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Dominant clusters of increasing *n*-corona product graph

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Abstract

The streamlining of dominating clusters from the family of product graphs can be well organized using graph domination. A minimal dominating set D comprises the lowest possible integer of graph vertices, where alternative graph vertices are connected with at least one minimal dominating set. The Increasing Corona Product (ICP) of order one for K_n and family of prisms are modelled and their domination number $\gamma(G)$, dominating generation number $\gamma_{\mathcal{G}}$, dominating cluster number $\gamma_{\zeta}(G)$ and split domination number $\gamma_s(G)$ are obtained in this research work.

Subject Classification: *05C69, 05C76.*

Keywords: *Domination number, Dominance generation number, Dominant clusters, Split domination number, Corona product.*

1. Introduction

Graphs model real world situations by considering the interactions between sets of systems. For example, communications between people in social networks, connections between servers and clients in computer networks; interactions between proteins in protein networks, etc. In these situations, graph-based models have been proposed on deep learning of

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these problems. The primary goal of this work is to generate *n* times CP of the same network which has an ascending order of 1. Using the domination of graphs, it is possible to identify a dominance cluster in this structure. The clustering technique helps to simplify the processing of large datasets and has numerous applications in the fields of biological data analysis, image processing, pattern recognition and social networks, etc.

Domination of graphs were first discussed by Berge [2]. In graph theory domination of graphs is the huge research area. Any vertex that is not in $V-D$ is linked to a minimum of one vertex in subset *D*, which is a DS [4] for a graph *G*. The number of vertices in this MDS is the graph's Kulli and Bidarahall introduced the concept of split domination in graphs in 1997, defining both the split dominating set and its split domination number [10]. They came up with a variety of results for the split domination number of several standard graphs and discovered the relationship between split domination number and other variables, including domination number, connected domination number and vertex coverage number. It is applied in the real time situation where a mass connected network need to be disconnected. Different kind of split domination were discussed by researchers. The split domination number in edge-middle graph was discussed by Goudar, Prasad, and Venkanagouda [20]. The authors Mohamed and Sabitha [9] examined the accurate split domination number of fuzzy graph in the year 2021. The researchers Mullai et al. obtained the split domination number of neutrosophic graphs [11]. In 2022, Raja and Praisy discussed the split domination for decomposition of path graphs [14]. In 1970, Frucht and Harry [6] investigated about the corona product of two graphs. Domination of various type are investigated to many kind of corona product graphs in recent times. The certified domination number of corona product [5] were analyzed by Durai Raj et al. in 2021. The author Alameri discussed F-coindex of some CP of two graphs in [1].The author Wardani et al. [21] elucidated the dominating sets of generalized corona product of path and cycles. Corona products of biochemical and phylogenetic networks-I discussed in [17]. In 2022, Iyer and Sundaram obtained the irregular χ for specific types of CP of graphs and Sierinski graphs [13].

2. Preliminaries

Definition 2.1: The domination *D* [8] is a subset of *V*, the node set of *G*, $\ni$ each $v \in D$ is linked with nodes in $\langle V-D \rangle$, which is also known as dominating set and abbreviated as DS.

Definition 2.2: The DS is considered as a split dominating set (SDS) [16] of G, if $\langle V-D\rangle$ becomes disconnected. SDN (γ_s) of G is the minimal cardinal number of the SDS. [10]

Definition 2.3: A **prism graph** CL_n, [7] and an **antiprism graph** Q_n [12] corresponding to the skeleton of prism and an antiprism belong to a family of generalized Petersen graphs.

Definition 2.4: An **n-crossed prism graph** [15] R_n is a graph containing couple of independent cycles. Additional edges are added between corresponding vertices on the two cycles, forming intersecting cross connections, resulting in the crossed prism structure.

Definition 2.5: Permutation: The notation nP_r represents the count of distinct arrangements that can be formed by selecting r items from a total of n unique items. The value is calculated using the formula: nP_r is $n!/(n-r)!$ [3].

Definition 2.6: Graphs G and H can be combined to form a corona product[18], it involves taking one graph G as the center and making copies $|V(G)|$ of the other graph H around it. In the emerging graph, each i^{th} vertex in the center graph is connected to all the vertices in its corresponding i^{th} copy of the other graph.

