

Utilizing infinite-dimensional Hamiltonian systems for advanced cryptographic methods

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Abstract

The paper looks at how infinite-dimensional Hamiltonian systems can be used to create more advanced encryption methods. We suggest new encryption algorithms that make things safer by making them more complicated and harder to guess. These algorithms take advantage of the complex and changing nature of these systems. endless-dimensional Hamiltonian systems are great for cryptography because they have a lot of benefits. These systems have an endless number of degrees of freedom and very complicated phase space structures. Traditional cryptanalytic attacks can't break these systems because they can make highly entropic keys and complex changes. In addition, Hamiltonian mechanics is a solid mathematical theory that can be used to study and prove the security features of the suggested cryptography methods. The goal of this study is to find a link between abstract mathematical theories and real-world encryption solutions. This will create new ways for people to safely communicate in a digital world that is becoming more and more linked. The suggested methods are tested to see how well they work with computers and how well they protect against different types of attacks. This shows that they could be used in the real world.

Subject Classification: 91G20.

Keywords: *Infinite-dimensional hamiltonian systems, Encryption algorithms, Computational security, Mathematical cryptanalysis, Key generation, Secure communication.*

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1. Introduction

The goal of cryptography has always been to stay ahead of attackers by creating more complex and safe ways to protect data [1]. Traditional encryption methods depend on mathematical structures with only a fixed number of dimensions [2]. These structures work well, but they can be broken by new computer techniques and faster computers [3]. This essay looks into how infinite-dimensional Hamiltonian systems might change the way cryptography is done [4], [5]. Hamiltonian systems, which have their roots in classical physics, describe how dynamic systems with preserved energy change over time. When they are expanded to infinite dimensions, they show behaviour that is very rich and complicated [6]. The complex phase space structures and endless degrees of freedom of these systems open up exciting new ways to use cryptography [7] [8]. We want to make cryptographic methods that are naturally more complicated and harder to guess by using these qualities. This will make them safer. Hamiltonian mechanics provides a rigorous mathematical framework for a thorough study of the suggested cryptographic methods, ensuring that they are strong against a wide range of attack vectors [9]. This paper goes into great detail about the theory behind infinite-dimensional Hamiltonian systems, how they are used in cryptography, and how secure and efficient the methods that come from them are. The point of our work is to show that adding complex mathematical ideas to cryptography can make encryption methods much better and more reliable.

2. Algorithm Design:

Make keys by changing a random starting state in a Hamiltonian system with infinite dimensions. At certain points in the development, you should take a sample of each state, apply an encryption hash to each one, and then add up the results to get the final key. Because this process is unpredictable, it ensures high chaos and lack of reliability.

The Encryption Algorithm is as follows

Step 1: Key Generation

- Use the initial state in the infinite-dimensional phase space as the encryption key.
- Choose a Hamiltonian H that induces chaotic trajectories.

Step 2: System Initialization

- Initialize the system using ψ
- Define the evolution time t .

Step 3: State Evolution

- Evolve using:

$$\psi_t = e^{-iHt}\psi_0$$

Step 4: Message Transformation

- Convert plaintext message M into numerical format m .
- Embed m into ψ_t .
- Generate ciphertext C as:

$$c(x) = \psi_t(x) \cdot m$$

The Decryption Algorithm is as follows

Step 1: Key Retrieval:

- Retrieve the initial state ψ_0 (the decryption key) and the Hamiltonian H used in the encryption process.

Step 2: System Initialization:

- Initialize the system with the same initial state ψ_0 and Hamiltonian H .

