

A neutrosophic fuzzy Pseudo in δ -algebras

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Abstract

This work's objective is to present Neutrosophic fuzzy pseudo δ -subalgebras, Neutrosophic fuzzy pseudo δ -ideal and Neutrosophic fuzzy pseudo δ' -ideal in δ -algebras.

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Several properties and propositions are investigated. Additionally, number of examples are provided to highlight the ideas presented in this paper.

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1. Introduction

Two classes of abstract algebras, known as BCK-algebras and BCI-algebras, were introduced by Imai and Iseki. [1], [2] Another significant extension of BCK-algebras is d-algebras, which were initially introduced by Neggers and Kim [3]. They explored various relationships between d-algebras and BCK-algebras, as well as connections between d-algebras and oriented digraphs. The concept of neutrosophic sets was introduced by Smarandache [4] in 1999. Subsequently, the notion of neutrosophic fuzzy sets is being studied by numerous mathematicians in various fields. In 2021, the notion of δ -algebra is given by Mahmood and Hassan [5]. Jabber and Khalil introduced the notion of a pseudo rho-algebras in fields [6]. Further research on fuzzy sets can be found in [7–11]. Non-classical sets such as neutrosophic sets [12,13] and others [14, 15] are provided. Presenting Neutrosophic fuzzy pseudo δ -subalgebras, Neutrosophic fuzzy pseudo δ -ideal, and Neutrosophic fuzzy pseudo δ -ideal in δ -algebras is the aim of this work. Numerous attributes and claims are examined. Furthermore, several instances are given to illustrate the concepts discussed in this work.

2. Fundamental

The basic definitions required for this research are covered.

Definition 2.1: [5] A triple $(V, *, 0)$ is called a δ -algebra (δ -A) if $0 \in V$ and fulfilling the following assumptions (i) $\tau * \tau = 0$, (ii) $0 * \tau = 0$, (iii) If $\tau * \kappa = 0$ and $\kappa * \tau = 0 \rightarrow \tau = \kappa$, for all $\tau, \kappa \in V$, (iv) $\forall \tau \neq \kappa \in V - \{0\} \rightarrow \tau * \kappa = \kappa * \tau \neq 0$, (v) $\forall \tau \neq \kappa \in V - \{0\} \rightarrow (\tau * (\tau * \kappa)) * (\kappa * \tau) = 0$.

Definition 2.2: [5] A neutrosophic set M over the universal V is defined by $M = \{ \langle \tau, M_H(\tau), M_G(\tau), M_I(\tau) \rangle \mid \tau \in V \}$, where $M_H(\tau): V \rightarrow [0,1]$, $M_G(\tau): V \rightarrow [0,1]$ & $M_I(\tau): V \rightarrow [0,1]$ are functions. Values $M_H(\tau)$, $M_G(\tau)$ and $M_I(\tau)$ refer to the degree of membership, non-membership, and indeterminacy of τ to M , respectively.

Definition 2.3: [5] Let $\emptyset \neq K \subseteq V$, where $(V, *, 0)$ is a (δ -A), K is said to be δ -subalgebra (δ -SA) of V if $\tau * \kappa \in K$, for any $\tau, \kappa \in K$.

Definition 2.4: [5] Let $(V, *, 0)$ be a $(\delta - A)$ and $\emptyset \neq K \subseteq V$. K is said to be a δ -ideal of V if (i) $\tau, \kappa \in K \rightarrow \tau * \kappa \in K$, (ii) $\tau * \kappa \in K$ and $\kappa \in K \rightarrow \tau \in K$, $\forall \tau, \kappa \in V$.

3. A Neutrosophic Fuzzy Pseudo δ -Subalgebras in δ -Algebras.

Here, we look into and examine a number of new concepts, like $(NF\delta - I)$, δ - homomorphism, $(NF\delta - SA)$, $(NF\delta - I)$, and. Some basic properties are given too.