Definition 2.7: The **ICP** of order 1 involves placing G_1 in the center and surrounding it with $|V(G_1)|$ copies of G_2. Each vertex in G_1 is connected to all the vertices in its corresponding vertices of G_2, repeated for subsequent generations up to G_{n-1} generations [19].

Definition 2.8: If all the vertices in any generation of increasing n-corona product of graph are dominating vertices then such generation is known as dominance generation. The number of dominance generation is known as **dominating generation number** and denoted by $\gamma_{\mathcal{G}}$. Each graph present in the dominating generations are defined to be dominating clusters, whose cardinality is said to be **dominating cluster number** γ_{ζ}.

3. Computation of Domination Numbers

The process of proofs begins with finding dominant generations. The dominating clusters are observed from these dominating generations. Hence the dominating set is made up of the number of vertices present in the dominant clusters. In this section the domination numbers (DN) and split domination numbers (SDN) are derived for the increasing n-corona product (ICP) of complete, prism graph families such as Prism, Antiprsm and Crossed prism graphs.

Theorem 3.1: *The DN for the increasing n-corona product of complete graph K_n is given by*

$$\gamma(K_{r-1}\circ_{+1}^{n}K_r)=\begin{cases}2(\sum_j\prod_{i=3}^{3j+2}i),\ j=0,1,2,\dots,\frac{n-3}{3} & \text{for } n\equiv 0 \bmod 3\\ 2(\sum_j\prod_{i=2}^{3j}i)+1,\ j=1,2,\dots,\frac{n-1}{3} & \text{for } n\equiv 1 \bmod 3\\ 2(\sum_j\prod_{i=3}^{3j+1}i)+1,\ j=1,2,\dots,\frac{n-2}{3} & \text{for } n\equiv 2 \bmod 3.\end{cases}$$

Proof: The notation and the cardinality of the vertex set, edge set of complete graph is refered from [19]

$V=\{v_1^{(1)}\}\cup\{v_k^{(l)}/l=2,3,\dots,n$ and $k=1,2,\dots,2(l!)-1,2(l!)\}$ is the representation of graphs. Let $\mathcal{G}=\{1,2,\dots,n\}$ indicates the generations of graph.

Case 1: When $n\equiv 0\ mod\ 3$ the dominating generations are $\mathcal{G}=\{3d+2 : d=0,1,2,\dots,\frac{n-3}{3}\}$ and $\gamma_{\mathcal{G}}=|\mathcal{G}|=\frac{n}{3}$. Thus $\gamma_{\zeta}(K_n)=\sum_{r=2}^{3d+2}2(r-1)!$, $d=0,1,\dots,\frac{n-3}{3}$. The vertices in the dominating generation are the same as that of dominating set D. Hence D is the MDS.

$$\begin{aligned}D &= \{v_k^{(l)}: l=2,5,8,\dots,n-4,n-1,k=1,2,\dots,2(l!)-1,2(l!)\}.\\ |D| &= (n-1)(n-2)\dots3.2.2+(n-4)(n-7)\dots3.2.2+2.2\\ |D| &= 2[(n-1)!+(n-4)!+\dots+5!+2!]=2\sum_j\prod_{i=2}^{3j+2}i,\ j=0,1,2,\dots,\frac{n-3}{3}\\ &= \gamma(K_{r-1}\circ_{+1}^{n}K_r).\end{aligned}$$

Case 2: When $n\equiv 1\ mod\ 3$ we have the dominating generations as $\mathcal{G}=\{3d : d=1,2,\dots,\frac{n-1}{3}\}$. Hence $\gamma_{\mathcal{G}}=|\mathcal{G}|=\frac{n-1}{3}$. Thus $\gamma_{\zeta}=\sum_{r=3}^{3d}2(r-1)!$, $d=1,2,\dots,\frac{n-1}{3}$.

