Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 28 (2025), No. 4-B, pp. 1375–1383
DOI : 10.47974/JDMSC-2277

Some parameters for the resize graph of $G_2(4)$

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Abstract

Topological indices provide important insights into the structural characteristics of molecular graphs. The present investigation proposes and explores a creative graph on a finite group G, which is known as the RIG. This graph is designated as $\Gamma^{RS}_{G_2(4)}$ indicating a simple undirected graph containing elements of G. Two distinct vertices are regarded as nearly the same if and only if their sum yields a non-trivial involution element in G. RIGs have been discovered in various finite groups. We examine several facets of the RIG by altering the graph through the conjugacy classes of G. Furthermore, we investigate the topological indices as applications in graph theory applying the distance matrix of the $G_2(4)$ group.

Subject Classification: *05C90, 05C09, 20D05, 05C25, 05C69.*

Keywords: *Application of graph theory, Graphical indices, Resize graph, Cliques.*

1. Introduction

The fundamental strategy for understanding group structure includes performing yield an assessment on a graph. In accordance with the latest findings in [1], [2] and [3]. If Γ is a simple graph, then its vertex set is P(G), the power set of the elements of G. Two sets S, T ∈ P(G) are viewed as neighboring nations in Γ if and only if there exist components α ∈ S and β ∈ T such that the result αβ is related to I(G). A RIG is the subgraph of Γ with vertex set $G \subseteq P(G)$, represented by Γ_G^{RI}, denotes the resize graph, which is a subgraph of Γ with vertex set G-conjugacy(classes) $\subseteq P(G)$. Jund and Salih [7] originally provided the results of the involution graph and its specific features. Aubad and Salih examined the structure of the result involution graphs in [8]. The

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researchers noticed that for n > 4, the graphs Γ_{Sn}^{RI}, and Γ_{An}^{RI} are connected and feature a diameter, radius, and girth of 3. Also, when G represents a finite dihedral or quaternion group, they defined specific features for Γ_{G}^{RI}. For every Mathieu sporadic simple group, the investigation of the results involution graph structure for the Higman-Sims group (HS) & the McLaughlin group (ML) appears in [6]. Substantial details regarding the Γ_{G}^{RI} and Γ_{G}^{RS} for the Suzuki group (Suz) could be obtained in [4]. TI is a mathematical model, a branch of graph theory [4, 5] that represent the crucial elements of graphs, which are frequently used in various fields such as chemistry, biology, and network studying [24], [25], [26], [27]. These indexes give major insights concerning the graph's level of detail, connectivity, and other intrinsic attributes. In chemical graph theory, topological indices facilitate the assessment of the physical, chemical, and biological characteristics of molecules. Social network analysis provides insights into the structure and behavior of the network.in this paper we study the topological index of the resize graph for expectational group of lie type [9, 10] with properties.

2. The preliminary elections

First, we define a few terms and state a few initial results that will be necessary later in the work.

A non-trivial element φ in G is said to be an involution element [7] if $\varphi^2 = e$. The set of all involution elements in G is denoted by $I(G)$. It is clear that the size of $I(G)$ is the sum of the size of involution conjugacy classes in G and denoted by t. Recall that the number of vertices of $\Gamma_{G}^{RI}$is t and the number of edges of Γ_{G}^{RI} can be obtained in the following result.

Definition 2.1: [9] Assume that G is any finite group. The result involution graph Γ_{G}^{RI}. is a graph whose two vertices are near if they yield an element in $I(G)$.

Definition 2.2: [7], [9] A resize graph consists of a network whose vertices represent all of the conjugacy classes in the group G. Two vertices are near if the representatives of those conjugacy classicize connected in Γ_{G}^{RI}.

