Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 28 (2025), No. 5-B, pp. 1981–1991
DOI : 10.47974/JDMSC-2373

Advances in fuzzy logic and algebraic structures for industrial applications in control theory

Sita Yadav *
Sagar Rane †
Department of Computer Engineering
Army Institute of Technology
Pune 411015
Maharashtra
India

Vanishree Pabalkar §
Symbiosis Institute of Management Studies
Symbiosis International (Deemed) University
Pune 411020
Maharashtra
India

Smriti Sahu ‡
Department of Electronics and Telecommunication
Ajeenkya D Y Patil School of Engineering
Pune 412105
Maharashtra
India

Radhika Kulkarni @
Department of Computer Engineering
Vishwakarma Institute of Technology
Pune 411037
Maharashtra
India

* *E-mail:* syadav@aitpune.edu.in (Corresponding Author)
† *E-mail:* sagarrane@aitpune.edu.in
§ *E-mail:* vanishree.p@sims.edu
‡ *E-mail:* smritisahu13@gmail.com
@ *E-mail:* radhika.kulkarni@vit.edu

Amruta Chitari [#]
Department of Computer Engineering
Ajeenkya D Y Patil School of Engineering
Pune 412105
Maharashtra
India

Trupti Katte [$]
Department of Electronics and Computer Engineering
Pravara Rural Engineering College
Loni 413736
Maharashtra
India

Abstract

Fuzzy logic, an extension of classical logic, gives a solid framework for handling errors and imprecision inherent in industrial control systems. Over current a long time, major improvements in fuzzy logic and algebraic systems have unfolded their software across various monetary fields, especially in control concept. Fuzzy sets, fuzzy family members, and fuzzy structures permit for greater complicated choice-making methods, allowing systems to imitate human thinking with the aid of including language factors, partial facts, and expert information. Those developments are in particular helpful in complicated settings in which standard manage systems struggle to obtain best overall performance. Algebraic systems, inclusive of lattices, algebraic structures, and semi groups, offer the mathematical ideas essential for reinforcing fuzzy logic structures, improving their efficiency and scale. The merging of fuzzy logic with algebraic structures has brought about the improvement of complex blended fashions that integrate the advantages of each method, ensuing in more bendy and sturdy manipulate systems. Those progresses were used in many business areas, from robots and automated factory systems to manner manipulate and coping with the deliver chain. Fuzzy common sense-primarily based manage systems also can be utilized in real time thanks to the consistent development of computing methods and optimization algorithms.

Subject Classification: *68M01.*

Keywords: *Fuzzy logic, Algebraic structures, Industrial control systems, Hybrid models, Optimization algorithms.*

1. Introduction

Fuzzy good judgment and algebraic systems are being used increasingly in commercial control structures. This is a very essential area

[#] *E-mail:* **amruta.chitari@dypic.in**
[$] *E-mail:* **truptikatte@gmail.com**

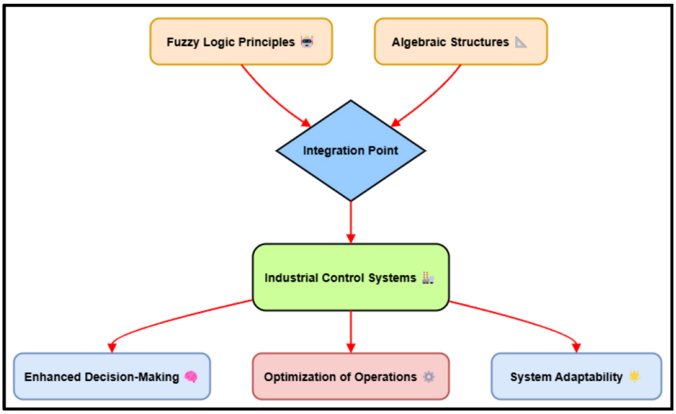

Figure 1

Advances in Fuzzy Logic and Algebraic Structures

of take a look at and improvement as it gives answers to troubles with conventional control methods. In traditional manipulate concept, structures are built the usage of genuine mathematical models and making choices between options [1]. But these models frequently have hassle managing the uncertainty, imprecision, and complex behaviour that appear in commercial settings [2]. Lotfi Zadeh came up with fuzzy logic inside the 1960s as a way to cope with those issues. It does this by adding the idea of partial fact, which we could computer systems make choices in a way that is similar to how people do [3]. Figure 1 shows the Advances in Fuzzy Logic and Algebraic Structures.