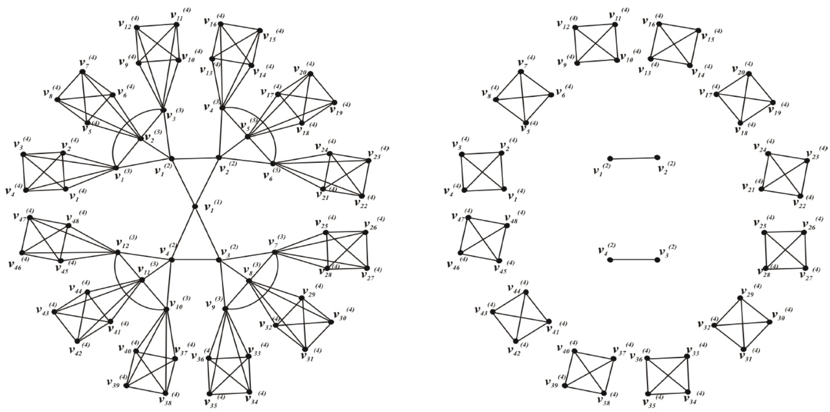
In this case $\mathscr{G}-\mathcal{G}=\{3d-1\}\cup\{3d+1\}$. Since the vertex set corresponding to the dominating generation is not adjacent with the origin vertex, the dominating set is given as

$$\begin{aligned} D &= \{v_1^{(1)}\} \cup \{v_k^{(l)} : l = 3, 6, \ldots, n-4, n-1 \text{ and } k = 1, 2, \ldots, 2(l!)-1, 2(l!)\}. \\ |D| &= (n-1)(n-2)\ldots 3.2.2 + (n-4)(n-7)\ldots 3.2.2 + \ldots + 3.2.2 + 1 \\ |D| &= 2[(n-1)! + (n-4)! + \ldots + 6! + 3!] + 1 \\ |D| &= 2\left(\sum_j \prod_{i=2}^{3j} i\right) + 1, j = 1, 2, \ldots, \frac{n-1}{3} = \gamma(K_{r-1} \circ_{+1}^{n} K_r). \end{aligned}$$

Case 3: When $n \equiv 2 \, mod \, 3$ the dominating generations are $\mathcal{G} = \{3d+1 : d = 0, 1, 2, \ldots, \frac{n+1}{3}\}$ and hence $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n-2}{3}$. Thus $\gamma_{\zeta} = \sum_{r=3d+1} 2(r-1)!$.

In this case $\mathscr{G} - \mathcal{G} = \{3d-1\} \cup \{3d+1\}$. The dominating set is given as

$$\begin{aligned} D &= \{v_1^{(1)}\} \cup \{v_k^{(l)} : l = 4, \ldots, n-4, n-1, k = 1, 2, \ldots, 2(l!)-1, 2(l!)\}. \\ |D| &= (n-1)(n-2)\ldots 3.2.2 + (n-4)(n-7)\ldots 3.2.2 + \ldots + 4.3.2.2 + 1 \\ |D| &= 2[(n-1)! + (n-4)! + \ldots + 6! + 3!] + 1 \\ |D| &= 2\left(\sum_j \prod_{i=2}^{3j+1} i\right) + 1, j = 1, 2, \ldots, \frac{n-2}{3} \\ &= \gamma(K_{r-1} \circ_{+1}^{n} K_r). \end{aligned}$$



(a) (b)

Figure 1

Increasing 4-Corona product graph

Corollary 3.1.1: *The SDN of increasing corona product of complete graph is* $\gamma_s(K_{r-1} \circ_{+1}^{n} K_r) = \gamma(K_{r-1} \circ_{+1}^{n} K_r)$. *See Figure 1.*