3. Practical part: The resize of $G_2(4)$:

This section contains all the details concerning the graph $\Gamma_{G_2(4)}^{RS}$. The vertices set of $\Gamma_{G_2(4)}^{RS}$ are with classes and degrees: {1Ⓐ, 2Ⓐ.2Ⓑ, 3Ⓐ, 3Ⓑ, 4Ⓐ, 4Ⓑ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 6Ⓑ, 7Ⓐ, 8Ⓐ, 8Ⓑ, 10Ⓐ, 10Ⓑ, 10Ⓒ, 10Ⓓ, 12Ⓐ, 12Ⓑ, 12Ⓒ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ}. thus size ($\Gamma_{G_2(4)}^{RS}$) =424. we held computerized using the program (Gap) [11], Online Atlas [13], YAGS [12] On the previous information in order to understand the behavior of $\Gamma^{RS}_{G_2(4)}$ We provide the discs for the vertices of $\Gamma^{RS}_{G_2(4)}$graph as shown in the following Table 1:

Table 1
Connected vertices with related discs

Vertices	(Δ_1) Vertices	(Δ_2) Vertices	(Δ_3) Vertices
1Ⓐ	{2Ⓐ, 2Ⓑ}	{3Ⓐ, 3Ⓑ, 4Ⓐ, 4Ⓑ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 6Ⓑ, 7Ⓐ, 8Ⓐ, 8Ⓑ, 10Ⓐ, 10Ⓑ, 10Ⓒ, 10Ⓓ, 12Ⓐ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ}	{12Ⓑ, 12Ⓒ}
2Ⓐ	{1Ⓐ, 2Ⓑ, 3Ⓑ, 4Ⓐ, 4Ⓑ, 5Ⓐ, 5Ⓑ, 6Ⓑ, 6Ⓓ, 8Ⓐ, 10Ⓒ, 10Ⓓ}	{3Ⓐ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 7Ⓐ, 8Ⓑ, 10Ⓐ, 10Ⓑ, 12Ⓐ, 12Ⓑ, 12Ⓒ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ}	-------
3Ⓐ	{2Ⓑ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 8Ⓐ, 10Ⓐ, 10Ⓑ, 12Ⓐ, 12Ⓑ, 12Ⓒ}	{1Ⓐ, 2Ⓐ, 3Ⓑ, 4Ⓐ, 4Ⓑ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 6Ⓑ, 7Ⓐ, 8Ⓑ, 10Ⓒ, 10Ⓓ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ}	-------
2Ⓑ 3Ⓑ, 5Ⓑ, 5Ⓒ, 5Ⓓ, 7Ⓐ, 8Ⓐ, 10Ⓐ, 10Ⓑ, 10Ⓒ, 10Ⓓ, 12Ⓐ	$V(\Gamma^{RS}_{G_2(4)}) \setminus \Delta_2$	Δ_2(2Ⓑ)= {12Ⓑ, 12Ⓒ}, Δ_2(3Ⓑ)= Δ_2(5Ⓒ)= Δ_2(7Ⓐ)= Δ_2(10Ⓒ)= Δ_2(10Ⓓ)={1Ⓐ, 3Ⓐ}. Δ_2(5Ⓑ)= Δ_2(8Ⓐ)= {1Ⓐ, 3Ⓑ}. Δ_2(5Ⓓ)= {1Ⓐ, 3Ⓒ}. Δ_2(10Ⓐ)= Δ_2(10Ⓑ)= Δ_2(12Ⓐ)= {1Ⓐ, 2Ⓐ}.	-------
4Ⓐ, 8Ⓑ	$V(\Gamma^{RS}_{G_2(4)}) \setminus \Delta_2$	Δ_2(4Ⓐ)= {1Ⓐ, 3Ⓐ, 3Ⓑ, 3Ⓒ, 3Ⓔ , 9Ⓐ}. Δ_2(8Ⓑ)= {1Ⓐ, 3Ⓐ, 3Ⓑ, 3Ⓓ, 4Ⓐ, 4Ⓑ}	
4Ⓑ, 4Ⓒ, 5Ⓐ	$V(\Gamma^{RS}_{G_2(4)}) \setminus \Delta_2$	Δ_2(4Ⓑ)= {1Ⓐ, 3Ⓐ, 3Ⓑ, 3Ⓒ, 3Ⓓ}. Δ_2(4Ⓒ)= Δ_2(5Ⓐ)= {1Ⓐ, 3Ⓐ, 3Ⓑ, 3Ⓒ, 9Ⓐ}	
6Ⓐ, 6Ⓑ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ	$V(\Gamma^{RS}_{G_2(4)}) \setminus \Delta_2$	Δ_2(6Ⓐ)= Δ_2(6Ⓑ)= {1Ⓐ, 3Ⓐ, 3Ⓑ}. Δ_2(13Ⓐ)= Δ_2(13Ⓑ)= Δ_2(15Ⓐ)= Δ_2(15Ⓑ)= Δ_2(15Ⓒ)= Δ_2(15Ⓓ)= Δ_2(21Ⓐ)= Δ_2(21Ⓑ) = {1Ⓐ, 2Ⓐ, 3Ⓐ}	
12Ⓑ, 12Ⓒ	$V(\Gamma^{RS}_{G_2(4)}) \setminus \{\Delta_2 \cup \Delta_3\}$	Δ_2(12Ⓑ)= Δ_2(12Ⓒ)= {2Ⓐ, 2Ⓑ}	{1Ⓐ}

