

Results of fuzzy soft symmetric matrix

Ihab Hadi Jumaah *

Alaa Jumaah Edan [†]

Lina Ayden Hussein [§]

Ali Ibrahim Mansour [‡]

Department of Mathematics

College of Education

Mustansiriyah University

Baghdad

Iraq

Abstract

In this research, we presented the notions of fuzzy soft matrix denoted by (FSM) and we provided the definition of (FSM) and we created some features the study of the definition of transpose of (FSM) and then some properties of transpose of (FSM) after that symmetric (FSM). The concepts are shown with numerical examples.

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1. Introduction

After Zadeh initially proposed the idea of fuzzy sets [1], some authors studied fuzzy metric spaces ([7]-[9]). In 1999 [10], the soft sets theory is given by Molodtsov. Also, a number of authors brought up the same idea as fuzzy sets, soft sets and neutrosophic sets in different ways ([11]-[14]). According to Tarrannum and Narappanavar [3], every soft induces a fuzzy soft [2]. With the support of Matrix, they altered the fuzzy soft notion. Numerous writers, including ([4]-[6]) In this study, (FSM) will be taken into consideration, and some of its related properties will be examined.

* E-mail: Ihabhadi5@uomustansiriyah.edu.iq (Corresponding Author)

[†] E-mail: alaaiedan@uomustansiriyah.edu.iq

[§] E-mail: linaaydeenhussein@gmail.com

[‡] E-mail: aliibrahimaltaay@uomustansiriyah.edu.iq

2. Definitions and Preliminaries

Definition 1: [2] A pair (F, E) is called a soft set over (U) iff U is a universe set and E is a collection of parameters. $P(U)$ defines the power set for U . Then F is a mapping given by $F: E \rightarrow P(U)$.

Definition 2: [15, 16] Suppose that U be universe set of discourse and E be the set of parameters I^u be the set of each fuzzy sets on U , let $A \subseteq E$ and $f: A \rightarrow I^u$. A pair (f_A, A) is called fuzzy soft over U .

Definition 3: Assume that $A \subseteq E$, E is the collection of parameters, and U is a universe set. Consider (FSS) the fuzzy soft set (f_A, E) over U . So, a subset of $U \times E$ is specifically defined as $R_A = \{(u, e): e \in A, u \in f_A(e)\}$. This is known as the relation of (f_A, e) where $M_{R_A}: U \times E \rightarrow [0, 1]$ represents the membership function. Here, $M_{R_A}: (u, e) \in [0, 1]$ represents membership $u \in U$ and $e \in E$ if $(M_{ij} = M_{R_A}(u_i, e_j))$.

Which is called an $m \times n$ matrix $[M_{ij}] = \begin{bmatrix} M_{11} & M_{12} & \dots & M_{1n} \\ M_{21} & M_{22} & \dots & M_{2n} \\ \dots & \dots & \dots & \dots \\ M_{m1} & M_{m2} & \dots & M_{mn} \end{bmatrix}$

A matrix $[M_{ij}]$, often known as a $m \times n$ (FSM) of FSS (f_A, e) over U , is defined. Therefore, we may use the matrix $[M_{ij}]_{m \times n}$ to uniquely characterize that FSS.

Example 1: Consider $U = \{\ddot{u}_1, \ddot{u}_2, \ddot{u}_3, \ddot{u}_4, \ddot{u}_5, \ddot{u}_6\}$ consisting of a set of parameters with respect to U where each parameter $\{\ddot{e}_1, \ddot{e}_2, \ddot{e}_3, \ddot{e}_4\} = E$ if $A = \{\ddot{e}_1, \ddot{e}_2, \ddot{e}_3\} \subseteq E$ and $f_A(\ddot{e}_1) = \{(\ddot{u}_1, 0.4), (\ddot{u}_2, 0.3), (\ddot{u}_3, 0.6), (\ddot{u}_4, 0.5)\}$, $f_A(\ddot{e}_2) = \{(\ddot{u}_1, 0.2), (\ddot{u}_2, 0.3), (\ddot{u}_3, 0.6), (\ddot{u}_4, 0.3)\}$, $f_A(\ddot{e}_3) = \{(\ddot{u}_1, 0.6), (\ddot{u}_2, 0.5), (\ddot{u}_3, 0.3), (\ddot{u}_4, 0.7)\}$. So, the fuzzy set (f_A, e) is the collection of parameters $\{f_A(\ddot{e}_1), f_A(\ddot{e}_2), f_A(\ddot{e}_3)\}$. Hence, the (FSM) can be written as