Theorem 3.2: *The DN of increasing n-corona product of prism graph* $CL_{r-1} \circ_{+1}^{n} CL_r$, $n \geq 4$ *and* $4 \leq r \leq n-2$ *is given by*

$$\gamma(CL_{r-1} \circ_{+1}^{n} CL_r) = \begin{cases} \sum_{\kappa} 2^{\kappa-2} (\prod_{i=3}^{\kappa} i) + \gamma(CL_3), & \kappa = 3j+2 \text{ and } \\ & j = 1, 2, \ldots, \frac{n-3}{3}, \text{ for } n \equiv 0 \bmod 3 \\ \sum_{\kappa} 2^{\kappa-2} (\prod_{i=3}^{\kappa} i), & \kappa = 3j \text{ and } j = 1, 2, \ldots, \frac{n-1}{3}, \\ & \text{for } n \equiv 1 \bmod 3 \\ \sum_{\kappa} 2^{\kappa-2} (\prod_{i=3}^{\kappa} i), & \kappa = 3j+1 \text{ and } j = 1, 2, \ldots, \frac{n-2}{3}, \\ & \text{for } n \equiv 2 \bmod 3. \end{cases}$$

Proof: The increasing n-corona product of prism graph $CL_n = (V, E)$ in which vertex set $|V| = \sum_j (\prod_{i=3}^{j} (2i))$, $j = 3, 4, \ldots, n$. The vertices are denoted as

$V = \{v_{\kappa}^{(l)} : l = 1, 2, \ldots, n-2$ and $\kappa = 1, 2, \ldots, l!-1, l!\}$. There exists $\mathscr{G}$ generation of vertices in which $\{l = 1, 2, \ldots, n-2\}$ indicates the corresponding generations. The total number of edges present are $|E| = 9$ $+ \sum_{r=2}^{n-2} 2^{r-1} (\prod_{i=3}^{r+1} i)$ $[5(r+2)]$.

Case 1: When $n \equiv 0 \; mod \; 3$ the dominating generation is obtained from $\mathcal{G} = \{3d : d = 1, 2, \ldots, \frac{n-3}{3}\}$. $\mathscr{G} - \mathcal{G}$ = $\{3d+1, 3d-1\} d$ dominated by $\mathcal{G}$. Thus $\gamma_{\mathcal{G}} = |\mathcal{G}|$ = $\frac{n-3}{3}$. Hence $\gamma_{\zeta} = \sum_{r=3}^{3d} 2^{(r-3)}$ $(\prod_{i=3}^{r-1} i)$, $d = 1, 2, \ldots, \frac{n-3}{3}$. The vertex set related to the dominating generation is failed to dominate the vertex from origin graph CL_3. Hence the MDS is

$$\begin{aligned} D &= \{v_{\kappa}^{(l)} : l = 3, 6, \ldots, n-6, n-3 \text{ and } \kappa = 1, 2, \ldots, l!-1, l!\} \cup \\ &\quad \{v_{n+3}^{(1)}, \ldots, v_{m}^{(1)}\}, i = 4q+1, m = 4q+3 \text{ and } q = 1, 2, \ldots, \frac{n-4}{4}. \\ |D| &= 2^{n-3}(n-1P_{n-3}) + 2^{n-6}(n-4P_{n-6}) \ldots + 2^{3}(5P_3) + \gamma(CL_3) \\ |D| &= \sum_{\kappa} 2^{\kappa-2} \left(\prod_{i=3}^{\kappa} \right) + \gamma(CL_3), \kappa = 3j+2, j = 1, 2, \ldots, \frac{n-3}{3} \\ &= \gamma(CL_{r-1} \circ_{+1}^{n} CL_r). \end{aligned}$$

Case 2: When $n \equiv 1 \; mod \; 3$ we can get the dominating generation as follows $\mathcal{G} = \{3d + 1 : d = 0, 1, 2, \ldots, \frac{n-4}{3}\}$ each generation belonging to $\mathscr{G} - \mathcal{G}$ = $\{3d\} \cup \{3d+2\}$ is adjacent with members of $\mathcal{G}$. Hence the dominating generation number is given as $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n-1}{3}$.

$$\gamma_\zeta = \sum_{r=1}^{3d+1} 2^{(r-3)}\left(\prod_{i=3}^{r-1} i\right), d = 0,1,2,\ldots,\frac{n-4}{3}$$
$$D = \{v_\kappa^{(l)} : l = 1,4,\ldots,n-3 \text{ and } \kappa = 1,2,\ldots,l!-1,l!\}.$$