4. Topological indices of $\Gamma_{G_2(4)}^{RS}$:

Topological indices [15], [21], [22] are Divided into two branches:

i- Degree based topological indices (DBTI) of $\Gamma_{G_2(4)}^{RS}$

ii- Distance based topological indices for $\Gamma_{G_2(4)}^{RS}$

4.1. *Degree based topological indices (DBTI) of $\Gamma_{G_2(4)}^{RS}$:[14], [16], [18], [20], [23]*

Following the mathematical calculation of various frequently employed indices of that sort, we will then generate a Table 2, with the values of all necessary indices. Assume that U=$\Gamma_{G_2(4)}^{RS}$, then we have:

Table 2

(DBTI) of $\Gamma_{G_2(4)}^{RS}$

Topological Index lows	Value
Z_1 (U) = $\sum_{\mu \in V(U)} (\deg(\mu))^2 = \sum_{\mu\rho \in E(U)} [deg\ (\mu) + \deg(\rho)]$	23676
Z_2 (U) = $\sum_{\mu\rho \in E(U)} \deg(\mu)\deg(\rho)$	330712
$F(U) = \sum_{\mu \in V(U)} (\deg(\mu))^3 = \sum_{\mu\rho \in E(U)} [deg\ (\mu)^2 + \deg(\rho)^2]$	669260
$\bar{Z}_1$ (U)= $\sum_{\mu\rho \notin E(U)} [deg\ (\mu) + \deg(\rho)]$	**2612**
$\bar{Z}_2$(U)= $\sum_{\mu\rho \notin E(U)} \deg(\mu)\deg(\rho)$	**17002**
RZ_2 (U) = $\sum_{\mu\rho \in E(U)} [\deg(\mu) - 1][\deg(\rho) - 1]$	307460
NK(U)=$\prod_{\mu \in V(U)} deg(\mu)$.	236*32*52*715 *13*2912
$\prod_1(U)$=$\prod_{\mu \in V(U)} \deg(\mu)^2 = \prod_{\mu\rho \in E(U)}(deg\ (\mu) + \deg(\rho)$	272*34*54*730 *132*2924
$\prod_1(U) = \prod_{\mu\rho \in E(U)} deg(\mu)deg(\rho)$	2932*342*540*7420*13^26*29348
R(U)= $\sum_{\mu\rho \in E(U)} \frac{1}{\sqrt{deg(\mu)deg(\rho)}}$	0.622686
RR(U)= $\sum_{\mu\rho \in E(U)} \sqrt{deg(\mu)deg(\rho)}$	10.8973
RRR(U)= $\sum_{\mu\rho \in E(U)} \sqrt{[deg(\mu) - 1][deg(\rho) - 1]}$	11782.3
ABC(U)= $\sum_{\mu\rho \in E(U)} \frac{\sqrt{(deg\ (\mu) + \deg(\rho) - 2}}{\sqrt{deg(\mu)deg(\rho)}}$	11353.7
H(U)= $\sum_{\mu\rho \in E(U)} \frac{2}{deg(\mu) + deg(\rho)}$	113.188
ZAI(U)=$\left(\frac{deg(\mu)deg(\rho)}{deg(\mu)+deg(\rho)-2}\right)3$	15.377
GA(U)= $\sum_{\mu\rho \in E(U)} \frac{2\sqrt{(deg\ (\mu)\deg(\rho)}}{\sqrt{deg(\mu)+deg(\rho)}}$	1294024.39004
SCI(U)= $\sum_{\mu\rho \in E(U)} \frac{1}{\sqrt{deg(\mu)+deg(\rho)}}$	297.629

4.2 *Distance based topological indices:[17], [19]*

In this section we provide specific calculation about that part of TI presented in the following Table 3 below:

Table 3

Certain Distance based topological indices for $\Gamma^{RS}_{G_2(4)}$.

