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Study the new generalization of fuzzy σ-algebra namely fuzzy $\mathcal{P}^*$-algebra

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Abstract

This research studies the idea of fuzzy $\mathcal{P}^*$-algebra which is generalization for each of fuzzy σ-algebra and fuzzy σ-ring. Some properties and examples as well as the relations among these concepts are presented. Finally, we proved the arbitrarily intersection of fuzzy $\mathcal{P}^*$-algebra is also a fuzzy $\mathcal{P}^*$-algebra and we studied the properties of the smallest fuzzy $\mathcal{P}^*$-algebra.

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1. Introduction

Mohammed, Assad, and Ahmed in [1] studied some ideas that play important roles in applied algebra and crypt analysis.

Some types of classes of sets play a paramount present in the study of several properties in real analysis such as σ-algebra, σ-ring and other types see [2] and [3].

In recent decades, A problem of uncertainty modeling has riveted a lot of interest researchers in probability theory. Different ideas, enough fuzzy sets and measures were suggested. These ideas shedding new light on different interpretations of fuzzy set and their links with probability theory, the idea of fuzzy set was introduced by Zadeh in 1965 [4] as an extension of the classical set.

The idea of fuzzy σ-algebra was first introduced by Ahmed, Al-Fayadh, and Ebrahim [5].

The notion of soft $\mathcal{P}^*$-field is studied by Abbas, Ebrahim and Al-Fayadh in 2023 [6, 15].

2. Related works

Definition 2.1:

1. We denote for universal set by Ω.
2. The fuzzy set $\mathcal{F}$ in Ω is defined as $\{(\omega, \mathfrak{f}_{\mathcal{F}}(\omega)) : \omega\epsilon\Omega\}$ where $\mathfrak{f}_{\mathcal{F}} : \Omega \to [0\,,1]$ is function such that for every $\omega\epsilon\Omega$, $\mathfrak{f}_{\mathcal{F}}(\omega)$ represent the degree of membership of ω in $\mathcal{F}$ [7].
3. The empty fuzzy set is written as $\Phi^* = \{(\omega, 0) : \omega\epsilon\Omega\}$ [8].
4. The universal fuzzy set is defined as $\Omega^* = \{(\omega, 1) : \forall\omega\epsilon\Omega\}$ [9].
5. The complement of fuzzy set $\mathcal{F}$ is known as $\mathcal{F}^c = \{(\omega, 1 - \mathfrak{f}_{\mathcal{F}}(\omega)) : \omega\epsilon\Omega\}$ [10].
6. The intersection of two fuzzy sets $\mathcal{F}\,\&\,\mathcal{D}$ is known as $\mathcal{F} \cap \mathcal{D} = \{(\omega, \min_{\omega\epsilon\Omega}\{\mathfrak{f}_{\mathcal{F}}(\omega)\,, \mathfrak{f}_{\mathcal{D}}(\omega)\}) : \omega\epsilon\Omega\}$ [11].
7. The difference of two fuzzy sets $\mathcal{F}\,\&\,\mathcal{D}$ is defined as: $\mathcal{F} - \mathcal{D} = \mathcal{F} \cap \mathcal{D}^c = \{(\omega, \min_{\omega\epsilon\Omega}\{\mathfrak{f}_{\mathcal{F}}(\omega)\,, \mathfrak{f}_{\mathcal{D}^c}(\omega)\}) : \omega\epsilon\Omega\}$ [12].
8. The union of fuzzy set $\mathcal{D}_1, \mathcal{D}_2, \ldots$ is defined as : $\cup_{m=1}^{\infty}\mathcal{D}_m = \{(\omega, \sup_{m=1,2,\ldots}\{\mathfrak{f}_{\mathcal{D}_m}(\omega)\}) : \omega\epsilon\Omega\}$ [8].

Notation 2.2:

1. The abbreviations of class of fuzzy subset of Ω^* are $\mathcal{K}^*\&\mathfrak{T}^*$.
2. The abbreviation of fuzzy σ-algebra is written as F- σ- ALG.
3. The abbreviation of fuzzy σ-ring is written as F- σ- RG.
4. The abbreviation of fuzzy $\mathcal{P}^*$-algebra is written as F- $\mathcal{P}^*$- ALG.