The dominating set associated with dominating generation gives the minimal dominating set

$$|D| = (2n-2)(2n-4)\ldots 6 + (2n-8)(2n-10)\ldots 6 + \ldots + 12.10.8.6 + 6$$
$$|D| = 2^{n-3}(n-1P_{n-3}) + 2^{n-6}(n-4P_{n-6})\ldots + 2^7(9P_7) + 2^3(6P_4) + 2(3P_1)$$
$$|D| = \sum_i 2^{3i-2}(3iP_{(3i-2))}, i = 1,2,\ldots,\frac{n-2}{3}$$
$$|D| = \sum_\kappa 2^{\kappa-2}\left(\prod_{i=3}^{\kappa} i\right), \kappa = 3j \text{ and } j = 1,2,\ldots,\frac{n-2}{3} = \gamma(CL_{r-1} \circ_{+1}^{n} CL_r).$$

Case 3: When $n \equiv 2 \, mod \, 3$ the required dominating generations are $\mathcal{G} = \{3d + 2 / d = 0,1,2, \ldots, \frac{n-5}{3}\}$. The adjacent generations $\mathscr{G} - \mathcal{G} = \{3d\} \cup \{3d+1\}$ are dominated by $\mathcal{G}$. The dominating generation number is $\gamma_{\mathcal{G}} = \frac{n-2}{3}$.

$$\gamma_\zeta = \sum_{r=2}^{3d+2} 2^{(r-3)}\left(\prod_{i=3}^{r-1} i\right), d = 0,1,2,\ldots,\frac{n-5}{3}$$
$$D = \{v_\kappa^{(l)} : l = 2,5,\ldots,n-6,n-3 \text{ and } \kappa = 1,2,\ldots,l!-1,l!\}.$$
$$|D| = (2n-2)(2n-4)\ldots 6 + (2n-8)(2n-10)\ldots 6 + \ldots 14.12.10.8.6 + 8.6$$
$$|D| = 2^{n-3}(n-1P_{n-3}) + 2^{n-6}(n-4P_{n-6})\ldots + 2^5(7P_5) + 2^2(4P_2)$$
$$|D| = \sum_i 2^{3i-2}(3i+1P_{(3i-1)}), i = 1,2,\ldots,\frac{n-1}{3}$$
$$|D| = \sum_\kappa 2^{\kappa-2}\left(\prod_{i=3}^{\kappa} i\right), \kappa = 3j+1 \text{ and } j = 1,2,\ldots,\frac{n-1}{3} = \gamma(CL_{r-1} \circ_{+1}^{n} CL_r).$$

Corollary 3.2.1: *The SDN of increasing* $n-$ *corona product of prism graph is* $\gamma_s(CL_{r-1} \circ_{+1}^{n} CL_r) = \gamma(CL_{r-1} \circ_{+1}^{n} CL_r)$.

Theorem 3.3: *The DN of increasing n-corona product of antiprism graph* $Q_{r-1} \circ_{+1}^{n} Q_r$, $n \geq 5$ *and* $5 \leq r \leq n-2$ *is given by*

$$\gamma(Q_{r-1} \circ_{+1}^{n} Q_r) = \begin{cases} \sum_k 2^{k-2}(\prod_{i=4}^{k} i), & k=3j+2 \text{ and } j=1,2,\ldots,\frac{n-3}{3} \\ & \text{for } n \equiv 0 \bmod 3 \\ \sum_k 2^{k-2}(\prod_{i=4}^{k} i)+\gamma(Q_4), & k=3j \text{ and } j=1,2,\ldots,\frac{n-4}{3} \\ & \text{for } n \equiv 1 \bmod 3 \\ \sum_k 2^{k-2}(\prod_{i=4}^{k} i), & k=3j+1 \text{ and } j=1,2,\ldots,\frac{n-1}{3} \\ & \text{for } n \equiv 2 \bmod 3. \end{cases}$$

Proof: The increasing n-corona product of antiprism graph $Q_{r-1} \circ_{+1}^{n} Q_r = (V,E)$ in which vertex set $|V| = \sum_j(\prod_{i=4}^{j}(2i))$, $j=4,5,\ \ldots,\ n$. The vertices are denoted as $V=\{v_k^{(l)} : l=1,2,\ \ldots,\ n-3 \ \ and \ \ k=1,2,\ \ldots,\ l!-1, l!\}$. There exists $\mathcal{G}$ generation of vertices in which $\{l=1,2,\ \ldots,\ n-3\}$ indicates the generations. The number of edges $|E| = 16+\sum_r 2^{r-2}\left(\prod_{i=4}^{r+2} i\right)(r+3)$ where $r=2,3,\ \ldots,\ n-3$.