Definition 3.1: Let $(V, *, \bullet, 0)$ is pseudo δ - algebra (briefly, $P\delta - A$) and $M = \{ \langle \tau, M_H(\tau), M_G(\tau), M_I(\tau) \rangle \mid \tau \in V \}$ be an (NFS) of V . We say M is a Neutrosophic fuzzy pseudo δ^* - subalgebra of V (briefly, $NFP\delta^* - SA$) if ; (i) $M_H(\tau * \kappa) \geq \min\{M_H(\tau), M_H(\kappa)\}$, (ii) $M_G(\tau * \kappa) \leq \max\{M_G(\tau), M_G(\kappa)\}$, (iii) $M_I(\tau * \kappa) \geq \min\{M_I(\tau), M_I(\kappa)\}$, $\forall \tau, \kappa \in V$. Also, we say M is a Neutrosophic fuzzy pseudo δ^* - subalgebra of V (briefly, $NFP\delta^* - SA$) if such that (i) $M_H(\tau \bullet \kappa) \geq \min\{M_H(\tau), M_H(\kappa)\}$, (ii) $M_G(\tau \bullet \kappa) \leq \max\{M_G(\tau), M_G(\kappa)\}$, (iii) $M_I(\tau \bullet \kappa) \geq \min\{M_I(\tau), M_I(\kappa)\}$, $\forall \tau, \kappa \in V$. If M is both $(NFP\delta^* - SA)$ & $(NFP\delta^* - SA)$ of V , we say that M is a Neutrosophic fuzzy pseudo δ - subalgebra (briefly, $NFP\delta - SA$) of V .

Example 3.2: Assume that $V = \{0, r, s, p\}$ is a set, and let $(*)$ and $(\bullet)$ be defined on V as indicated by the tables (1) and (2), respectively.

Table 1
 $*: V \times V \rightarrow V$

*	0	r	s	p
0	0	0	0	0
r	r	0	s	r
s	s	s	0	s
p	p	r	r	0

Table 2
 $\bullet: V \times V \rightarrow V$

$\bullet$	0	r	s	p
0	0	0	0	0
r	s	0	s	r
s	r	s	0	r
p	p	r	s	0

Then $(V, *, \bullet, 0)$ is a $(P\delta - A)$. let $M_H = \begin{pmatrix} 0 & r & s & p \\ 0.5 & 0.3 & 0.3 & 0.5 \end{pmatrix}$ be (NFS) for membership of M , $M_G = \begin{pmatrix} 0 & r & s & p \\ 0.3 & 0.6 & 0.6 & 0.3 \end{pmatrix}$ is (NFS) for non-membership of M & $M_I = \begin{pmatrix} 0 & r & s & p \\ 0.2 & 0.1 & 0.1 & 0.2 \end{pmatrix}$ is (NFS) for indeterminate of M . Then M is $(NFP\delta^* - SA)$ of V , M is a $(NFP\delta^* - SA)$ of V , therefore M is $(NFP\delta - SA)$ of V .

Definition 3.3: Let $(V, *, \bullet, 0)$ be $P\delta - A$ and $M = \{ \langle \tau, M_H(\tau), M_G(\tau), M_I(\tau) \rangle \mid \tau \in V \}$ be an (NFS) of V . We say M is a Neutrosophic fuzzy pseudo δ^* - ideal of V (briefly, $NFP\delta^* - I$) if such that (i) $M_H(\tau * \kappa) \geq \min\{M_H(\tau), M_H(\kappa)\}$ and $M_H(\tau) \geq \min\{M_H(\tau * \kappa), M_H(\kappa)\}$, (ii) $M_G(\tau * \kappa) \leq \max\{M_G(\tau), M_G(\kappa)\}$ and $M_G(\tau) \leq \max\{M_G(\tau * \kappa), M_G(\kappa)\}$, (iii) $M_I(\tau * \kappa) \geq \min\{M_I(\tau), M_I(\kappa)\}$ & $M_I(\tau) \geq \min\{M_I(\tau * \kappa), M_I(\kappa)\}$, $\forall \tau, \kappa \in V$. Also, we say M is a Neutrosophic fuzzy pseudo δ^* - ideal of V (briefly, $NFP\delta^* - I$) if such that (i) $M_H(\tau \bullet \kappa) \geq$

$\min\{M_H(\tau), M_H(\kappa)\}$ and $M_H(\tau) \geq \min\{M_H(\tau \bullet \kappa), M_H(\kappa)\}$, (ii) $M_G(\tau \bullet \kappa) \leq \max\{M_G(\tau), M_G(\kappa)\}$ and $M_G(\tau) \leq \max\{M_G(\tau \bullet \kappa), M_G(\kappa)\}$, (iii) $M_J(\tau \bullet \kappa) \geq \min\{M_J(\tau), M_J(\kappa)\}$ and $M_J(\tau) \geq \min\{M_J(\tau \bullet \kappa), M_J(\kappa)\}$, $\forall \tau, \kappa \in V$. If M is both a $(NFP\delta^* - I)$ and a $(NFP\delta - I)$ of V , we say that M is a Neutrosophic fuzzy pseudo δ -ideal (briefly, $NFP\delta - I$) of V .