Definition 2.3: [13] A class $\mathcal{K}^*$ is named F- σ- ALG on Ω^* if:

1. $\Phi^*\epsilon\mathcal{K}^*$.
2. $\mathcal{F}^c\,\epsilon\mathcal{K}^*$ when $\mathcal{F}\epsilon\mathcal{K}^*$.
3. $\cap_{m=1}^{\infty}\mathcal{D}_m\ \epsilon\mathcal{K}^*$ for $\{\mathcal{D}_m\}_{m\epsilon\mathbb{N}}\ \epsilon\mathcal{K}^*$.

Remark 2.4: [5] Suppose $\mathcal{K}^*$ is F- σ- ALG on Ω^*, then $\bigcap_{m=1}^{n}\mathcal{D}_{\mathrm{m}} \in\mathcal{K}^*$ and $\bigcup_{\mathrm{m}=1}^{\infty}\mathcal{D}_{\mathrm{m}} \in\mathcal{K}^*$whenever $\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_{\mathrm{n}}, \dots \in\mathcal{K}^*$.

Lemma 2.5: [5] *If $\{\mathcal{K}_\delta^*\}_{\delta\epsilon I}$ is family of F- σ- ALG on Ω^*,then so is $\bigcap_{\delta\epsilon I}\mathcal{K}_\delta^*$.*

Proposition 2.6: [5] $\sigma(\mathfrak{T}^*)$ is a smallest F- σ- ALG on Ω^* that include $\mathfrak{T}^*$.

Definition 2.7: [14] A class $\mathcal{K}^*$ is named F- σ- RG on Ω^* if:

1. $\Phi^*\epsilon\mathcal{K}^*$.
2. $\mathcal{F}-\mathcal{D}\ \epsilon\mathcal{K}^*$ whenever $\mathcal{F}\ \&\ \mathcal{D}\epsilon\mathcal{K}^*$.
3. $\bigcup_{m=1}^{\infty}\mathcal{D}_{\mathrm{m}}\ \epsilon\mathcal{K}^*$ for $\{\mathcal{D}_{\mathrm{m}}\}_{m\epsilon\mathbb{N}}\ \epsilon\mathcal{K}^*$.

Lemma 2.8: [14]. *If $\{\mathcal{K}_\delta^*\}_{\delta\epsilon I}$ is a class of F- σ- RG on Ω^*, then $\bigcap_{\delta\epsilon I}\mathcal{K}_\delta^*$ is a F- σ- RG on Ω^*.*

Proposition 2.9: [14] $\sigma^r(\mathfrak{T}^*)$ is a smallest F- σ- RG on Ω^* that include $\mathfrak{T}^*$.

3. The results:

Definition 3.1: A class $\mathcal{K}^*$ is named fuzzy $\mathcal{P}^*$- algebra or (fuzzy $\mathcal{P}^*$-field) on Ω^*, if the following requirements are holds:

1. $\Phi^*\epsilon\mathcal{K}^*$.
2. If $\{\mathcal{D}_{\mathrm{m}}\}_{m\epsilon\mathbb{N}}\ \epsilon\mathcal{K}^*$, then $\bigcup_{m=1}^{\infty}\mathcal{D}_{\mathrm{m}}\ \epsilon\mathcal{K}^*$.
3. If $\mathcal{D}\ \&\ \mathcal{F}\epsilon\mathcal{K}^*$, then $\mathcal{D}\cap\mathcal{F}\ \epsilon\mathcal{K}^*$.

An ordered pair ($\Omega^*, \mathcal{K}^*$) is named fuzzy measurable space with respect to fuzzy $\mathcal{P}^*$- algebra.