Case 1: When $n \equiv 0 \, mod \, 3$ the dominating generation is obtained as $\mathcal{G}=\{3d + 2 : d=0,\ 1,\ 2,\ \ldots,\ \frac{n-6}{3}\}$. $\mathscr{G}-\mathcal{G} = \{3d+1,\ 3d+3\}$ generations are dominated by $\mathcal{G}$. Thus the dominating generation number is $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n-3}{3}$. The dominating cluster number derived by means of dominating generations is

$$\begin{aligned} \gamma_\zeta &= \sum_{r=2}^{3d+2} 2^{(r-4)}\left(\prod_{i=4}^{r-1} i\right), d=0,1,2,\ldots,\frac{n-6}{3} \\ D &= \{v_k^{(l)} : l=5,8,\ldots,n-4,n-1 \text{ and } k=1,2,\ldots,l!-1,l!\}. \\ |D| &= (2n-2)(2n-4)\ldots 8+(2n-8)(2n-10)\ldots 8+\ldots+10.8 \\ |D| &= 2^{n-4}(n-1P_{n-4})+2^{n-7}(n-4P_{n-7})\ldots+2^5(8P_5)+2^2(5P_2) \\ |D| &= \sum_k 2^{k-2}\left(\prod_{i=4}^{k} i\right), k=3j+2 \text{ and } j=1,2,\ldots,\frac{n-3}{3} \\ &= \gamma(Q_{r-1} \circ_{+1}^{n} Q_r). \end{aligned}$$

Case 2: When $n \equiv 1 \, mod \, 3$ the dominating generation are given as $\mathcal{G}=\{3d : d=1,\ 2,\ \ldots,\ \frac{n-4}{3}\}$, each generation belonging to $\mathscr{G}-\mathcal{G} = \{3d-1\} \cup \{3d+1\}$ is adjacent with members of $\mathcal{G}$. $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n-4}{3}$.

$$\gamma_\zeta = \sum_{r=3}^{3d} 2^{(r-4)}(\prod_{i=4}^{r-1} i), d=1,2,\ldots,\frac{n-4}{3}.$$

Since the dominating set associated with dominating generation is failed to dominate the vertices of origin graph Q_4, the obtained minimal dominating set is

$$\begin{aligned} D &= \{v_1^{(1)}, v_3^{(1)}\} \cup \{v_k^{(l)} : l = 6, 9, \ldots, n-4, n-1 \text{ and } k = 1, 2, \ldots, l!-1, l!\}. \\ |D| &= (2n-2)(2n-4)\ldots 8 + (2n-8)(2n-10)\ldots 8 + \ldots + (12)(10)(8) + \gamma(Q_4) \\ |D| &= 2^{n-4}(n-1P_{n-4}) + 2^{n-7}(n-4P_{n-7})\ldots + 2^7(9P_7) + 2^3(6P_3) + \gamma(Q_4) \\ |D| &= \sum_i 2^{3i-2}(3iP_{(3i-2)}) + \gamma(Q_4),\ i = 1, 2, \ldots, \frac{n-4}{3} \\ |D| &= \sum_k 2^{k-2}\left(\prod_{i=4}^{k} i\right) + \gamma(Q_4),\ k = 3j \text{ and } j = 1, 2, \ldots, \frac{n-4}{3} = \gamma(Q_{r-1} \circ_{+1}^{n} Q_r). \end{aligned}$$

Case 3: When $n \equiv 2 \bmod 3$ the required dominating generation are $\mathcal{G} = \{3d+1 : d = 0,\ 1,\ 2,\ \ldots,\ \frac{n-5}{3}\}$. The adjacent generations $\mathscr{G} - \mathcal{G}$ = $\{3d \cup 3d + 2\}$ are dominated by $\mathcal{G}$. The dominating generation number is $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n-2}{3}$.