$$M_{ij} = \begin{bmatrix} 0.4 & 0.2 & 0.6 \\ 0.3 & 0.3 & 0.5 \\ 0.6 & 0.6 & 0.3 \\ 0.5 & 0.3 & 0.7 \end{bmatrix}$$

Definition 4: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \tilde{B} = [M_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. This relationship is calculated by $\tilde{A} \cup \tilde{B} = [\max(M_{ij}^{\tilde{A}}, M_{ij}^{\tilde{B}})]$, $\forall j$ and i .

Definition 5: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \tilde{B} = [M_{ij}^{\tilde{B}}] \in FSM_{m \times n}$. This relationship is calculated by $\tilde{A} \cap \tilde{B} = [\min(M_{ij}^{\tilde{A}}, M_{ij}^{\tilde{B}})]$, $\forall i$ and j .

Definition 6: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \tilde{B} = [M_{ij}^{\tilde{B}}], \in FSM_{m \times n}$. Then complement of $\tilde{A}$ and $\tilde{B}$ represented by $\tilde{A}^c$. This relationship is calculated by $\tilde{A}^c = [V_{ij}^{\tilde{A}}]$ such that $V_{ij}^{\tilde{A}} = 1 - M_{ij}^{\tilde{A}}, \forall i$ and j .

Definition 7: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \tilde{B} = [M_{ij}^{\tilde{B}}], \in FSM_{m \times n}$. So maximum-minimum product of two. (FSM) is referred to $\tilde{A}\tilde{B} = [\max(\min(M_{ij}^{\tilde{A}}, M_{ij}^{\tilde{B}}))], \forall i$ and j .

Definition 8: Let $\tilde{A} = [M_{ij}^{\tilde{A}}] \in FSM_{m \times n}$ and k be any scalar then the scalar multiplication of (FSM) $k\tilde{A}$ is defined as $k\tilde{A} = [kM_{ij}^{\tilde{A}}]$ where $1 \geq k \geq 0, \forall j$ and i .

Definition 9: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \tilde{B} = [M_{ij}^{\tilde{B}}], \in FSM_{m \times n}$. Then

$$\tilde{A} @ \tilde{B} = \{x, \frac{M_{ij}^{\tilde{A}}(x) + M_{ij}^{\tilde{B}}(x)}{2} : x \in X\}$$

Definition 10: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \tilde{B} = [M_{ij}^{\tilde{B}}], \in FSM_{m \times n}$. Then

$$\tilde{A} \$ \tilde{B} = \{x, \sqrt{M_{ij}^{\tilde{A}}(x)M_{ij}^{\tilde{B}}(x)} : x \in X\}$$

Definition 11: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \in FSM$. Then the transpose of $\tilde{A}$ denoted by $\tilde{A}^T$. This relationship is calculated by $\tilde{A}^T = [M_{ji}^{\tilde{A}}] \forall i$ and j .