Example 3.4: By using Tables(1&2), we consider that $(V, *, \bullet, 0)$ is a $(P\delta - A)$. Suppose that $M_H = \begin{pmatrix} 0 & r & s & p \\ 0.3 & 0.2 & 0.2 & 0.3 \end{pmatrix}$ is a (NFS) for membership of M , $M_G = \begin{pmatrix} 0 & r & s & p \\ 0.4 & 0.6 & 0.6 & 0.4 \end{pmatrix}$ is a (NFS) for non-membership of M and $M_J = \begin{pmatrix} 0 & r & s & p \\ 0.3 & 0.2 & 0.2 & 0.3 \end{pmatrix}$ is a (NFS) for indeterminate of M . Hence M is a $(NFP\delta^* - I)$ of V also M is $(NFP\delta^* - I)$ of V . Therefore M is a $(NFP\delta - I)$ of V .

Definition 3.5: Let $(V, *, \bullet, 0)$ be $P\delta - A$ & $M = \{< \tau, (M_H(\tau), M_G(\tau), M_J(\tau)) > | \tau \in V\}$ be an (NFS) of V . We say M is a Neutrosophic fuzzy pseudo δ^* -ideal of V (briefly, $NFP\delta^* - I$) if such that (i) $M_H(0) \geq M_H(\kappa)$, $M_G(0) \leq M_G(\kappa)$ and $M_J(0) \geq M_J(\kappa)$, (ii) $M_H(\tau * \kappa) \geq \min\{M_H(\tau), M_H(\kappa)\}$, $M_G(\tau * \kappa) \leq \max\{M_G(\tau), M_G(\kappa)\}$ and $M_J(\tau * \kappa) \geq \min\{M_J(\tau), M_J(\kappa)\}$, $\forall \tau, \kappa \in V$. Also, we say M is a Neutrosophic fuzzy pseudo $\delta^\bullet$ -ideal of V (briefly, $NFP\delta^\bullet - I$) if such that $M_H(0) \geq M_H(\kappa)$, $M_G(0) \leq M_G(\kappa)$ & $M_J(0) \geq M_J(\kappa)$, $M_H(\tau \bullet \kappa) \geq \min\{M_H(\tau), M_H(\kappa)\}$, $M_G(\tau \bullet \kappa) \leq \max\{M_G(\tau), M_G(\kappa)\}$ & $M_J(\tau \bullet \kappa) \geq \min\{M_J(\tau), M_J(\kappa)\}$, $\forall \tau, \kappa \in V$. If M is both a $(NFP\delta^* - I)$ & a $(NFP\delta^\bullet - I)$ of V , we say that M is a Neutrosophic fuzzy pseudo δ -ideal (briefly, $NFP\delta - I$) of V .

Example 3.6: By using Tables(1&2) in Example 3.2, we get M is a $(NFP\delta^* - I)$ of V , also M is a $(NFP\delta^\bullet - I)$ of V . Hence M is a $(NFP\delta - I)$ of V .

Definition 3.7: Let $M = \{< \tau, M_H(\tau), M_G(\tau), M_J(\tau) > | \tau \in V\}$ be an (NFS) of V . For any $\sigma \in [0, 1]$, define the sets $P(M_H, \sigma)$, $Q(M_G, \sigma)$ and $Y(M_J, \sigma)$ by $P(M_H, \sigma) = \{\tau \in V | M_H(\tau) \geq \sigma\}$, $Q(M_G, \sigma) = \{\tau \in V | M_G(\tau) \leq \sigma\}$ and $Y(M_J, \sigma) = \{\tau \in V | M_J(\tau) \geq \sigma\}$ respectively.