	Topological Index lows	Value
1	W(U)= $\sum_{\{\mu,\rho\}\subseteq V(U)} (c(\mu))$	570
2	W_p(U)=c(U, 3)	2
3	TW(U)=$\sum_{\{\mu,\rho\}\subseteq V_1(U)} (c(\mu))$	0
4	WW(U)=$\sum_{\{\mu,\rho\}\subseteq V(U)} [c(\mu,\rho) + c(\mu,\rho)^2]$	1292
5	RCW(U)=$\sum_{\{\mu,\rho\}\subseteq V(U)} \frac{1}{diam(u)-1+c(\mu,\rho)}$	167.233
6	H_{old}(U)=$\sum_{\{\mu,\rho\}\subseteq V(U)} \frac{1}{c(\mu,\rho)^2}$	441.722
7	H (U)=$\sum_{\{\mu,\rho\}\subseteq V(U)} \frac{1}{c(\mu,\rho)}$	459.667
8	Ψ(U)=$\sum_{\mu\in V(U)} E(\mu)$, E=eccentricity	57920
9	$Ƶ_1$(U)=$\sum_{\mu\in V(U)} E(\mu)$	729656
10	$Ƶ_2$(U)=$\sum_{\mu\rho\in E(U)} E(\mu)\, E(\rho)$	49944
11	Ψ^c(U)=$\sum_{\mu\in V(U)} E(\mu)$deg (μ)	1043216.66667

5. Theoretical part

In this part, we prove the following essential theorem that give some properties of the graph $\Gamma^{RI}_{G_2(4)}$:

Theorem 1: *The $\Gamma^{RS}_{G_2(4)}$ is the elements relating to the connections are displayed in the next Table 4:*

Table 4

$\Gamma^{RS}_{G_2(4)}$ properties

Diam($\Gamma^{RS}_{G_2(4)}$)	R($\Gamma^{RS}_{G_2(4)}$)	g($\Gamma^{RS}_{G_2(4)}$)	ω ($\Gamma^{RS}_{G_2(4)}$)	χ ($\Gamma^{RS}_{G_2(4)}$)
3	2	3	27	27

Proof: For any vertex in the graph $\Gamma^{RS}_{G_2(4)}$, the sum of all of a diameter of their discs equaled 32. As a result, the graph $\Gamma^{RS}_{G_2(4)}$ has been connected. Additionally, based on the quantity of discs at each vertex of the graph, we can assume that the eccentricity of the vertices are, respectively, equal to {3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 2, 2, 2, 2, 2, 2, 2, 2}. Consequently, we obtain that Diam($\Gamma^{RS}_{G_2(4)}$)=3 and R($\Gamma^{RS}_{G_2(4)}$)=2. Since the vertices {2Ⓐ, 2Ⓑ, 4Ⓐ} from a cyclic of length 3 then g($\Gamma^{RS}_{G_2(4)}$)=3. In $\Gamma^{RS}_{G_2(4)}$ there are 8-cliques which is laid out as follows:

Q_1 = {3Ⓑ, 4Ⓐ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 6Ⓑ, 7Ⓐ, 8Ⓐ, 8Ⓑ, 10Ⓐ, 10Ⓑ, 10Ⓒ, 10Ⓓ, 12Ⓐ, 12Ⓑ, 12Ⓒ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ} with size 27.

Q_2 = {2Ⓑ, 3Ⓑ, 4Ⓐ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 6Ⓑ, 7Ⓐ, 8Ⓐ, 8Ⓑ, 10Ⓐ, 10Ⓑ, 10Ⓒ, 10Ⓓ, 12Ⓐ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ} with size 26.