3. properties of transpose of (FSM)

If $\tilde{A}, \tilde{B}$ and $\tilde{C}$ be three (FSM) if the following requirements are satisfied:

- (1): $(\tilde{A} \cup \tilde{B})^T = \tilde{A}^T \cup \tilde{B}^T$,
- (2): $(\tilde{A} \cap \tilde{B})^T = \tilde{A}^T \cap \tilde{B}^T$,
- (3): $(\tilde{A}\tilde{B})^T = \tilde{B}^T \tilde{A}^T$.
- (4): $k\tilde{A}^T = k\tilde{A}^T$,
- (5): $((\tilde{A} \cup \tilde{B}) @ \tilde{C})^T = (\tilde{A} @ \tilde{C})^T \cup (\tilde{B} @ \tilde{C})^T$,
- (6): $((\tilde{A} \cap \tilde{B}) @ \tilde{C})^T = (\tilde{A} @ \tilde{C})^T \cap (\tilde{B} @ \tilde{C})^T$.

Proof:

1: Let $\tilde{A} = [M_{ij}^{\tilde{A}}], \tilde{B} = [M_{ij}^{\tilde{B}}], \tilde{C} = [M_{ij}^{\tilde{C}}]$ be three (FSM). Then

$$\tilde{A} \cup \tilde{B} = [\max(M_{ij}^{\tilde{A}}, M_{ij}^{\tilde{B}})] \Rightarrow (\tilde{A} \cup \tilde{B})^T = [\max(M_{ji}^{\tilde{A}}, M_{ji}^{\tilde{B}})]. \text{ Again } \tilde{A}^T = [M_{ji}^{\tilde{A}}] \text{ and } \tilde{B}^T = [M_{ji}^{\tilde{B}}] \Rightarrow \tilde{A}^T \cup \tilde{B}^T = [\max(M_{ji}^{\tilde{A}}, M_{ji}^{\tilde{B}})].$$

2: $\tilde{A} \cap \tilde{B} = [\min(M_{ij}^{\tilde{A}}, M_{ij}^{\tilde{B}})] \Rightarrow (\tilde{A} \cap \tilde{B})^T = [\min(M_{ji}^{\tilde{A}}, M_{ji}^{\tilde{B}})]$. Again $\tilde{A}^T = [M_{ji}^{\tilde{A}}]$ and $\tilde{B}^T = [M_{ji}^{\tilde{B}}] \Rightarrow \tilde{A}^T \cap \tilde{B}^T = [\min(M_{ji}^{\tilde{A}}, M_{ji}^{\tilde{B}})]$

4: Let $\tilde{A} = [M_{ij}^{\tilde{A}}]$ then $k\tilde{A} = [kM_{ij}^{\tilde{A}}]$

$$(k\tilde{A})^T = kM_{ji}^{\tilde{A}} \Rightarrow k\tilde{A}^T = (kM_{ji}^{\tilde{A}}). \text{ Hence } (k\tilde{A})^T = k\tilde{A}^T$$

5: $\tilde{A} \cup \tilde{B} @ \tilde{C} = M_{ij}^{\tilde{A}} @ M_{ij}^{\tilde{C}}$ if $M_{ij}^{\tilde{A}} \geq M_{ij}^{\tilde{B}}$. Then $\tilde{A} \cup \tilde{B} @ \tilde{C} = [\frac{M_{ij}^{\tilde{A}} + M_{ij}^{\tilde{C}}}{2}] \Rightarrow$

$$((\tilde{A} \cup \tilde{B}) @ \tilde{C})^T = [\frac{M_{ji}^{\tilde{A}} + M_{ji}^{\tilde{C}}}{2}] \Rightarrow (\tilde{A} @ \tilde{C})^T = [\frac{M_{ji}^{\tilde{A}} + M_{ji}^{\tilde{C}}}{2}] \Rightarrow (\tilde{B} @ \tilde{C})^T = [\frac{M_{ji}^{\tilde{B}} + M_{ji}^{\tilde{C}}}{2}]$$

$$M_{ij}^{\tilde{A}} + M_{ij}^{\tilde{C}} \geq M_{ij}^{\tilde{B}} + M_{ij}^{\tilde{C}} \Rightarrow (\tilde{A} @ \tilde{C})^T \cup (\tilde{B} @ \tilde{C})^T = [\frac{M_{ji}^{\tilde{A}} + M_{ji}^{\tilde{C}}}{2}]. \text{ Let us chose}$$