$$\begin{aligned} \gamma_{\zeta} &= \sum_{r=1}^{3d+1} 2^{(r-4)}\left(\prod_{i=d}^{r-1} i\right),\ d = 0, 1, \ldots, \frac{n-5}{3} \\ D &= \{v_k^{(l)} : l = 4, 7, \ldots, n-4, n-1 \text{ and } k = 1, 2, \ldots, l!-1, l!\}. \\ |D| &= (2n-2)(2n-4)\ldots 8 + \ldots (14)(12)(10)(8) + (10)(8) \\ |D| &= 2^{n-4}(n-1P_{n-4}) + 2^{n-7}(n-4P_{n-7}) + 2^4(7P_4) + 2^1(4P_1) \\ |D| &= \sum_i 2^{3i+1}(3i+1P_{(3i+4)}),\ i = 0, 1, 2, \ldots, \frac{n-5}{3} \\ |D| &= \sum_k 2^{k-2}\left(\prod_{i=4}^{k} i\right),\ k = 3j+1 \text{ and } j = 1, 2, \ldots, \frac{n-1}{3} \\ &= \gamma(Q_{r-1} \circ_{+1}^{n} Q_r). \end{aligned}$$

Corollary 3.3.1: *The SDN of increasing n-corona product of antiprism graph is* $\gamma_s(Q_{r-1} \circ_{+1}^{n} Q_r) = \gamma(Q_{r-1} \circ_{+1}^{n} Q_r)$.

Theorem 3.4: *For an increasing n-corona product of crossed prism graph* $R_{r-1} \circ_{+1}^{n} R_r$, *n is even*, $n \geq 6$ *and* $6 \leq r \leq n-2$, the domination number is

$$\gamma(R_{r-1} \circ_{+1}^{n} R_r) = \begin{cases} \sum_k 4^{k-1}(\prod_{i=1}^{k} i), & k = 3j+1 \text{ and } j = 1,2,\ldots,\frac{n}{6}, \\ & \text{for } n \equiv 0 \bmod 6 \\ \sum_k 4^{k-1}\left(\prod_{i=2}^{k} i\right), & k = 3j+2 \text{ and } j = 1,2,\ldots,\frac{n-2}{6} \\ & \text{for } n \equiv 2 \bmod 6 \\ \sum_k 4^{k-1}\left(\prod_{i=2}^{k} i\right) + \gamma(R_4), & k = 3j \text{ and} j = 1,2,\ldots,\frac{n-4}{6} \\ & \text{for } n \equiv 4 \bmod 6. \end{cases}$$

Proof: Let $R_{r-1} \circ_{+1}^{n} R_r$ be the n ICP of crossed prism graph having set of V nodes and E edges. $|V| = \sum_j(\prod_{i=2}^{j}(4i))$, $j = 2,3,\ \ldots,\ \frac{n}{2}$, the vertex set obtained by the ICP of graph is given by $V = \{v_k^{(l)} : 1 \le i \le \frac{n-2}{2}$ and $1 \le k \le l!\}$. For crossed prism graph we have $n = 2m$, $m = 1,2,\ldots$ There exists $\mathcal{G}$ generations of vertices in which $l = \{1,2,\ldots,\frac{n-2}{2}\}$ refers the generations. The total of edges is

$$|E| = 12 + \sum_{r=2}^{\frac{n-2}{2}} 4^{r-1}(r!)[10(r+1)].$$

Case 1: When $n \equiv 0 \, mod \, 6$ the dominating generations will be taken as $\mathcal{G} = \{3d+1 : d = 0,1,\ \ldots,\ \frac{n}{6}\}$. The complement $\mathscr{G} - \mathcal{G} = \{3d,\ 3d+2\}$ is adjacent with $\mathcal{G}$. Thus the dominating generation number is $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n}{6}$.

