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Result for the series in the partition (12,7,4)

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Abstract

$\mathcal{R}$ is a commute ring and has unity ring, $\mathcal{F}$ free $\mathcal{R}$-module and $\mathcal{D}_i\mathcal{F}$ divided algebra, $\wedge\mathcal{F}$ is the exterior algebra for the partition λ/μ. The series of the complex terms of the partition (12,7,4) studied in this work, by the place polarization injective of maps and Capelli identities.

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1. Introduction

$\mathcal{K}_{\lambda/\mu}\mathcal{F} = \operatorname{Im}(d'_{\lambda/\mu})$, we want the following, [1].

$$\partial_{32}^{(2)}\,\partial_{21} = \partial_{21}\,\partial_{32}^{(2)} + \partial_{32}\,\partial_{31} \qquad \ldots(1)$$

$$\partial_{32}\,\partial_{21}^{(2)} = \partial_{21}^{(2)}\,\partial_{32} + \partial_{21}\,\partial_{31} \qquad \ldots(2)$$

$$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31} \qquad \ldots(3)$$

$$\partial_{32}\,\partial_{31} = \partial_{31}\,\partial_{32} \qquad \ldots(4)$$

We employed the same notion in [2] for the case (12,7,4). Also we can extend that for $\mathcal{PSL}(2,\mathfrak{U})$ and $\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U}$ where $\mathfrak{U}$ = 31, 37, and Zp^nZp^n, Zp^mp^n [3-5], authors in [6-11] study RCLS-Modules, SR-Modules and E-Modules, also we apply the idea for these type.

2. The series with its result

The series of Complex terms for the partition (12,7,4) has the following:

$$\mathcal{D}_{14}\mathcal{F}\otimes\mathcal{D}_{7}\mathcal{F}\otimes\mathcal{D}_{2}\mathcal{F} \longrightarrow \begin{array}{c}\mathcal{D}_{14}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{3}\mathcal{F}\\ \oplus \\ \mathcal{D}_{13}\mathcal{F}\otimes\mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{2}\mathcal{F}\end{array} \longrightarrow \begin{array}{c}\mathcal{D}_{13}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{4}\mathcal{F}\\ \oplus \\ \mathcal{D}_{12}\mathcal{F}\otimes\mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{3}\mathcal{F}\end{array} \longrightarrow \mathcal{D}_{12}\mathcal{F}\otimes D_{7}\,F\otimes D_{4}\,F$$

Consider the following silhouette

$$\begin{tikzcd}
\mathcal{D}_{14}\mathcal{F}\otimes\mathcal{D}_{7}\mathcal{F}\otimes\mathcal{D}_{2}\mathcal{F} \arrow[r, "\mathfrak{n}_1"] \arrow[d, "\mathfrak{m}_1"'] & \mathcal{D}_{13}\mathcal{F}\otimes\mathcal{D}_{8}\mathcal{F}\otimes\mathcal{D}_{2}\mathcal{F} \arrow[d, "\mathfrak{m}_2"] \\
\mathcal{D}_{14}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{3}\mathcal{F} \arrow[r, "\mathfrak{n}_2"] & \mathcal{D}_{13}\mathcal{F}\otimes\mathcal{D}_{6}\mathcal{F}\otimes\mathcal{D}_{4}\mathcal{F}
\end{tikzcd}$$

Silhouette (1)

Where, $\mathfrak{n}_1(x) = \partial_{21}(x)$; $\mathfrak{m}_1(x) = \partial_{32}(x)$, $\mathfrak{m}_2(x) = \partial_{32}^{(2)}(x)$; $\mathcal{F}$, and $\mathfrak{n}_2(x) = \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(x)$.

Proposition 2.1: The commute of silhouette (1).

Proof:

$(\mathfrak{m}_2 \circ \mathfrak{n}_1)(x) = (\partial_{32}^{(2)}\partial_{21})(x) = \left(\partial_{21}\,\partial_{32}^{(2)} + \partial_{31}\partial_{32}\right)(x)$ by (1) and (4)

$$= \left(\left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)\partial_{32}\right)(x)) = (\mathfrak{n}_2 \circ \mathfrak{m}_1)(x)$$

Also, consider the following silhouette

$$\begin{tikzcd}
\mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "\mathfrak{c}_1"] \arrow[d, "\mathfrak{m}_2"'] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "\mathfrak{m}_3"] \\
\mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \arrow[r, "\mathfrak{c}_2"] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_4\mathcal{F}
\end{tikzcd}$$

Silhouette (2)

Where, $\mathfrak{c}_1(x) = \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(x)$; $\mathfrak{m}_3(x) = \partial_{32}(x)$, and $\mathfrak{c}_2(x) = \partial_{21}(x)$.

Proposition 2.2: The commute of silhouette (2).