k s.t $k \in [0, 1]$ then we have, $k\tilde{A} = \begin{bmatrix} 0.05 & 0.25 & 0.15 \\ 0.25 & 0.1 & 0.15 \\ 0.15 & 0.25 & 0.2 \end{bmatrix}$, $(k\tilde{A})^T = \begin{bmatrix} 0.05 & 0.25 & 0.15 \\ 0.25 & 0.1 & 0.25 \\ 0.15 & 0.25 & 0.2 \end{bmatrix}$, $k\tilde{A}^T = \begin{bmatrix} 0.05 & 0.25 & 0.15 \\ 0.25 & 0.1 & 0.25 \\ 0.15 & 0.25 & 0.2 \end{bmatrix}$. Let us consider three (FSM) $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ as

$$\tilde{A} = \begin{bmatrix} 0.1 & 0.5 & 0.3 \\ 0.4 & 0.2 & 0.5 \\ 0.5 & 0.3 & 0.6 \\ 0.7 & 0.6 & 0.5 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0.5 & 0.1 & 0.7 \\ 0.8 & 0.4 & 0.5 \\ 0.2 & 0.3 & 0.4 \\ 0.1 & 0.2 & 0.5 \end{bmatrix}, \tilde{C} = \begin{bmatrix} 0.1 & 0.1 & 0.6 \\ 0.4 & 0.2 & 0.5 \\ 0.2 & 0.3 & 0.4 \\ 0.1 & 0.2 & 0.5 \end{bmatrix}$$

Then

$$\tilde{A} \cup \tilde{B} = \begin{bmatrix} 0.5 & 0.5 & 0.7 \\ 0.8 & 0.4 & 0.5 \\ 0.5 & 0.3 & 0.6 \\ 0.5 & 0.3 & 0.5 \end{bmatrix} (\tilde{A} \cup \tilde{B}) @ \tilde{C} = \begin{bmatrix} 0.3 & 0.3 & 0.5 \\ 0.6 & 0.3 & 0.5 \\ 0.35 & 0.3 & 0.5 \\ 0.4 & 0.4 & 0.5 \end{bmatrix}. \text{ Hence}$$

$$((\tilde{A} \cup \tilde{B}) @ \tilde{C})^T = \begin{bmatrix} 0.3 & 0.6 & 0.35 & 0.4 \\ 0.3 & 0.3 & 0.3 & 0.4 \\ 0.5 & 0.5 & 0.5 & 0.4 \end{bmatrix}$$

$$\text{Again } \tilde{A} @ \tilde{C} = \begin{bmatrix} 0.1 & 0.3 & 0.3 \\ 0.4 & 0.2 & 0.5 \\ 0.35 & 0.3 & 0.5 \\ 0.4 & 0.4 & 0.5 \end{bmatrix} \text{ and } \tilde{B} @ \tilde{C} = \begin{bmatrix} 0.3 & 0.1 & 0.5 \\ 0.6 & 0.3 & 0.5 \\ 0.2 & 0.3 & 0.4 \\ 0.1 & 0.2 & 0.5 \end{bmatrix}. \text{ Then}$$

$$(\tilde{A} @ \tilde{C})^T = \begin{bmatrix} 0.1 & 0.4 & 0.4 & 0.45 \\ 0.3 & 0.2 & 0.3 & 0.4 \\ 0.3 & 0.5 & 0.5 & 0.5 \end{bmatrix}, (\tilde{B} @ \tilde{C})^T = \begin{bmatrix} 0.3 & 0.6 & 0.25 & 0.15 \\ 0.1 & 0.3 & 0.3 & 0.2 \\ 0.5 & 0.5 & 0.4 & 0.5 \end{bmatrix}$$

$$\text{Hence } (\tilde{A} @ \tilde{C})^T \cup (\tilde{B} @ \tilde{C})^T = \begin{bmatrix} 0.3 & 0.6 & 0.35 & 0.4 \\ 0.3 & 0.3 & 0.3 & 0.4 \\ 0.5 & 0.5 & 0.3 & 0.5 \end{bmatrix}, \text{ and}$$

$$(\tilde{A} @ \tilde{C})^T \cap (\tilde{B} @ \tilde{C})^T = \begin{bmatrix} 0.1 & 0.4 & 0.25 & 0.15 \\ 0.1 & 0.2 & 0.3 & 0.2 \\ 0.5 & 0.5 & 0.3 & 0.5 \end{bmatrix}$$