$$\gamma_\zeta = \sum_r 2^{(r-4)}\left(\frac{r-2}{2}\right)!,\ r = 3d+1 \text{ and } d = 0,1,\ldots,\frac{n}{6}.$$

$$D = \{v_k^{(l)} : l = 1,4,\ldots,\frac{n-4}{2}, k = 1,2,\ldots,l!-1,l!\}$$

$$|D| = (2n-4)(2n-8)\ldots8 + (2n-16)(2n-20)\ldots8 + \ldots + (20)(16)(12)(8) + 8$$

$$|D| = \sum_k 4^{k-1}\left(\prod_{i=1}^{k} i\right),\ k = 3j+1 \text{ and } j = 1,2,\ldots,\frac{n}{6} = \gamma(R_{r-1} \circ_{+1}^{n} R_r).$$

Case 2: $n \equiv 2 \, mod \, 6$ the dominating generation are $\mathcal{G} = \{3d+2 : d = 0,1,\ \ldots,\ \frac{n-8}{6}\}$ the vertices of adjacent generations $\mathscr{G} - \mathcal{G}$ are dominated by $\mathcal{G}$. Hence the dominating generation number is $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n-2}{6}$.

$$\gamma_\zeta = \sum_r 2^{(r-4)}\left(\frac{n-2}{2}\right)!,\ r = 3d+1 \text{ and } d = 0,1,\ldots,\frac{n-8}{6}\}$$

$$D = \{v_k^{(l)} : l = 2,5,\ldots,\frac{n-8}{6}, k = 1,2,\ldots,l!-1,l!\}$$

$$|D| = (2n-4)(2n-8)\ldots8 + (2n-16)(2n-20)\ldots8 + \ldots + (12)(8)$$

$$|D| = 4^{\frac{n-4}{2}}(m-1)(m-2)\ldots 2+\ldots+4^2(3)(2)$$
$$|D| = \sum_k 4^{k-1}\left(\prod_{i=2}^{k} i\right), k = 3j+2 \text{ and } j = 1,2,\ldots,\frac{n-2}{6}$$
$$= \gamma(R_{r-1} \circ_{+1}^{n} R_r).$$

Case 3: The dominating generation when $n \equiv 4 \; mod \; 6$ are given by $\mathcal{G} = \{3d : d = 1,2, \ldots, \frac{n-4}{6}\}$. $\gamma_{\mathcal{G}} = |\mathcal{G}| = \frac{n-4}{6}$

$$\gamma_{\zeta} = \sum_r 2^{(r-4)}\left(\frac{n-2}{2}\right)!, r = 3d \text{ and } d = 1,2,\ldots,\frac{n-4}{6}$$

The vertices from $\mathcal{G}$ adjacent with $\mathscr{G} - \mathcal{G} = \{3d-1, 3d+1\}$ but it is failed to dominate the vertices of initial generation. Hence the associated minimal dominating set is

$$D = \{v_k^{(l)} / l = 3,6,\ldots,\frac{n-4}{6}, k = 1,2,\ldots,l!-1,l!\} \cup \{v_1^{(1)}, v_8^{(1)}\}$$
$$|D| = (2n-4)(2n-8)\ldots 8+(2n-16)(2n-20)\ldots 8+\ldots+16.12.8+\gamma(R_4)$$
$$|D| = 4^{\frac{n-4}{2}}(m-1)(m-2)\ldots 2+\ldots+4^3(4)(3)(2)+\gamma(R_4)$$
$$|D| = \sum_k 4^{k-1}\left(\prod_{i=2}^{k} i\right), k = 3j \text{ and } j = 1,2,\ldots,\frac{n-4}{6}+\gamma(R_4)$$
$$= \gamma(R_{r-1} \circ_{+1}^{n} R_r).$$

Corollary 3.4.1: *The SDN of increasing $n-$ corona product of crossed prism graph is* $\gamma_s(R_{r-1} \circ_{+1}^{n} R_r) = \gamma(R_{r-1} \circ_{+1}^{n} R_r)$.

4. Conclusion

This paper is organised around the characterization of the *n*-corona product of complete and prism graph families. This study models and formulates the *n*-CP of the same graphs having an ascending order of one in their sizes and provides several upper bounds on domination. Further calculating the *n*-corona product for different graphs with systematic or random order and determining the various types of domination's in this area is wide open for future work.

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