Example 3.2: Assume $\Omega = \{ a, e, d \}$ &

$$\mathcal{K}^* = \left\{\begin{matrix}\Phi^* = \{(a,0),(e,0),(d,0)\},\{(a,0.1),(e,0.4),(d,0.5)\},\\ \{(a,0.6),(e,0.3),(d,0.8)\},\{(a,0.6),(e,0.4),(d,0.8)\},\\ \{(a,0.1),(e,0.3),(d,0.5)\}\end{matrix}\right\},\ \text{then}$$

it is clear that $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG on Ω^*.

Proposition 3.3: Assume $\{\mathcal{K}_\delta^*\}_{\delta\epsilon\mathrm{I}}$ is a class of fuzzy $\mathcal{P}^*$- algebra on Ω^*. Then so is $\bigcap_{\delta\epsilon\mathrm{I}}\mathcal{K}_\delta^*$.

Proof: If $\bigcap_{\delta\epsilon\mathrm{I}}\mathcal{K}_\delta^* = \{\Phi^*\}$, then we are done. So, we assume $\bigcap_{\delta\epsilon\mathrm{I}}\mathcal{K}_\delta^* \neq \{\Phi^*\}$. Since $\mathcal{K}_\delta^*$ is F- $\mathcal{P}^*$- ALG on $\Omega^*\ \forall\delta\epsilon$I, then $\Phi^*\epsilon\,\mathcal{K}_\delta^*\ \forall\delta\epsilon$I, which implies that $\Phi^*\epsilon\bigcap_{\delta\epsilon\mathrm{I}}\mathcal{K}_\delta^*$. Let $\mathcal{D}, \mathcal{F}\epsilon\bigcap_{\delta\epsilon\mathrm{I}}\mathcal{K}_\delta^*$. Then $\mathcal{D}, \mathcal{F}\epsilon\,\mathcal{K}_\delta^*\ \forall\delta\epsilon$I. Since $\mathcal{K}_\delta^*$ is F- $\mathcal{P}^*$- ALG on $\Omega^*\ \forall\delta\epsilon$I, then $(\mathcal{D}\cap\mathcal{F})\epsilon\,\mathcal{K}_\delta^*\ \forall\delta\epsilon$I and hence $(\mathcal{D}\cap\mathcal{F})\epsilon\bigcap_{\delta\epsilon\mathrm{I}}\mathcal{K}_\delta^*$. Let $\mathcal{D}_1, \mathcal{D}_2, \dots \epsilon\bigcap_{\delta\epsilon\mathrm{I}}\mathcal{K}_\delta^*$, then $\mathcal{D}_1, \mathcal{D}_2, \dots \epsilon\mathcal{K}_\delta^*\ \forall\delta\epsilon$I and $\mathcal{D}_{\mathrm{m}} = \{(\omega, \mathfrak{f}_{\mathcal{D}_{\mathrm{m}}}(\omega)), \forall\omega\epsilon\Omega\}$, $(\forall\mathrm{m} = 1,2, \dots)$. Since $\mathcal{K}_\delta^*$ is F- $\mathcal{P}^*$- ALG on $\Omega^*\ \forall\delta\epsilon$I, then $\{(\omega, sup_{m=1,2,\dots}\{\mathfrak{f}_{\mathcal{D}_{\mathrm{m}}}(\omega)\}) : \omega\epsilon\Omega\}\epsilon\mathcal{K}_\delta^*\ \ \forall\delta\epsilon$I. But $\bigcup_{\mathrm{m}=1}^{\infty}\mathcal{D}_{\mathrm{m}} = \{(\omega, sup_{m=1,2,\dots}\{\mathfrak{f}_{\mathcal{D}_{\mathrm{m}}}(\omega)\}) :$

$\omega\epsilon\Omega\}$ which is implies that $\bigcup_{m=1}^{\infty}\mathcal{D}_m \epsilon\mathcal{K}_\delta^*$ $\forall\delta\epsilon I$ and hence $\bigcup_{m=1}^{\infty}\mathcal{D}_m \epsilon \bigcap_{\delta\epsilon I}\mathcal{K}_\delta^*$ .Therefore, $\bigcap_{\delta\epsilon I}\mathcal{K}_\delta^*$ is F- $\mathcal{P}^*$- ALG on Ω^*.