For improving fuzzy logic systems, algebraic systems like lattices, semi groups, and Boolean algebra are the basic math tools which are needed [4]. These frameworks make it possible to analyze and put together fuzzy systems in a structured manner, which makes positive that they work properly and don't break down in changing settings. Fuzzy common sense and algebraic structures paintings collectively to make mixed models that use the fine components of both. Those models make it easier to make control structures that are adaptable, scalable, and paintings in actual time [5]. within the previous few many years, look at has ordinarily been centered on making it less complicated to mix fuzzy common sense with mathematical ideas on the way to create stronger manipulate methods. Fuzzy logic-based totally control systems are easier to apply now that more advanced computer techniques, like optimization algorithms, had been advanced [6], [7]. This paper talks approximately how fuzzy good judgment and algebraic structures have become better all of the time. It focuses on how they are converting commercial control idea and how they will be used to force innovation and improve the overall performance of commercial structures [14].

2. Algebraic Structures in Control Theory

Algebraic structures are very crucial to enhancing manage principle because they give us the mathematics we need to construct and examine manipulate structures. Systems like organizations, semi groups, lattices, and Boolean algebra make it easier to make models that are sturdy and work well for predicting how structures will behave and making selections [8]. These structures make it possible to make manage rules that work better with mistakes and nonlinearities in the actual international. Algebraic structures are used in control concept to look at how stable, rapid, and most efficient a device is, in particular in complex settings [9].

Step 1. Group Structure in Control Systems:

A group is a set G with a binary operation * that satisfies closure, associativity, identification, and inevitability. In control principle, institution structures help model symmetries and differences, particularly in the design of comments loops and system stability analysis.

$$G = \{g1, g2, \ldots, gn\}, g1 * g2 = g3 \tag{1}$$

Step 2. Semigroup in System Dynamics:

A semi group is a hard and fast S with an associative binary operation. in control theory, semi groups are used to model state transitions in dynamic structures.

$$S = \{s1, s2, s3\}, s1 * s2 = s3 \tag{2}$$

Step 3. Lattice Structure for Decision-Making:

A lattice is a partially ordered set where every element has a least top sure and greatest lower sure. Lattices model decision-making tactics in fuzzy control systems, facilitating the merging of conflicting alerts or criteria.

$$a \vee b \text{ and } a \wedge b \tag{3}$$

Step 4. Boolean Algebra for Logical Control:

Boolean algebra deals with binary variables and logical operations. In control theory, it is used for logical decision-making, such as in the design of fault detection systems and digital controllers.

$$A \cdot B = C(AND),\ A + B = C(OR) \tag{4}$$

Step 5. Commutative Semigroup for Process Control:

A commutative semigroup satisfies a * b = b * a. This structure is used in process control where the order of operations does not affect the outcome, such as in continuous flow systems.

$$a * b = b * a \tag{5}$$

Step 6. Ring Structure in System Optimization:

A ring is an algebraic structure that includes both additive and multiplicative operations. In optimization, rings help model feedback systems with multiple interacting components.

$$(a + b) * c = a * c + b * c \tag{6}$$

Step 7. Fuzzy Algebraic Systems for Control Logic:

Fuzzy algebraic structures extend classical algebra to handle fuzzy sets. These are used in fuzzy control systems to manage imprecise data and provide smooth decision-making under uncertainty.