Remark 3.8: Let $M = \{< \tau, M_H(\tau), M_G(\tau), M_J(\tau) > | \tau \in V\}$ be an (NFS) of V . Then $M = \{< \tau, M_H(\tau), M_G(\tau), M_J(\tau) > | \tau \in V\}$ is $NFP\delta^* - I$ [resp., $NFP\delta^\bullet - I$, and $NFP\delta - I$] of V if and only if $P(M_H, \sigma)$, $Q(M_G, \sigma)$ and $Y(M_J, \sigma) \in P\delta^*I(V)$ [resp., $P\delta^\bullet I(V)$, and $P\delta I(V)$], where $P\delta^*I(V)$ [resp., $P\delta^\bullet I(V)$, and $P\delta I(V)$] is the set of all $P\delta^*I$ [resp., $P\delta^\bullet I$, and $P\delta I$] of V , whenever $P(M_H, \sigma) \neq \emptyset$ (resp. $Q(M_G, \sigma) \neq \emptyset$ and $Y(M_J, \sigma) \neq \emptyset$), where $\sigma \in [0, 1]$.

Example 3.9: Assume that $M_I(r) = \begin{cases} \tau, & \text{if } r \in I \\ \kappa, & \text{o.w.} \end{cases}$, where $\tau, \kappa \in [0, 1]$ with $\tau > \kappa$. Then $M_I(r)$ is an $NFP\delta^* - I$ [resp., $NFP\delta^\bullet - I$, and $NFP\delta - I$] of V .

Theorem 3.10: If $M = \{ \langle \tau, M_H(\tau), M_G(\tau), M_J(\tau) \rangle \mid \tau \in V \}$ is a (NFS) of V , then the two items are equivalent: (1) M is a $(NFP\delta^* - SA)$ [resp. $(NFP\delta^* - SA)$, $(NFP\delta - SA)$] of V , (2) $P(M_H, \sigma)$ [resp. $Q(M_G, \sigma)$, and $Y(M_J, \sigma)$] is a $(P\delta^* - SA)$ [resp. $(P\delta^* - SA)$, $(P\delta - SA)$] of V when $P(M_H, \sigma) \neq \emptyset$ [resp. $Q(M_G, \sigma) \neq \emptyset$, and $Y(M_J, \sigma) \neq \emptyset$].

Proof: It is clearly.

Theorem 3.11: Suppose that $M = \{ \langle \tau, M_H(\tau), M_G(\tau), M_J(\tau) \rangle \mid \tau \in V \}$ is (NFS) of V . If each one of $P(M_H, \sigma)$, $Q(M_G, \sigma)$ and $Y(M_J, \sigma)$ is $(P\delta - I)$, then (1) $M_H(\tau) \geq \min\{M_H(\tau * \kappa), M_H(\kappa)\}$, (2) $M_G(\tau) \leq \max\{M_G(\tau * \kappa), M_G(\kappa)\}$, (3) $M_J(\tau) \geq \min\{M_H(\tau * \kappa), M_H(\kappa)\}$, (4) $M_H(\tau) \geq \min\{M_H(\tau \bullet \kappa), M_H(\kappa)\}$, (5) $M_G(\tau) \leq \max\{M_G(\tau \bullet \kappa), M_G(\kappa)\}$, (6) $M_J(\tau) \geq \min\{M_H(\tau \bullet \kappa), M_H(\kappa)\}$, $\forall \tau, \kappa \in V$.

