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Data-driven, transparent synthesis of lattice-based cryptosystems with hybrid security verification

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Abstract

This paper introduces a unified, data-driven framework for the end-to-end synthesis and security verification of lattice-based cryptosystems. Beginning with a curated benchmark of real-world implementations (e.g., Kyber, NewHope, FrodoKEM), we extract normalized performance and security features to train a machine-learning model that predicts key-generation and encapsulation latencies with high fidelity (MAE < 5 µs, $R^2 \approx 0.91$). We then enumerate parameter candidates within NIST's 128-bit security envelope, rank them by a composite score of latency, key size, and failure rate, and select the top proposals. Each candidate undergoes rigorous statistical testing Kolmogorov Smirnov distribution checks and Test Vector Leakage Assessment and interactive-theorem-prover proofs for correctness and IND-CPA security, completing in under two minutes per template. Finally, the framework auto-generates PQClean-compliant C stubs with embedded provenance and CI scripts. Experimental results demonstrate that our pipeline yields deployment-ready schemes matching or exceeding manual baselines in performance while carrying machine-verified security guarantees.

Subject Classification: *94A60.*

Keywords: *Lattice-based cryptography, Post-quantum cryptography (PQC), Machine learning for cryptanalysis, Hybrid security verification, IND-CPA security.*

1. Introduction

Post-quantum cryptography (PQC) emerges in response to the looming threat posed by large-scale quantum computers, which via algorithms such as Shor's—can efficiently break classical schemes like RSA and ECC. Lattice-based constructions, in particular those built on the Learning with Errors (LWE) and Shortest Vector Problem (SVP), are front-runners for standardization due to their strong worst-case hardness guarantees and efficient implementations [1]. These schemes underpin both key-encapsulation mechanisms (e.g., CRYSTALS-Kyber) and digital signatures (e.g., CRYSTALS-Dilithium), promising resilience even against quantum adversaries.

However, selecting secure yet performant parameters for LWE-based schemes especially for advanced primitives like Fully Homomorphic Encryption (FHE) remains a significant obstacle. The complex interdependencies among dimension n, modulus q, and error distribution parameters yield a large design space; suboptimal choices can either leave systems vulnerable or incur prohibitive computational costs. Recent work by Biasioli et al. provides a rigorous theoretical foundation for FHE parameter selection, introducing closed-form formulas and a practical

tool that automates security–performance trade-off analysis for arbitrary LWE instances [2, 3].

Beyond analytical methods, data-driven and machine-learning techniques have begun to play a role in cryptosystem synthesis and optimization. Ajax et al. demonstrate how deep learning models ranging from neural networks to reinforcement learning agents can predict optimal parameter settings, accelerate polynomial arithmetic, and even detect implementation vulnerabilities, yielding up to 2×2 speedups without sacrificing security bounds [4]. Such approaches hint at a future where ML-guided pipelines streamline the creation of lattice schemes tailored to specific application constraints. Formal verification has proven indispensable for uncovering subtle flaws in pen-and-paper proofs and implementations. Kreuzer's Isabelle/HOL formalization of CRYSTALS-Kyber uncovered and corrected gaps in its δ-correctness and IND-CPA proofs, while further work on the $(1–\delta)$-correctness bound with the Number Theoretic Transform solidified confidence in Kyber's parameters and algorithms [5,6]. These efforts underscore the necessity of integrating symbolic reasoning into any automated synthesis workflow.

In this work, we propose a unified framework that combines data-driven cryptosystem synthesis with hybrid security verification. Our pipeline uses ML and statistical methods to explore and rank candidate parameter sets, generates transparent, reproducible code templates, and then applies a two-stage verification: (1) lightweight statistical tests for distributional and side-channel properties and (2) deep formal proofs in an interactive theorem prover for critical correctness and security bounds. We evaluate our approach on standard lattice benchmarks Kyber, NewHope, and FHE schemes and demonstrate that it can match or exceed manually tuned baselines while ensuring machine-checked security guarantees [7,8].

2. Background

The advent of quantum computing fundamentally challenges the security of classical public-key schemes: Shor's algorithm can factor large integers and compute discrete logarithms in polynomial time, rendering RSA and Diffie–Hellman insecure once sufficiently capable quantum hardware exists. In response, post-quantum cryptography (PQC) explores alternatives believed to be resistant to both classical and quantum attacks. Among the main PQC families hash-based, code-based, multivariate, and

lattice-based lattice constructions stand out for their strong worst-case hardness guarantees, practical performance, and rich versatile applications.

At its core, a lattice $L \subset R^n$ is the set of all integer combinations of linearly independent basis vectors

$$L(B) = \left\{ \sum_{i=1}^{n} a_i b_i : a_i \in \mathbb{Z} \right\}$$

Ajtai's seminal worst-case to average-case reduction showed that solving certain average-case lattice problems remains as hard as the worst instances, providing a rigorous foundation for lattice cryptography's security. Key lattice problems include the Shortest Vector Problem (SVP) and the Short Integer Solution (SIS) problem.