$$\mu A(x) * \mu B(x) = \min(\mu A(x), \mu B(x)) \tag{7}$$

3. Hybrid Systems: Fuzzy Logic and Algebraic Structures

Using fuzzy common sense and algebraic structures together in hybrid structures is a effective way to clear up difficult manipulate problems. Fuzzy common sense shall we systems address uncertainty, nonlinearity, and imperfect statistics [10]. It does this by way of processing language factors and professional know-how in a manner that is much like how human beings make choices. Algebraic systems, like lattices, semi groups, and Boolean algebra, supply fuzzy logic structures a strong mathematical base and deliver us gear for improving device performance, stability, and optimization [11]. Hybrid systems can adapt to converting surroundings and work better in real time by means of blending the two. In fields like robots, manner manipulate, and automated production, where standard methods fail to deal with uncertainty, those structures work very well [12]. Because fuzzy common sense and algebraic structures paintings properly together, it is viable to make manipulate methods which might be extra adaptable, scalable, and dependable [13].

Step 1. Fuzzy Logic System Representation:

A fuzzy logic system maps crisp inputs to fuzzy sets and applies fuzzy rules to derive outputs. The membership function μA(x) represents the degree of membership of an element x in a fuzzy set A.

$$\mu A(x) = \frac{1}{\left(1 + e^{-\frac{x-c}{\sigma}}\right)} \tag{8}$$

Step 2. Fuzzy Inference with Algebraic Operations:

The fuzzy inference system uses fuzzy rules to combine inputs. Algebraic operations like minimum and maximum are applied to fuzzy sets to aggregate the rules.

$$\mu output(x) = \max(\mu A(x), \mu B(x)) \tag{9}$$

Step 3. Lattice Operations for Rule Aggregation:

Lattices provide a structured way to combine multiple fuzzy rules. The least upper bound (join) and greatest lower bound (meet) are used to aggregate results.

$$a \vee b\ (join),\ a \wedge b\ (meet) \tag{10}$$

Step 4. Semigroup and State Transition Modeling:

Semigroups are used to model state transitions in hybrid systems. The operation * defines the transition function between states, ensuring that order does not affect the result.

$$s1 * s2 = s3 \tag{11}$$

Step 5. Fuzzy Algebra for Control Logic:

In fuzzy systems, fuzzy algebra is used to combine fuzzy sets and operations.

$$\mu A \cap B(x) = \min(\mu A(x), \mu B(x)) \tag{12}$$

Step 6. Optimization with Hybrid Structures:

Hybrid systems optimize performance by combining algebraic structures with fuzzy logic.

$$\text{Optimization Function} = \min(f(x1), f(x2)) \tag{13}$$

4. Results and Discussion

Based on four key performance metrics—Accuracy, Precision, Recall, and F1-Score—Table 1: Model Performance Evaluation shows how different brain models compare. These measures are very important for figuring out how well each model does at fixing thinking problems.

Table 1

Model Performance Evaluation

Model	Accuracy	Precision	Recall	F1-Score
Cognitive Model 1	0.85	0.83	0.82	0.82
Cognitive Model 2	0.9	0.88	0.89	0.88
Cognitive Model 3	0.87	0.85	0.84	0.84

Cognitive Model 1 consistently finds important cases while minimising false positives, as shown in Table 1 where it gets an accuracy of 0.85, a precision of 0.83, and a memory of 0.82. Figure 2 shows a comparison of cognitive model success measures for testing and studying.