Proof: Let (1) is not valid. Then $M_H(\tau_1) < \min\{M_H(\tau_1 * \kappa_1), M_H(\kappa_1)\}$ for some $\tau_1, \kappa_1 \in V$. Putting $\sigma_1 = \frac{1}{2[M_H(\tau_1) + \min\{M_H(\tau_1 * \kappa_1), M_H(\kappa_1)\}]}$ so $M_H(\tau_1) < \sigma_1 < \min\{M_H(\tau_1 * \kappa_1), M_H(\kappa_1)\}$. As a consequence of this, $\kappa_1 \in P(M_H, \sigma_1)$ and $\tau_1 * \kappa_1 \in P(M_H, \sigma_1)$, but $\tau_1 \notin P(M_H, \sigma_1)$. But this is an inconsistency. Hence (1) is held, and let (2) is not valid. Hence $M_G(\tau_2) > \max\{M_G(\tau_2 * \kappa_2), M_G(\kappa_2)\}$ for some $\tau_2, \kappa_2 \in V$. Putting $\sigma_2 = \frac{1}{2[M_G(\tau_2) + \max\{M_G(\tau_2 * \kappa_2), M_G(\kappa_2)\}]}$ so $M_G(\tau_2) > \sigma_2 > \max\{M_G(\tau_2 * \kappa_2), M_G(\kappa_2)\}$. As a consequence of this, we obtain $\kappa_2 \in Q(M_G, \sigma_2)$ and $\tau_2 * \kappa_2 \in Q(M_G, \sigma_2)$, but $\tau_2 \notin Q(M_G, \sigma_2)$. But this is an inconsistency. Hence (2) is held. Also Let (3) is not valid. Then $M_J(\tau_3) < \min\{M_J(\tau_3 * \kappa_3), M_J(\kappa_3)\}$ for some $\tau_3, \kappa_3 \in V$. Putting $\sigma_3 = \frac{1}{2[M_J(\tau_3) + \min\{M_J(\tau_3 * \kappa_3), M_J(\kappa_3)\}]}$, we have $M_J(\tau_3) < \sigma_3 < \min\{M_J(\tau_3 * \kappa_3), M_J(\kappa_3)\}$. Thus $\kappa_3 \in Y(M_J, \sigma_3)$ and $\tau_3 * \kappa_3 \in Y(M_J, \sigma_3)$, but $\tau_3 \notin Y(M_J, \sigma_3)$. But this is an inconsistency. Now, let (4) is not valid. Then $M_H(\tau_4) < \min\{M_H(\tau_4 \bullet \kappa_4), M_H(\kappa_4)\}$ for some $\tau_4, \kappa_4 \in V$. Putting $\sigma_4 = \frac{1}{2[M_H(\tau_4) + \min\{M_H(\tau_4 \bullet \kappa_4), M_H(\kappa_4)\}]}$ so $M_H(\tau_4) < \sigma_4 < \min\{M_H(\tau_4 \bullet \kappa_4), M_H(\kappa_4)\}$. As a consequence of this, we have $\kappa_4 \in P(M_H, \sigma_4)$ and $\tau_4 \bullet \kappa_4 \in P(M_H, \sigma_4)$, but $\tau_4 \notin P(M_H, \sigma_4)$. But this is an inconsistency. Hence (4) is held. Next let (5) is not valid. Hence $M_G(\tau_5) > \max\{M_G(\tau_5 \bullet \kappa_5), M_G(\kappa_5)\}$ for some $\tau_5, \kappa_5 \in V$. Putting $\sigma_5 = 1/2[M_G(\tau_5) + \max\{M_G(\tau_5 \bullet \kappa_5), M_G(\kappa_5)\}]$ thus $M_G(\tau_5) > \sigma_5 > \max\{M_G(\tau_5 \bullet \kappa_5), M_G(\kappa_5)\}$. As a consequence of this, we obtain $\kappa_5 \in Q(M_G, \sigma_5)$ and $\tau_5 \bullet \kappa_5 \in Q(M_G, \sigma_5)$, but $\tau_5 \notin Q(M_G, \sigma_5)$. But this is an inconsistency. Hence (5) is held. Let (6) is not valid. Then $M_J(\tau_6) < \min\{M_J(\tau_6 \bullet \kappa_6), M_J(\kappa_6)\}$ for some $\tau_6, \kappa_6 \in V$. Putting $\sigma_6 = 1/2[M_J(\tau_6) + \min\{M_J(\tau_6 \bullet \kappa_6), M_J(\kappa_6)\}]$ we have $M_J(\tau_6) < \sigma_6 < \min\{M_J(\tau_6 \bullet \kappa_6), M_J(\kappa_6)\}$. As a consequence of this, we have $\kappa_6 \in Y(M_J, \sigma_6)$ and $\tau_6 \bullet \kappa_6 \in Y(M_J, \sigma_6)$, but $\tau_6 \notin Y(M_J, \sigma_6)$. But this is an inconsistency. Hence (6) is held.

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