Q_3 = {3Ⓑ, 4Ⓐ, 4Ⓑ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 6Ⓑ, 7Ⓐ, 8Ⓐ, 8Ⓑ, 10Ⓐ, 10Ⓑ, 10Ⓒ, 10Ⓓ, 12Ⓐ, 12Ⓑ, 12Ⓒ, 13Ⓐ, 13Ⓑ, 15Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ} with size 25.

Q_4 = {2Ⓑ, 3Ⓑ, 4Ⓐ, 4Ⓑ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 6Ⓑ, 7Ⓐ, 8Ⓐ, 8Ⓑ, 10Ⓐ, 10Ⓑ, 10Ⓒ, 10Ⓓ, 12Ⓐ, 13Ⓐ, 13Ⓑ, 15 Ⓐ, 15Ⓑ, 15Ⓒ, 15Ⓓ, 21Ⓐ, 21Ⓑ} with size 24.

Q_5 = {2Ⓐ, 2Ⓑ, 3Ⓑ, 4Ⓐ, 4Ⓑ, 4Ⓒ, 5Ⓐ, 5Ⓑ, 6Ⓑ, 8Ⓐ, 10Ⓒ, 10Ⓓ} with size 12.

Q_6 = {3Ⓐ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 8Ⓐ, 10Ⓐ, 10Ⓑ, 12Ⓐ, 12Ⓑ, 12Ⓒ} with size 10.

Q_7 = {2Ⓑ, 3Ⓐ, 5Ⓒ, 5Ⓓ, 6Ⓐ, 8Ⓐ, 10Ⓐ, 10Ⓑ, 12Ⓐ} with size 9.

Q_8 = {1Ⓐ, 2Ⓐ, 2Ⓑ} with size 3.Thus, $\omega(\varGamma^{RS}_{G_2(4)})$=27.

Now we clam that $\chi(\varGamma^{RS}_{G_2(4)})$=27. To prove this clam, we look at the complement of the maximum cliques in $\varGamma^{RS}_{G_2(4)}$ which are: Q_1^c= {1Ⓐ, 2Ⓐ, 2Ⓑ, 3Ⓐ, 4Ⓑ}.

In case Q_1^c we assigned in the following manner as shown in Table 5:

Table 5
Similar vertex color

Vertex	1Ⓐ	2Ⓐ	2Ⓑ	3Ⓐ	4Ⓑ
Indicated vertex color	21Ⓐ	5Ⓒ	12Ⓑ	15Ⓐ	6Ⓐ

We establish that the resulting outcome is an application of the graph $\Gamma^{RI}_{G_2(4)}$ for the algebraic structures of the group $G_2(4)$:

Corollary 1: *The unique set {ξ,* (Dih_ξ)*} of the* $Dih_\xi \subseteq G_2(4)$*, created through specific u ∈ 2Ⓐ and vertices* $w \in V(\varGamma^{RI}_{G_2(4)})$ *of order ξ, where ξ ∈ {2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 15, 21}.*

Proof: Let U=2Ⓐ∪2Ⓑ , choose a random $x \in$ U, then $Dih_\alpha \cong < u , w >$ for all $w \in u$Letter under the condition that $uw \in$ U . Equivalency, $Dih_\alpha \cong < u , w >$ if $u \in$ U, $w \in \alpha$Letter and $(u , w) \in$ E($\Gamma^{\mathrm{RI}}_{Ly}$). However, $(u , w) \in$ E($\Gamma^{\mathrm{RI}}_{G_2(4)}$) requires that α Letter ∈ {Δ_1(2Ⓐ)∪ 2Ⓐ}\ {1Ⓐ} or {Δ_1(2Ⓑ)∪ 2Ⓑ}\ {1Ⓐ}, this shows the value of α.

5. Discussion and Conclusion

The resize graph of $G_2(4)$ was employed to serve the aim of the research, together with various generic properties associated to this graph. This work produced many different results. The connectedness, diameter, clique number, girth, and radius of the resize graph are all calculated. In addition, nearly every topological indices for the resize graph are included. The computational discrete algebra framework Gap has been used to identify specific components of such graph.

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Received February, 2025