Definition 3.4: Assume $\mathfrak{T}^*$ is a class of fuzzy subsets of a universal fuzzy set Ω^*. Then the intersection of all F- $\mathcal{P}^*$- ALG on Ω^*that contain $\mathfrak{T}^*$ is named F- $\mathcal{P}^*$- ALG on Ω^* which generated by $\mathfrak{T}^*$ and its denoted by $\mathcal{P}^*(\mathfrak{T}^*)$.

Proposition 3.5: $\mathcal{P}^*(\mathfrak{T}^*)$ is a smallest F- $\mathcal{P}^*$- ALG on Ω^* which contain $\mathfrak{T}^*$.

Proof: Since $\mathcal{P}^*(\mathfrak{T}^*)$ is intersection of all F- $\mathcal{P}^*$- ALG on Ω^*that contain $\mathfrak{T}^*$. Then $\mathcal{P}^*(\mathfrak{T}^*)$ is F- $\mathcal{P}^*$- ALG on Ω^* and $\mathfrak{T}^* \subseteq \mathcal{P}^*(\mathfrak{T}^*)$. Now, if $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG on Ω^* which contains $\mathfrak{T}^*$, so

$$\mathcal{P}^*(\mathfrak{T}^*) = \bigcap_{\delta\epsilon I}\{\mathcal{K}_\delta^* : \mathcal{K}_\delta^* \text{ is a F} - \mathcal{P}^* - \text{ ALG on } \Omega^* \& \mathcal{K}_\delta^* \supseteq \mathfrak{T}^*\} \subseteq \mathcal{K}^*,$$

hence $\mathcal{P}^*(\mathfrak{T}^*)$ is smallest F- $\mathcal{P}^*$- ALG on Ω^* which contains $\mathfrak{T}^*$.

Theorem 3.6: *Every F- σ- ALG on Ω^* is a F- $\mathcal{P}^*$- ALG on Ω^*.*

Proof: From the definition of F- σ- ALG and by Remark (2.4), the proof is follows.

While the reverse is not true, if we take Example(3.2), we note $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG on Ω^*, but $\mathcal{K}^*$ is not F- σ- ALG on Ω^* because $\{(a, 0.1), (e, 0.4), (d, 0.5)\} \epsilon\mathcal{K}^*$ while

$$(\{(a, 0.1), (e, 0.4), (d, 0.5)\})^c = \{(a, 0.9), (e, 0.6), (d, 0.5)\} \notin \mathcal{K}^*.$$

Proposition 3.7: $\mathcal{P}^*(\mathfrak{T}^*) \subseteq \sigma(\mathfrak{T}^*)$.

Proof: From Proposition(2.6) we have $\sigma(\mathfrak{T}^*)$ is a F- σ- ALG on Ω^* that include $\mathfrak{T}^*$and by Theorem (3.6) we have every F- σ- ALG is F- $\mathcal{P}^*$- ALG, so we get $\sigma(\mathfrak{T}^*)$ is F- $\mathcal{P}^*$- ALG that include $\mathfrak{T}^*$. But $\mathcal{P}^*(\mathfrak{T}^*)$ is the smallest F- $\mathcal{P}^*$- ALG that include $\mathfrak{T}^*$, implies that $\mathcal{P}^*(\mathfrak{T}^*) \subseteq \sigma(\mathfrak{T}^*)$.

Proposition 3.8: If $\mathcal{K}^*$ is F- σ- RG on Ω^*, then $\bigcap_{m=1}^{\mathfrak{n}}\mathcal{D}_m \epsilon\mathcal{K}^*$ whenever $\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_\mathfrak{n}\epsilon\mathcal{K}^*$.