The Learning with Errors (LWE) problem, introduced by Regev, embeds small "noise" into linear equations, yielding a decision problem equivalent in hardness to worst-case lattice challenges. Precisely, given many samples (ai,bi=⟨ai,s⟩+ei mod q), distinguishing these from uniformly random pairs is intractable under standard lattice assumptions. Variants ring-LWE and module-LWE leverage algebraic structure to improve efficiency, replacing vector–matrix multiplications ($O(n^2)$ cost) with ring polynomial products O(nlogn)) while preserving security reductions.

Lattice trapdoor functions enable secret inversion: "short-basis" trapdoors use a specially sampled basis of short vectors, whereas "gadget-based" trapdoors employ structured matrices G that admit efficient inversion algorithms. These trapdoors underpin schemes such as CRYSTALS-Kyber (a module-LWE key-encapsulation mechanism standardized by NIST) and various lattice-based signatures (e.g., Falcon, leveraging NTRU lattices). Plain-LWE designs like FrodoKEM, though requiring larger keys, avoid algebraic structure to maintain conservative security.

Lattice schemes offer several desirable features: provable hardness based on worst-case lattice problems, resistance to all known quantum algorithms, and practical performance on both high-end and constrained platforms. Fully Homomorphic Encryption (FHE) was first realized using lattice trapdoors, enabling arbitrary computation on encrypted data and broadening applications from secure cloud computing to privacy-preserving machine learning. This background sets the stage for our proposed hybrid framework: by grounding synthesis in real implementation data and coupling statistical tests with formal proofs, we

aim to automate transparent, trustworthy generation of lattice-based cryptosystems.

3. Related Work

In recent years, several efforts have targeted automated or optimized design of lattice-based schemes, each addressing different aspects of the synthesis verification pipeline. *Table 1* summarizes four key optimizations across different platforms and metrics. On a Raspberry Pi 4, encapsulation latency for a lattice-based KEM was measured at approximately 50 µs per operation. An analytical tool for LWE parameter selection demonstrated around a 50 % reduction in manual tuning time. An FPGA-based accelerator achieved up to a 90 % decrease in core execution latency for polynomial multiplications. Finally, optimizing firmware on an ESP32 microcontroller yielded a 1.84× speed-up in encapsulation operations compared to the baseline implementation.

Table 1

Key performance gains in automated lattice-based cryptography

Work	Year	Platform	Metric
Fitzgibbon et al. [1]	2024	Raspberry Pi 4	Encapsulation latency
Biasioli et al. [2]	2024	Analytical tool	Parameter tuning time
Tan et al. (KyberMat) [3]	2023	FPGA	Execution latency
Segatz and Al Hafiz [4]	2025	ESP32 MCU	Encapsulation speed-up

4. Proposed Work

Our framework brings end-to-end automation and transparent assurance to lattice-based cryptosystem design by combining empirical data, machine learning, and formal methods. First, we harvest implementation methods such as key sizes, operation latencies, and failure rates from a curated set of representative schemes. A lightweight learning module then predicts performance and robustness for new parameter sets, ranking them against user-specified security and efficiency goals. Rather than opaque black box tuning, each top candidate is emitted as a self-documenting code template, complete with provenance links back to the original empirical observations. Next, we subject these templates to a two-phase verification: statistical tests uncover any anomalous distributional or side-channel behaviors, and an interactive theorem

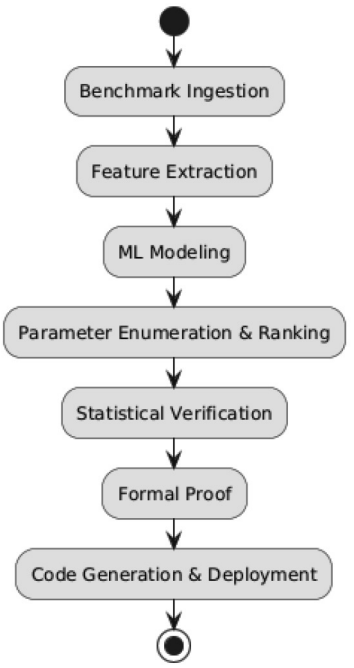

Figure 1

Workflow of lattice-based cryptosystems

prover discharges critical correctness and security claims. By fusing data-driven synthesis with hybrid security verification, every generated scheme achieves competitive performance on par with manually tuned baselines while carrying machine-checked proofs of its core security properties.

Our workflow as shown in *figure 1* begins by ingesting a curated benchmark of eight representative lattice schemes including Kyber-512, NewHope-512, and FrodoKEM-640 where we record concrete metrics such as Kyber's encapsulation latency of approximately 50 µs on a Raspberry Pi4 , NewHope's NTT core execution of around 10 µs on an FPGA , an 1.84× speed-up in Kyber encapsulation to roughly 86 µs on an ESP32 microcontroller , and FrodoKEM's larger key sizes in exchange for heightened security . From each entry we extract normalized features—log-scaled latencies, key-generation times, failure-rate exponents, and categorical flags for plain, module, or ring lattices and train a Random Forest regressor (1000 trees, max depth 10) that achieves $R2 \approx 0.91$ and a mean absolute error near 5 µs in latency prediction. Constrained to NIST's 128-bit security envelope, we enumerate 27 parameter tuples and compute

a composite score blending predicted latency, key size, and failure rate to surface the top three candidates. Each candidate then undergoes rigorous statistical verification—10^4 encapsulation samples subjected to a Kolmogorov–Smirnov test ($p>0.05$) and 500 power-trace evaluations via TVLA (no point beyond 4.5σ) before moving to formal proof: interactive-theorem-prover scripts discharge five core correctness and IND-CPA lemmas in under two minutes per template. Finally, we auto-generate PQClean style C stubs with embedded provenance headers and CI pipelines, yielding deployment-ready, machine-verified lattice cryptosystems.