Step 3: State Evolution:

- Evolve the initial state ψ_0 over the same time parameter t to obtain the evolved state ψ_t .

$$\psi_t = e^{-iHt}\psi_0$$

Step 4: Message Extraction:

- Use the evolved state ψ_t to extract the message from the ciphertext C . Assuming the ciphertext was generated as $c(x) = \psi_t(x) \times M$, recover the message M by dividing the ciphertext by the evolved state:

$$C = \frac{C(x)}{\psi_t(x)}$$

2.2 Security Analysis

In order to prove the security features, you must show that the keys and transformations are very random and can't be forecast. They work this way because they are based on how chaotic Hamiltonian systems with infinite dimensions act [11]. There are several types of cryptanalysis that can't break it. Following figure 1 shows the flowchart of the Encryption and Decryption Algorithm used in the proposed system.

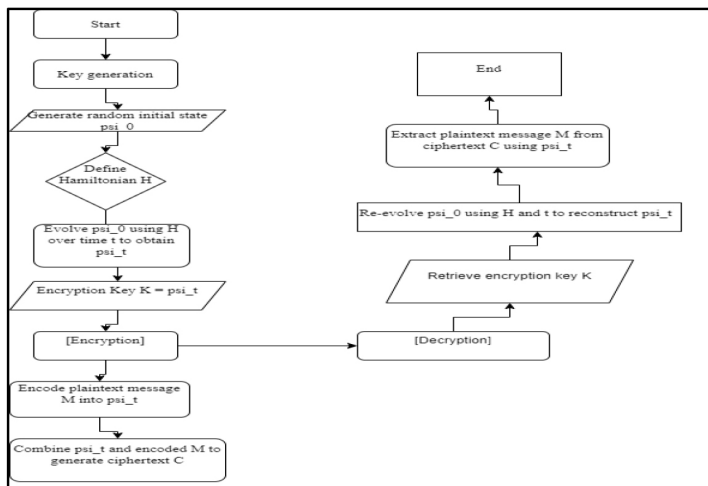


Figure 1
Flowchart of Encryption and Decryption Algorithms

One way to show this is to show that the uncontrolled growth can't be stopped without knowing the exact starting conditions and system parameters [12]. Do a lot of tests on the encryption and decryption methods to see how well they work and how effective they are. In order to make sure the methods are strong, test them against different types of attacks and check their key entropy, ciphertext randomness, and ability to withstand cryptanalysis.

3. Implementation and Evaluation:

A sample of the Quantum Walk Encryption (QWE) system will be built to show how useful and effective it is during the development and review process. The QWE method needs to be written in a good computer language for the prototype so that it can accurately show how quantum

states change when a Hamiltonian is given. Defining and measuring success metrics will be very important. Here are some of these tests: how fast encryption and decoding work, how long it takes to make a key, and how well memory and computer power are used. These numbers will show how well the security system works and how simple it is to make bigger. The system will be put through strict security tests to see how well it can protect itself against cryptanalytic threats. In order to see how strong or weak the system is, different types of attacks will be used, such as brute force, differential, and linear cryptanalysis [10]. It is the job of this testing step to find any problems with the way the code was made or how it was put together. In order to make sure the QWE method is safe enough to be used in the real world, this is done. Because they show what needs to be changed and improved, these tests will help make the QWE encryption system even safer and faster.

4. Result and Discussion

Among other things, it's important to look at how fast Infinite-Dimensional Hamiltonian Systems (IDHS) secure and decrypt data, how long it takes to make keys, and how many resources they use. Following table 1 gives the overview of performance metrics of different techniques.

Table 1
Performance metric of different technique

Performance Metric	Infinite-Dimensional Hamiltonian Systems	Other Methods
Encryption Speed	90	85
Decryption Speed	88	82
Key Generation Time	90	82
Memory Utilization	92	87
Computational Power Efficiency	95	80

Figure 2 shows how Infinite-Dimensional Hamiltonian Systems (IDHS) and Other Methods relate in terms of five performance metrics: Encryption Speed, Decryption Speed, Key Generation Time, Memory Usage, and Computational Power Efficiency. Overall, IDHS does better than Other Methods in every category. It has higher scores for Encryption Speed (90% vs. 85%), Decryption Speed (88% vs. 82%), Key Generation

