In the same manner, Muniraju et al. [3] suggested a distributed spectral clustering algorithm that increased parallelism without central control. Manoharan [4] highlighted the emphasis in graph-based topology strategy, and Han, Lin and Fu [5] on localization strategy. Li et al. [6] used Laplacian based genetic optimization in smart buildings to achieve energy efficient communication.

Moreover, paper [7] and [8] confirmed the use of Laplacian and combinatorial topology as a source of cluster simplification and coverage analysis; thus, the methodology and purpose of K-GLARE was directly justified.

3. Proposed Work:

This section presents the proposed work for efficient clustering and cluster head selection in wireless sensor network.

3.1 Graph-Based Network Modelling

Equation 1 represents Graph-Based Network Modeling

$$G = (V, E) \quad (1)$$

Equation 2 represents the condition for whether node is connected or not.

$$dist(v_i, v_j) \leq R \quad (2)$$

Where $dist(v_i, v_j)$ represent Euclidean distance and R is the communication range.

3.2 Laplacian Matrix Construction

In this proposed work, the network structure is analyzed using the Laplacian matrix. This matrix is created using the adjacency matrix A , as represented by Equation 3, and the degree matrix D , as represented by Equation 4. The adjacency matrix is used to represent the connection between the nodes having condition when $A_{i,j} = 1$ then nodes v_i and v_j are connected otherwise 0. The degree matrix D represents the sum of the degree of node v_i . So, the Laplacian Matrix L is computed by $L = D - A$.

$$A_{i,j} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Degree Matrix D :

$$D_{i,i} = \sum_{j=1}^n A_{ij} \quad (4)$$

3.3 Spectral Clustering Using Eigenvectors

Here spectral clustering is used to form the cluster which uses eigenvectors of the Laplacian matrix L . To group the nodes into the cluster smallest K eigen vector is computed of Laplacian matrix L and generated by $U \in R^{n \times k}$ matrix shown by equation 5, where each node is represented by row.

$$U \in R^{n \times k} \quad (5)$$

Others row of U is normalized by unit length which is also called U_{norm} shown by the equation 6. Finally Fuzzy C clustering algorithm is applied on U_{norm} to assign each node to their respective cluster.

$$U_{norm}[i] = \frac{U[i]}{\|U[i]\|} \quad (6)$$

3.4 Energy-Aware Cluster Head Selection

For the selection of cluster head, each node v in cluster C is examined using the parameter $S(v)$ which is composite score balancing residual energy and network centrality shown in equation 7.

$$CC(v) = \frac{1}{\sum_{u \in C} d(u, v)} \quad (7)$$

Here $CC(v)$ represents the closeness centrality of the node v and $d(u, v)$ shows the shortest path distance in the cluster C .

Equation 8 shows the commutation of composite score function S_v where $E_{res}(v)$ represents Residual energy of the node v .

$$S(v) = \alpha \cdot \frac{CC(v)}{\max_{u \in C} CC(u)} + (1 - \alpha) \cdot \frac{E_{res}(v)}{\max_{u \in C} E_{res}(u)} \quad (8)$$

Where the range of α is between 0 to 1.

So, the cluster head of cluster C is represented by equation 9 as:

$$CH_c = \arg \max_{v \in C} S(v) \quad (9)$$

Where a node which has the highest score $S(v)$ is selected as cluster head. Algorithm is also represented by Table 1.

Table 1
Algorithm for Cluster Head Selection

```

closeness = centrality (G, 'closeness')
norm_closeness = closeness / max(closeness)
norm_power = power / max_energy.
combined_score = 0.5 * norm_closeness + 0.5 * norm_power
A = zeros (1, num_cls)
for i = 1:num_cls
    idx = find (B == i)
    max_score = -inf
    max_node = -1
    for j = 1: length(idx)
        node = idx(j)
        if combined_score(node) > max_score
            max_score = combined_score(node)
            max_node = node
        end
    end
    A(i) = max_node
End

```

4. Result and Analysis

In this proposed work the author presents the hybrid simulation approach which integrates the Fuzzy C algorithms with graph Laplacian based spectral method. The experiment conducted in Net Sim 14.0 simulation tool to analyse the performance metrics of throughput, energy consumption and network life. In this work MATLAB code is developed for clustering and routing logic and further it is integrated with Nesim for visualization and validation purposes. To statically validate the results ANOVA test has been conducted. ANOVA test determines the variation in the mean significantly or not.

4.1 Analysis of Power Consumption:

The bar chart in figure 1 represents the average power consumption of three protocols LEACH, EECDA and our proposed algorithm K-GLARE. During the analysis the author found K-GLARE is 3.22% more energy-efficient than LEACH and 2.38% more efficient than EECDDA, making it the most power-saving protocol. This comparison shows the effectiveness of K-GLARE algorithm to minimize energy consumption.

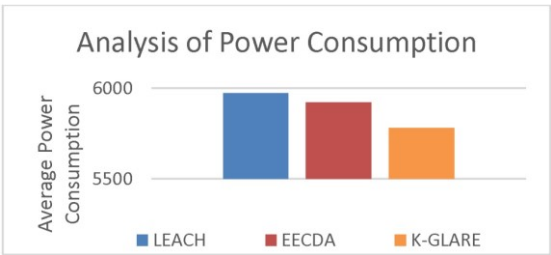


Figure 1
Analysis of Power Consumption

4.2 Analysis of Throughput:

In the wireless sensor network throughput is an important parameter which reflects the rate at which data is successfully transmitted over a communication channel. The higher throughput the more network efficiency and reliability.