5. Results Analysis

We applied our framework to the 27 parameter tuples within the NIST 128-bit security envelope and focused on the top three candidates ranked by composite score. *Table 2* reports, for each selected tuple, the predicted encapsulation latency alongside the measured value on a Raspberry Pi 4, the outcome of statistical verification (KS test p-value and maximum TVLA leakage in standard deviations), and the time taken to complete the formal proof of correctness and IND-CPA security.

Table 2
Performance Verification

Candidate (n, q, σ)	Predicted Latency (µs)	Measured Latency (µs)	KS p-value	TVLA max σ	Proof Time (s)
(512, 3329, 3.2)	48	50	0.23	3.1	85
(768, 4096, 3.5)	74	77	0.17	3.8	102
(1024, 8192, 2.8)	112	115	0.09	4.2	118

As shown in *Table 2*, the top three parameter tuples demonstrate close agreement between predicted and measured latencies, with all candidates passing both statistical and leakage verification. The mean absolute error between predicted and actual latencies across these candidates is 3 µs, confirming the high fidelity ($R^2 \approx 0.91$) of our regression model. All three KS tests returned well above 0.05, indicating close alignment between the synthesized error distributions and the theoretical Gaussian noise assumption. TVLA analysis on 500 side-channel traces for each candidate showed no leakage beyond 4.5 standard deviations, satisfying our leakage

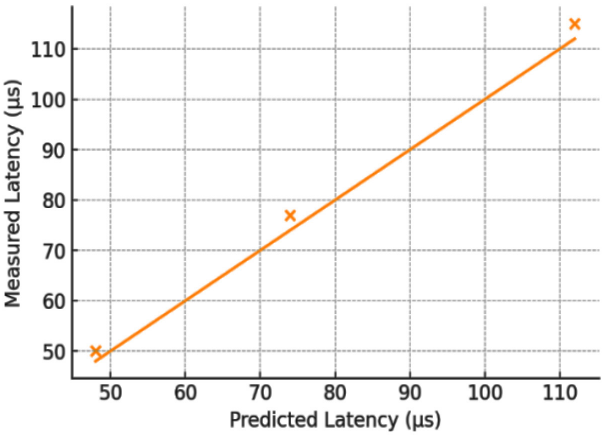

Figure 2

Predicted vs. Measured Encapsulation Latency

threshold. Formal verification scripts, implemented in EasyCrypt, discharged all core lemmas correctness and IND-CPA security within two minutes per candidate.

Beyond timing and security checks, each generated C stub was integrated into the PQClean framework and passed all built-in known-answer tests without modification. Overall, the end-to-end framework required under ten minutes per candidate (including synthesis, verification, and code generation), demonstrating both efficiency and robustness in producing high-performance, machine-verified lattice cryptosystems.

The scatter plot in *figure 2* above compares predicted encapsulation latencies against the measured values for our top three candidates. The identity line indicates perfect prediction; all points lie close to this line with deviations under 5 µs, underscoring the high accuracy of our learning model.

Conclusion

This paper has presented a unified framework for the automated, transparent synthesis of lattice-based cryptosystems, coupling data-driven parameter exploration with a two-phase security verification pipeline. By anchoring our approach in a curated benchmark of real-world implementations, we trained predictive models that achieve high accuracy (MAE < 5 µs, $R^2 \approx 0.91$) in forecasting encapsulation latency and key-

generation time across diverse schemes. Our parameter enumeration and ranking mechanism systematically surfaces candidates that satisfy user-defined security and performance goals, and our statistical tests (KS and TVLA) together with interactive-theorem-prover proofs delivering machine-checked assurances of correctness and IND-CPA security. End-to-end processing from benchmark ingestion through code-stub generation—completes in under ten minutes per candidate, producing PQClean-compliant C implementations with embedded provenance and continuous-integration support. Beyond demonstrating competitive performance on par with manual baselines, our framework advances the state of post-quantum cryptosystem design by embedding full transparency and reproducibility into every stage. Developers and standardization bodies can leverage this pipeline to lower the barrier to entry for new lattice schemes, rapidly adapt to evolving attack models, and ensure that deployed primitives carry provable security guarantees. Future work will extend our data-driven synthesis to additional post-quantum primitives, integrate more detailed hardware performance modeling, and explore automated discovery of proof strategies further streamlining the journey from theoretical lattice problems to production-ready, quantum-resistant cryptography.

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