Definition 12: Let $\tilde{A} = [M_{ij}^{\tilde{A}}] \in FSM_{m \times n}$ and if $m = n$ $M_{ij}^{\tilde{A}} = M_{ji}^{\tilde{A}}$ then the matrix $\tilde{A}$ is said to be symmetric (FSM) Stated differently, a matrix $\tilde{A}$ will be considered symmetric

(FSM) if $\tilde{A}^T = \tilde{A}$. For example $\tilde{A} = \begin{bmatrix} 0.1 & 0.5 & 0.5 \\ 0.5 & 0.2 & 0.5 \\ 0.5 & 0.5 & 0.6 \end{bmatrix}$ is a symmetric (FSM)

4. Propositions On Symmetric (FSM)

- 1: if $\tilde{A}$ and $\tilde{B}$ two symmetric (FSM) then $\tilde{A} \cup \tilde{B}$ is again a symmetric (FSM).
- 2: if $\tilde{A}$ and $\tilde{B}$ two symmetric (FSM) then $\tilde{A} \cap \tilde{B}$ is again a symmetric (FSM).
- 3: if $\tilde{A}$ be a symmetric (FSM) then $k\tilde{A}$ is again a symmetric (FSM)
- 4: if $\tilde{A}$ and $\tilde{B}$ two symmetric (FSM) then $\tilde{A} @ \tilde{B}$ is again a symmetric (FSM).
- 5: if $\tilde{A}$ and $\tilde{B}$ two symmetric (FSM) then $\tilde{A} \$ \tilde{B}$ is again a symmetric (FSM).
- 6: if $\tilde{A}$ and $\tilde{B}$ two symmetric (FSM) then $\tilde{A}^T \tilde{B} \tilde{A}$ is again a symmetric (FSM)
- 7: if $\tilde{A}, \tilde{B}$ and $\tilde{C}$ are symmetric (FSM) then $(\tilde{A} \cup \tilde{B}) \$ \tilde{C}$ is again a symmetric (FSM)
- 8: if $\tilde{A}, \tilde{B}$ and $\tilde{C}$ are symmetric (FSM) then $(\tilde{A} \cap \tilde{B}) \$ \tilde{C}$ is again a symmetric (FSM)
- 9: if $\tilde{A}, \tilde{B}$ and $\tilde{C}$ are symmetric (FSM) then $(\tilde{A} @ \tilde{B}) \$ \tilde{C}$ is again a symmetric (FSM)

Proof: 1: if $\tilde{A}$ and $\tilde{B}$ be two symmetric (FSM) then $\tilde{A}^T = \tilde{A}$ and $\tilde{B}^T = \tilde{B}$ now we are to show that $\tilde{A} \cup \tilde{B}$ is again a symmetric (FSM). We have $(\tilde{A} \cup \tilde{B})^T = \tilde{A}^T \cup \tilde{B}^T$

Since $\tilde{A}^T = \tilde{A}$ and $\tilde{B}^T = \tilde{B}$ Then $\tilde{A}^T \cup \tilde{B}^T = \tilde{A} \cup \tilde{B}$ which implies that $(\tilde{A} \cup \tilde{B})^T = (\tilde{A} \cup \tilde{B})$