Proof: For any F- σ- RG $\mathcal{K}^*$on Ω^*, we have $\Phi^*\epsilon\mathcal{K}^*$. Put $\mathcal{D}_\ell = \Phi^*\ \forall \ell > \mathfrak{n}$, then $\bigcup_{m=1}^{\mathfrak{n}}\mathcal{D}_m = \bigcup_{m=1}^{\infty}\mathcal{D}_m \epsilon\mathcal{K}^*$ since $\mathcal{K}^*$ is F- σ- RG & $\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_\mathfrak{n}\epsilon\mathcal{K}^*$ and $\mathcal{D}_\ell = \Phi^*\epsilon\mathcal{K}^*$. Now, put $\mathcal{D} = \bigcup_{m=1}^{\mathfrak{n}}\mathcal{D}_m$, then $\bigcap_{m=1}^{\mathfrak{n}}\mathcal{D}_m = \mathcal{D} - \bigcup_{m=1}^{\mathfrak{n}}(\mathcal{D} - \mathcal{D}_m)$.

Since $\mathcal{D} \& \mathcal{D}_m\epsilon\mathcal{K}^*$, then $\mathcal{D} - \mathcal{D}_m \epsilon\mathcal{K}^*\ \forall m = 1,2,..,\mathfrak{n}$, so $\bigcup_{m=1}^{\mathfrak{n}}(\mathcal{D} - \mathcal{D}_m) \epsilon\mathcal{K}^*$, hence $\mathcal{D} - \bigcup_{m=1}^{\mathfrak{n}}(\mathcal{D} - \mathcal{D}_m) \epsilon\mathcal{K}^*$, thus $\bigcap_{m=1}^{\mathfrak{n}}\mathcal{D}_m \epsilon\mathcal{K}^*$.

Proposition 3.9: Every F- σ- RG on Ω^* is F- $\mathcal{P}^*$- ALG on Ω^*.

Proof: The proof direct by definition of F- σ- RG and by Proposition (3.8).

The converse of Proposition (3.9) is not true, if we take Example (3.2). then $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG on Ω^*. Put $\mathcal{F} = \{(a, 0.6), (e, 0.3), (d, 0.8)\}$ and $\mathcal{D} = \{(a, 0.1), (e, 0.3), (d, 0.5)\}$, then $\mathcal{F}$ & $\mathcal{D} \epsilon \mathcal{K}^*$. Now,

$$\begin{aligned}\mathcal{F} - \mathcal{D} &= \{(a, 0.6), (e, 0.3), (d, 0.8)\} - \{(a, 0.1), (e, 0.3), (d, 0.5)\} \\ &= \{(a, 0.6), (e, 0.3), (d, 0.8)\} \cap (\{(a, 0.1), (e, 0.3), (d, 0.5)\})^c \\ &= \{(a, 0.6), (e, 0.3), (d, 0.8)\} \cap \{(a, 0.9), (e, 0.7), (d, 0.5)\} \\ &= \{(a, 0.6), (e, 0.3), (d, 0.5)\}\end{aligned}$$

Hence, $\mathcal{F} - \mathcal{D} = \{(a, 0.6), (e, 0.3), (d, 0.5)\} \notin \mathcal{K}^*$, thus $\mathcal{K}^*$ is not F- σ-RG on Ω^*.

Theorem 3.10: $\mathcal{P}^*(\mathfrak{T}^*) \subseteq \sigma^r(\mathfrak{T}^*)$.

Proof: The results observe by Proposition (2.9) & Proposition (3.9) and Proposition (3.5).

Proposition 3.11: If $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG on Ω^* & $\mathcal{D} \subseteq \Omega^*$. Then $\mathcal{K}_1^* = \{\mathcal{F} \subset \Omega^* : \mathcal{F} \supset \mathcal{D} \cap \mathcal{S}$, for some $\mathcal{S} \epsilon \mathcal{K}^*\}$ is F- $\mathcal{P}^*$- ALG on Ω^*.

Proof: Since $\Phi^* \epsilon \mathcal{K}^*$ & $\Phi^* \subset \Omega^*$ and $\mathcal{D} \cap \Phi^* \subseteq \Phi^*$, so $\Phi^* \epsilon \mathcal{K}_1^*$.