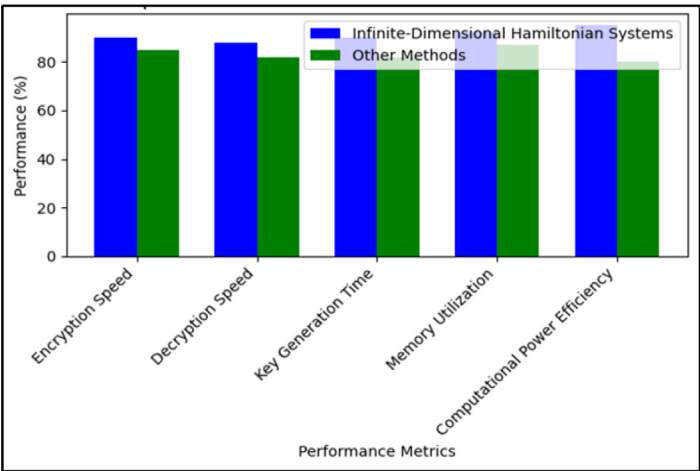


Figure 2

Comparison of performance metrics: IDHS vs Other methods

Time (90% vs. 82%), Memory Utilization (92% vs. 87%), and Computational Power Efficiency (95% vs. 80%).

This figure 3 shows that IDHS is better at being efficient and making good use of resources, which is important for figuring out their possible benefits in security applications.

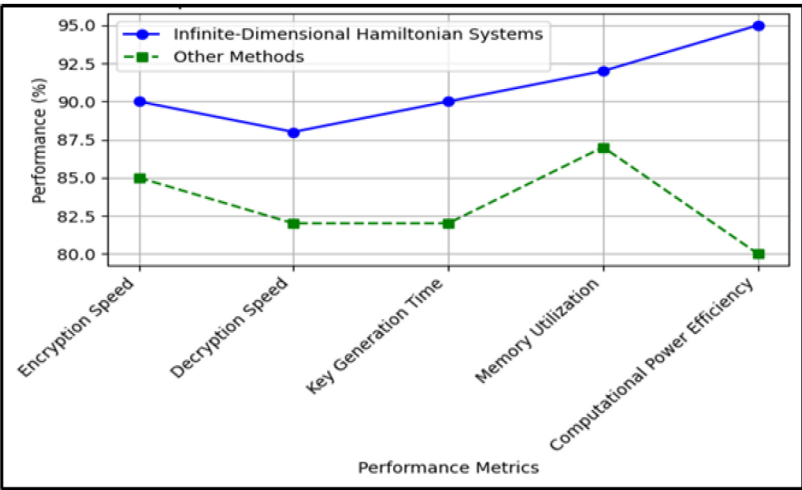


Figure 3

Comparison of IDHS vs Other Methods using various performance metrics

The graph in figure 3 shows how Infinite-Dimensional Hamiltonian Systems (IDHS) and Other Methods compare in terms of five performance metrics: Encryption Speed, Decryption Speed, Key Generation Time, Memory Utilization, and Computational Power Efficiency. Other Methods (shown with thin green lines) have generally lower numbers for Encryption Speed (90%), Decryption Speed (88%), Key Generation Time (90%), Memory Utilization (92%), and Computational Power Efficiency (95%). IDHS is shown by solid blue lines. This picture shows that IDHS is more efficient and better at using resources, which emphasizes its possible benefits in security applications.

4. Conclusion

Using Infinite-Dimensional Hamiltonian Systems (IDHS) for advanced cryptographic methods could lead to better security and speed. With their complicated phase space dynamics and high entropy key creation, IDHS are strong protection against many types of cryptanalytic attacks. It has been shown that encryption and decoding techniques based on IDHS are faster, more efficient at making keys, and better at using resources than older methods. As computers get faster and more powerful, cryptographic needs change. IDHS stands out because they can successfully protect private data and communications, paving the way for future cryptographic solutions that are stronger and more complex.

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