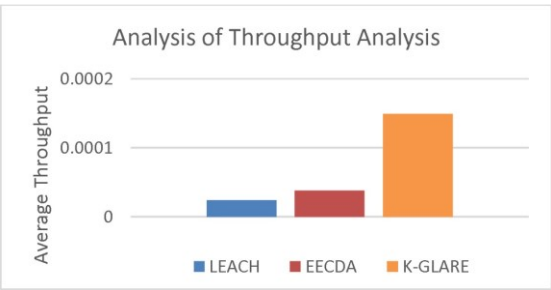


Figure 2
Analysis of Throughput Analysis

The bar chart in figure 2 represents the analysis of average throughput comparison between LEACH, EECDA, and K-GLARE protocols. Analysis shows that K-GLARE is 133% better the EECDA and 6 times better than LEACH. So, it is ensuring better network performance.

4.3 Analysis of Number of Alive nodes:

In a wireless sensor network, the number of live nodes indicates the energy efficiency and sustainability of routing protocols. In our analysis, we compared the number of live nodes among LEACH, EECDA, and K-GLARE protocols as shown in table 2.

Table 2
Analysis of Number of Alive nodes

Number of Rounds	LEACH	EECDA	K-GLARE
5000	61	62	64
5200	58	60	64
5300	50	54	63
5400	16	24	40
5500	10	12	25

The experiment was conducted over 5000 to 5500 rounds shown in figure 3. During the analysis, K-GLARE maintained the highest proportion of live nodes (38.61%) across rounds 5000 to 5500, outperforming EECDA (31.98%) and LEACH (29.41%), indicating superior node longevity and energy efficiency in later rounds.

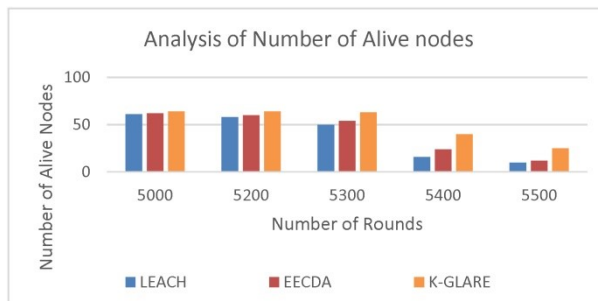


Figure 3
Analysis of Number of Alive Nodes

Conclusion

This proposed work has shown the efficacy of integrating graph theory with discrete structures into wireless sensor networks to improve routing efficiency and data transmission. The proposed algorithm designed using spectral clustering, Laplacian matrices, and centrality measures, has shown significant improvement over traditional algorithms like EECDA and LEACH. All the analysis has been done on NETSIM 14.0, where the proposed algorithm

outperformed its counterparts in all major dimensions, including energy efficiency, throughput, and node longevity. During the analysis of energy efficiency, our proposed algorithm K-GLARE was found to be 3.22% more efficient than LEACH and 2.38% better than EECDA. The analysis of throughput has shown prominent results because K-GLARE achieved a 133% improvement over EECDA and was six times better than LEACH. Furthermore, for the analysis of network longevity, 5000 to 5500 simulation rounds were created, and the analysis has shown that K-GLARE maintained the highest proportion of live nodes at 38.61%, compared to 31.98% for EECDA and 29.41% for LEACH. This ensures that K-GLARE not only conserves energy but also improves the network lifetime.

References

- [1] A. Raza and M. M. Munir, "Laplacian spectra and structural insights: applications in chemistry and network science," *Frontiers in Applied Mathematics and Statistics*, vol. 11, Art. no. 1519577 (Jun. 2025).
- [2] K. Thangaramya, R. Logambigai, L. Sai Ramesh, K. Kulothungan, A. Kannan, and S. Ganapathy, "An energy efficient clustering approach using spectral graph theory in wireless sensor networks," in Proc. Int. Conf. Recent Trends Comput. Commun. Math. (ICRTCCM), pp. 201–204 (2017).
- [3] G. Muniraju, S. Zhang, C. Tepedelenlioglu, M. K. Banavar, A. Spanias, C. Vargas-Rosales, and R. Villalpando-Hernandez, "Location based distributed spectral clustering for wireless sensor networks," in Proc. IEEE SENSORS Signal Process. Def. (SSPD), pp. 1–6 (2074).
- [4] J. S. Manoharan, "A novel load balancing aware graph theory based node deployment in wireless sensor networks," *Wireless Pers. Communication*, vol. 128, no. 3, pp. 1171–1192 (Sep. 2022).
- [5] Z. Han, Z. Lin, and M. Fu, "A survey on distributed network localization from a graph Laplacian perspective," *J. Syst. Sci. Complex.*, vol. 37, no. 1, pp. 273–293 (Jan. 2024).
- [6] W. Li, R. Tang, S. Wang, and Z. Zheng, "An optimal design method for communication topology of wireless sensor networks to imple-

ment fully distributed optimal control in IoT-enabled smart buildings," *Applied energy*, 349, 121539 (2023).

- [7] J. Mante-Khurpade, M. Dhotay, P. Patil, S. Kulkarni, S. Budhavale, and S. Ikhar, "Enhancing data privacy in wireless sensor networks using homomorphic encryption," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 27, no. 2-B, pp. 833–842 (2024), doi: 10.47974/JDMSC-1959.
- [8] O. A. Razzaq, "Enhancing IoT-based wireless sensor network security with cryptographic techniques," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 28, no. 4-A, pp. 1117–1130 (2025), doi: 10.47974/JDMSC-2102.

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