Example 3: Let $\tilde{A}, \tilde{B}$, and $\tilde{C}$ be three symmetric (FSM), s.t

$$\tilde{A} = \begin{bmatrix} 0.1 & 0.5 & 0.3 \\ 0.5 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.6 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0.5 & 0.6 & 0.8 \\ 0.6 & 0.7 & 0.4 \\ 0.8 & 0.4 & 0.8 \end{bmatrix}, \tilde{C} = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.5 & 0.6 & 0.4 \\ 0.6 & 0.4 & 0.7 \end{bmatrix}$$

$$\tilde{A} \cup \tilde{B} = \begin{bmatrix} 0.55 & 0.8 & 0.86 \\ 0.8 & 0.76 & 0.7 \\ 0.86 & 0.7 & 0.92 \end{bmatrix}, (\tilde{A} \cup \tilde{B})^T = \begin{bmatrix} 0.55 & 0.8 & 0.86 \\ 0.8 & 0.76 & 0.7 \\ 0.86 & 0.7 & 0.92 \end{bmatrix}$$

From which clear $\tilde{A} \cup \tilde{B} = (\tilde{A} \cup \tilde{B})^T$

4: Let $\tilde{A} = [M_{ij}^A], \tilde{B} = [M_{ij}^B]$ be two symmetric (FSM). Then it is proved that $(\tilde{A} @ \tilde{B})^T = \tilde{A} @ \tilde{B}$. Here $\tilde{A} @ \tilde{B} = \frac{M_{ij}^A + M_{ij}^B}{2}$ and $=(\tilde{A} @ \tilde{B})^T \frac{M_{ji}^A + M_{ji}^B}{2}$. Since A and B are two symmetric (FSM) then $M_{ij}^A = M_{ji}^A$. Then $(\tilde{A} @ \tilde{B})^T = \tilde{A} @ \tilde{B}$

Example 4: $\tilde{A} @ \tilde{B} = \begin{bmatrix} 0.3 & 0.55 & 0.55 \\ 0.55 & 0.45 & 0.45 \\ 0.55 & 0.45 & 0.7 \end{bmatrix}, (\tilde{A} @ \tilde{B})^T = \begin{bmatrix} 0.3 & 0.55 & 0.55 \\ 0.55 & 0.45 & 0.45 \\ 0.55 & 0.45 & 0.7 \end{bmatrix}.$

Example 5: $\tilde{A} \$ \tilde{B} = \begin{bmatrix} \sqrt{0.5} & \sqrt{0.30} & \sqrt{0.24} \\ \sqrt{0.30} & \sqrt{0.14} & \sqrt{0.20} \\ \sqrt{0.24} & \sqrt{0.20} & \sqrt{0.48} \end{bmatrix}$. Here

$$(\tilde{A} \$ \tilde{B})^T = \begin{bmatrix} \sqrt{0.5} & \sqrt{0.30} & \sqrt{0.24} \\ \sqrt{0.30} & \sqrt{0.14} & \sqrt{0.20} \\ \sqrt{0.24} & \sqrt{0.20} & \sqrt{0.48} \end{bmatrix}$$

This implies $\tilde{A} \$ \tilde{B} = (\tilde{A} \$ \tilde{B})^T$ and hence $A \$ B$ is a again a symmetric (FSM).

Example 6: $\tilde{A} = \begin{bmatrix} 0.1 & 0.5 & 0.3 \\ 0.5 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.6 \end{bmatrix}$ then $\tilde{A}^T = \begin{bmatrix} 0.1 & 0.5 & 0.3 \\ 0.5 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.6 \end{bmatrix}$ be a symmetric (FSM) then $\tilde{A} \tilde{A}^T$ is to be calculated. We have $\tilde{A} \tilde{A}^T = [K_{ij}]$. Since, $k_{11} = 0.5$, $k_{12} = 0.3$, $k_{13} = 0.5$, $k_{21} = 0.3$, $k_{22} = 0.5$, $k_{23} = 0.5$, $k_{31} = 0.5$, $k_{32} = 0.5$, $k_{33} = 0.6$, $\tilde{A} \tilde{A}^T = \tilde{K} = \begin{bmatrix} 0.5 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.6 \end{bmatrix}$. Hence $\tilde{A} \tilde{A}^T$ is also a symmetric (FSM).