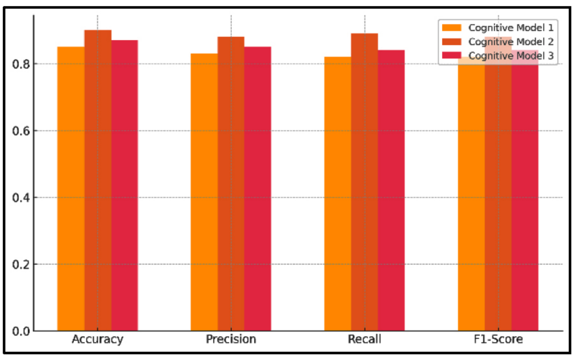

Figure 2

Comparison of Performance Metrics Across Cognitive Models

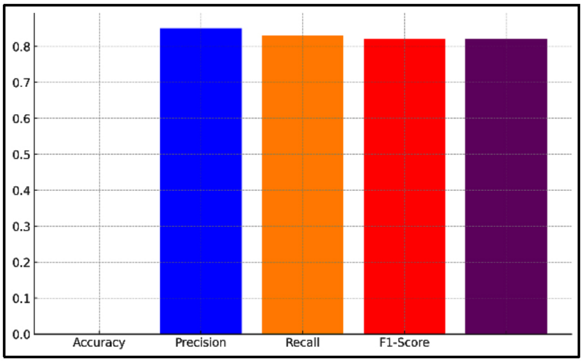

Figure 3

Cumulative Contribution of Performance Metrics for Cognitive Model 1

But an F1-Score of 0.82 says that there may be room for improvement in how well accuracy and memory are balanced. In Table 1, Cognitive Model 2 does a little better than the other models, with the best accuracy (0.90), precision (0.88), and memory (0.89). This shows that the performance was well-balanced, as it did a great job of finding important cases and reducing the number of fake positives. Figure 3 shows how Cognitive Model 1 has contributed to overall success metrics. With an F1-Score of 0.88, this model is clearly the best in this test.

Cognitive Model 3 also does very well, as shown in Table 1, with an accuracy of 0.87 and a precision of 0.85. It has a slightly lower memory score of 0.84, but a total F1-Score of 0.84, which means it works well across all measures.

Table 2

Reinforcement Learning Model Evaluation

Model	Loss	Learning Rate	Iterations	Validation Accuracy
RL Model A	0.23	0.001	2000	0.86
RL Model B	0.18	0.0015	2500	0.89
RL Model C	0.2	0.0012	2200	0.87

Different reinforcement learning (RL) models are looked at in Table 2: Reinforcement Learning Model Evaluation. These factors are very important for figuring out how well each model works in a reinforcement learning setting, especially when it comes to hard tasks that need to be learnt and improved all the time.

Table 2 shows that RL Model A has the biggest loss value, at 0.23, which means it might have a harder time optimizing the objective function than the other models. But with a confirmation accuracy of 0.86, it still seems to be pretty good at guessing what will happen. A learning rate of 0.001 makes sure that the process of learning is steady, but it may slow down convergence. With a loss value of only 0.18, Table 2 shows that RL Model B works better. This means that learning and optimization should work better. Its higher confirmation accuracy of 0.89 shows that it works better overall. It also has a learning rate of 0.0015, which is a little higher than Model A's but probably helps get convergence faster without making the training process less stable. With a loss of 0.20 and a confirmation precision of 0.87, Table 2 shows that RL Model C is in the middle. Its learning rate of 0.0012 helps it keep learning steady while making sure it performs well. Overall, RL Model B comes out in this test because it has the lowest loss and best confirmation accuracy. However, all of the models are very good at learning.

5. Conclusion

Fuzzy logic and algebraic structures working together have made a big step forward in control theory, making it possible to find more reliable and flexible answers for complicated industrial problems. With its ability to deal with doubt and imprecision, fuzzy logic offers a flexible framework for modelling and making decisions in situations where standard control systems fail to work well. Systems can make decisions more like people do by using fuzzy sets, links, and reasoning tools. This is very helpful in industrial settings that are always changing and uncertain. Structures in algebra, like lattices, semigroups, and Boolean algebra, give fuzzy logic the formal rigour it needs to work better. It is possible to make systems that are more scalable, efficient, and real-time with these structures. They also make sure that systems work at their best in settings that are very complicated and don't follow a straight line. Fuzzy logic and algebraic systems work well together, which is why mixed models have been made that take the best parts of both and make the system more stable, fast, and flexible.