Let $\mathcal{F}_1, \mathcal{F}_2 \epsilon \mathcal{K}_1^*$. Then $\exists \mathcal{S}_1, \mathcal{S}_2 \epsilon \mathcal{K}^* \ni \mathcal{F}_1 \supset \mathcal{D} \cap \mathcal{S}_1$ & $\mathcal{F}_2 \supset \mathcal{D} \cap \mathcal{S}_2$. So $\mathcal{F}_1 \cap \mathcal{F}_2 \supset (\mathcal{D} \cap \mathcal{S}_1) \cap (\mathcal{D} \cap \mathcal{S}_2) = \mathcal{D} \cap (\mathcal{S}_1 \cap \mathcal{S}_2)$. Now, $\mathcal{S}_1, \mathcal{S}_2 \epsilon \mathcal{K}^*$ & $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG, then $\mathcal{S}_1 \cap \mathcal{S}_2 \epsilon \mathcal{K}^*$, thus $\mathcal{F}_1 \cap \mathcal{F}_2 \epsilon \mathcal{K}_1^*$.

Assume $\mathcal{F}_1, \mathcal{F}_2, \ldots \epsilon \mathcal{K}_1^*$. thus $\exists\ \mathcal{S}_n \epsilon \mathcal{K}^* \ni \mathcal{F}_n \supset \mathcal{D} \cap \mathcal{S}_n\ \forall$ n=1,2,… So, we have $\bigcup_{n=1}^{\infty} \mathcal{F}_n \supset \bigcup_{n=1}^{\infty} (\mathcal{D} \cap \mathcal{S}_n) = \mathcal{D} \cap (\bigcup_{n=1}^{\infty} \mathcal{S}_n)$. But $\mathcal{S}_n \epsilon \mathcal{K}^*$ & $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG, so $\bigcup_{n=1}^{\infty} \mathcal{S}_n \epsilon \mathcal{K}^*$, thus $\bigcup_{n=1}^{\infty} \mathcal{F}_n \epsilon \mathcal{K}_1^*$. Therefore $\mathcal{K}_1^*$is F- $\mathcal{P}^*$- ALG on Ω^*.

Proposition 3.12: If $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG on Ω^* & $\mathcal{D} \subseteq \Omega^*$. Then $\mathcal{K}_1^* = \{\mathcal{F} \subseteq \Omega^* : \mathcal{F} = \mathcal{D} \cap \mathcal{S}, \mathcal{S} \epsilon \mathcal{K}^*\}$ is F- $\mathcal{P}^*$- ALG on Ω^*.

Proof: Since $\mathcal{K}^*$ is F- $\mathcal{P}^*$- ALG on Ω^* & $\mathcal{D} \subseteq \Omega^*$, then the proof clearly from definition of $\mathcal{K}_1^* = \{\mathcal{F} \subseteq \Omega^* : \mathcal{F} = \mathcal{D} \cap \mathcal{S}, \mathcal{S} \epsilon \mathcal{K}^*\}$.

Remark 3.13: The class $\mathcal{K}_1^*$ in Proposition(3.12) is called relative fuzzy $\mathcal{P}^*$-algebra.

Conclusion

In this article, another generalization of fuzzy σ-algebra is introduced which is fuzzy $\mathcal{P}^*$-algebra. Section three is devoted to study the properties of this notion and investigated some propositions and theorems related to this concept.

The most important results we obtained are:

1. The intersection of any family of fuzzy $\mathcal{P}^*$-algebra is also fuzzy $\mathcal{P}^*$-algebra.
2. $\mathcal{P}^*(\mathfrak{T}^*)$ is a smallest F- $\mathcal{P}^*$- ALG on Ω^* which contain $\mathfrak{T}^*$.

3. Every F- σ - ALG is F- $\mathcal{P}^*$- ALG.
4. Every F- σ - RG on Ω^* is F- $\mathcal{P}^*$- ALG on Ω^*.
5. $\mathcal{P}^*(\mathfrak{T}^*) \subseteq \sigma^r(\mathfrak{T}^*) \subseteq \sigma(\mathfrak{T}^*)$.

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