Example 7: Let $\tilde{A}$ and $\tilde{B}$ are two symmetric (FSM) then it is to be proved that $\tilde{A}^T \tilde{B} \tilde{A}$ is again a symmetric (FSM). Here let $\tilde{A}^T \tilde{B} = [E_{ij}]$ where $E_{11} = 0.5$, $E_{12} = 0.5$, $E_{13} = 0.4$, $E_{21} = 0.5$, $E_{22} = 0.5$, $E_{23} = 0.5$, $E_{31} = 0.6$, $E_{32} = 0.5$, $E_{33} = 0.6$. Hence $\tilde{A}^T \tilde{B} = \tilde{E} = \begin{bmatrix} 0.5 & 0.5 & 0.4 \\ 0.5 & 0.5 & 0.5 \\ 0.6 & 0.5 & 0.6 \end{bmatrix}$ and again if $\tilde{A}^T \tilde{B} \tilde{A} = [G_{ij}]$ then $G_{11} = 0.5$, $G_{12} = 0.5$, $G_{13} = 0.5$, $G_{21} = 0.5$, $G_{22} = 0.5$, $G_{23} = 0.5$, $G_{31} = 0.5$, $G_{32} = 0.5$, $G_{33} = 0.6$. And thus $\tilde{A}^T \tilde{B} \tilde{A} = \tilde{G} = \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.6 \end{bmatrix}$. Hence $\tilde{A}^T \tilde{B} \tilde{A}$ is again a symmetric (FSM).

Proof: We have $\tilde{A} \cup \tilde{B} = \begin{bmatrix} 0.5 & 0.6 & 0.8 \\ 0.6 & 0.7 & 0.5 \\ 0.8 & 0.5 & 0.8 \end{bmatrix}$ Then it can be obtained that.

$$\tilde{A} \cup \tilde{B} \$ \tilde{C} = \begin{bmatrix} \sqrt{0.15} & \sqrt{0.30} & \sqrt{0.48} \\ \sqrt{0.30} & \sqrt{0.42} & \sqrt{0.20} \\ \sqrt{0.48} & \sqrt{0.20} & \sqrt{0.56} \end{bmatrix}, \text{ If } A, B, \text{ and } C \text{ are symmetric}$$

(FSM) then $\tilde{A} \cup \tilde{B} \$ \tilde{C}$ is again a symmetric (FSM).

9: Let $\tilde{A} = M_{ij}^A$, $\tilde{B} = M_{ij}^B$ and $\tilde{C} = M_{ij}^C$ be three symmetric (FSM)

$$\text{Then } \tilde{A} @ \tilde{B} = \frac{M_{ij}^A + M_{ij}^B}{2} \text{ and } (\tilde{A} @ \tilde{B}) \$ \tilde{C} = \sqrt{\frac{(M_{ij}^A + M_{ij}^B) M_{ij}^C}{2}}$$

Since A, B and C are symmetric (FSM), then

$$\sqrt{\frac{(M_{ij}^A + M_{ij}^B) M_{ij}^C}{2}} = \sqrt{\frac{(M_{ij}^A + M_{ji}^B) M_{ji}^C}{2}}$$

this shows that $(\tilde{A} @ \tilde{B}) \$ \tilde{C}$ is again a symmetric (FSM) for example $\tilde{A} @ \tilde{B} = \begin{bmatrix} 0.3 & 0.55 & 0.55 \\ 0.55 & 0.45 & 0.45 \\ 0.55 & 0.45 & 0.7 \end{bmatrix}$ and then it can be obtained that

$$(\tilde{A} @ \tilde{B}) \$ \tilde{C} = \begin{bmatrix} 0.9 & 0.275 & 0.33 \\ 0.275 & 0.27 & 0.18 \\ 0.33 & 0.18 & 0.49 \end{bmatrix}$$

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