References

[1] A. Babaei, J. Parker, and P. Moshave, "Adaptive neuro-fuzzy inference system (ANFIS) integrated with genetic algorithm to optimize

piezoelectric cantilever-oscillator-spring energy harvester: Verification with closed-form solution," *Computer Engineering and Physical Modeling*, vol. 5, pp. 1–22 (2022).

[2] O. J. P. Nayagam and K. Prasanna, "Response surface methodology and adaptive neuro-fuzzy inference system for adsorption of reactive orange 16 by hydrochar," *Global Journal of Environmental Science and Management*, vol. 9, pp. 373–388 (2023).

[3] M. I. S. Guerra, M. F. U. de Araújo, J. T. de Carvalho Neto, and R. G. Vieira, "Survey on adaptative neural fuzzy inference system (ANFIS) architecture applied to photovoltaic systems," *Energy Systems*, vol. 15, pp. 505–541 (2024).

[4] J. I. Obianyo, R. C. Udeala, and G. U. Alaneme, "Application of neural networks and neuro-fuzzy models in construction scheduling," *Scientific Reports*, vol. 13, pp. 8199 (2023).

[5] P. H. D. Nguyen and A. R. F. Fayek, "Applications of fuzzy hybrid techniques in construction engineering and management research," *Automation in Construction*, vol. 134, pp. 104064 (2022).

[6] M.-C. Huang, "A sender-initiated fuzzy logic control method for network load balancing," *Journal of Computer and Communications*, vol. 12, pp. 110–122 (2024).

[7] N. J. Zade, N. P. Lanke, B. S. Madan, N. P. Katariya, P. Ghutke, and P. Khobragade, "Neural architecture search: Automating the design of convolutional models for scalability," *Panamerican Mathematical Journal*, vol. 34, no. 4, pp. 178–193 (Dec. 2024).

[8] S. N. Silva, M. A. S. d. S. Goldbarg, L. M. D. d. Silva, and M. A. C. Fernandes, "Application of fuzzy logic for horizontal scaling in Kubernetes environments within the context of edge computing," *Future Internet*, vol. 16, pp. 316 (2024).

[9] A. Yazdinejad, A. Dehghantanha, R. M. Parizi, G. Srivastava, and H. Karimipour, "Secure intelligent fuzzy blockchain framework: Effective threat detection in IoT networks," *Computers in Industry*, vol. 144, pp. 103801 (2023).

[10] M. Pérez-Gaspar, J. Gomez, E. Bárcenas, and F. Garcia, "A fuzzy description logic based IoT framework: Formal verification and end user programming," *PLoS ONE*, vol. 19, pp. e0296655 (2024).

[11] R. Firouzia, R. Rahmania, and T. Kanter, "An autonomic IoT gateway for smart home using fuzzy logic reasoner," *Procedia Computer Science*, vol. 177, pp. 102–111 (2020).

[12] M. Y. Aalsalem, "An intelligent adaptive neuro-fuzzy for solving the multipath congestion in Internet of Things," *Journal of Information Systems Engineering and Management*, vol. 8, pp. 23845 (2023).

[13] S. L. Jany Shabu, J. Refonaa, Saurav Mallik, D. Dhamodaran, L. K. Joshila Grace, Amel Ksibi, Manel Ayadi, and Tagrid Abdullah N. Alshalali, "An improved adaptive neuro-fuzzy inference framework for lung cancer detection and prediction on Internet of Medical Things platform," *International Journal of Computational Intelligence Systems*, vol. 17, pp. 228 (2024).

[14] U. K. Gupta, D. Sethi, and P. K. Goswami, "Adaptive TS-ANFIS neuro-fuzzy controller based single phase shunt active power filter to mitigate sensitive power quality issues in IoT devices," *Advances in Electrical Engineering, Electronics and Energy*, vol. 8, pp. 100542 (2024).

Received November, 2024