Proof: $(\mathfrak{m}_3 \circ \mathfrak{c}_1)(x) = \partial_{32}\left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(x)\right) = \left(\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31}\right)(x)$

From (1) we gain $(\mathfrak{m}_3 \circ \mathfrak{c}_1) = (\mathfrak{c}_2 \circ \mathfrak{m}_2)(x)$

Now, consider the next silhouette, where, $\mathcal{b}(x) = \partial_{21}^{(2)}(x)$

$$\begin{tikzcd}
\mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "\mathfrak{n}_1"] \arrow[d, "\mathfrak{m}_1"'] \arrow[dr, phantom, "\mathcal{L}"] & \mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "\mathfrak{c}_1"] \arrow[d, "\mathfrak{m}_2"] \arrow[dr, phantom, "\mathcal{N}"] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "\mathfrak{m}_3"] \\
\mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[r, "\mathfrak{n}_2"'] \arrow[urr, "\mathcal{b}"] & \mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \arrow[r, "\mathfrak{c}_2"'] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_4\mathcal{F}
\end{tikzcd}$$

Silhouette (3)

Proposition 2.3: The commute of silhouette $\mathcal{L}$ in silhouette (3).

Proof: $(\mathcal{b} \circ \mathfrak{m}_1)(x) = \left(\partial_{21}^{(2)}\partial_{32}\right)(x) = \left(\partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31}\right)(x)$ by (3)

$$= \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\left(\partial_{21}(x)\right) = (\mathfrak{c}_1 \circ \mathfrak{n}_1)(x)$$

Proposition 2.4: The commute silhouette $\mathcal{N}$ in silhouette (3).

Proof: $(\mathfrak{m}_3 \circ \mathcal{b})(x) = \left(\partial_{32}\,\partial_{21}^{(2)}\right)(x) = \left(\partial_{21}\left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)\right)(x)$ by (3)

$$= (\mathfrak{c}_2 \circ \mathfrak{n}_2)(x)$$

Finally, consider the series

$$\mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\mathcal{v}_3} \begin{matrix} \mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix} \xrightarrow{\mathcal{v}_2} \begin{matrix} \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \end{matrix} \longrightarrow \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_4\mathcal{F}$$

So, we define the maps $\mathcal{v}_3(u) = (\partial_{21}(u), \partial_{32}(u))$,

$\mathcal{v}_2(u,t) = \left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(u) - \partial_{21}(t), \left(\frac{1}{2}\partial_{21}\,\partial_{32} + \partial_{31}\right)(t) - \partial_{32}^{(2)}(u)\right)$, and $\mathcal{v}_1(u) = \partial_{21}(t) + \partial_{32}(u)$

Proposition 2.5: $\mathrm{Im}(\mathcal{v}_3) \subseteq \ker(\mathcal{v}_2)$

Proof: $(\mathcal{v}_2 \circ \mathcal{v}_3)\,(x) = \mathcal{v}_2(\partial_{21}(x), \partial_{32}(x)) = (\partial_{32}\,\partial_{21}^{(2)}(x) - \partial_{31}\,\partial_{21}(x) - \partial_{21}^{(2)}\,\partial_{31}(x), \partial_{21}\,\partial_{32}^{(2)}(x) + \partial_{31}\,\partial_{32} - \partial_{32}^{(2)}\partial_{21}(x))$

From (2) and (3) we get, $\partial_{32}\,\partial_{21}^{(2)}(x) - \partial_{31}\,\partial_{21}(x) - \partial_{21}^{(2)}\,\partial_{31}(x)$

$= \partial_{21}^{(2)}\partial_{32}(x) + \partial_{21}\,\partial_{31}(x) - \partial_{21}\,\partial_{31}(x) - \partial_{21}^{(2)}\,\partial_{32}(x) = 0$, and

$$\partial_{21}\,\partial_{32}^{(2)}(x) + \partial_{31}\,\partial_{32} - \partial_{32}^{(2)}\partial_{21}(x) =$$
$$\partial_{32}^{(2)}\,\partial_{21}(x) - \partial_{32}\,\partial_{31}(x) + \partial_{32}\,\partial_{31}(x) - \partial_{32}^{(2)}\,\partial_{21}(x) = 0$$

So, $(\mathcal{v}_2 \circ \mathcal{v}_3)\,(x) = 0$. Thus $\mathrm{Im}(\mathcal{v}_3) \subseteq \ker(\mathcal{v}_2)$

Proposition 2.6: $\mathrm{Im}(\mathcal{v}_2) \subseteq \ker(\mathcal{v}_1)$

Proof: $(\mathcal{v}_1 \circ \mathcal{v}_2)\,(u,t) = \mathcal{v}_1\left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(u) - \partial_{21}^{(2)}(t), \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(t) - \partial_{32}^{(2)}(u)\right) = \partial_{21}^{(2)}\,\partial_{32}(t) + \partial_{21}\,\partial_{31}(t) - \partial_{21}\,\partial_{32}^{(2)}(x) + \partial_{32}^{(2)}\,\partial_{21}(u) - \partial_{32}\,\partial_{31}(x) - \partial_{32}\,\partial_{21}^{(2)}(t)$. Thus from (1), (2) and (3) we have

$(\mathcal{v}_1 \circ \mathcal{v}_2)\,(u,t) = \partial_{21}^{(2)}\,\partial_{32}(t) + \partial_{21}\,\partial_{31}(t) - \partial_{21}\,\partial_{32}^{(2)}(u) + \partial_{21}\,\partial_{32}^{(2)}(u)$
$+\partial_{32}\,\partial_{31}(u) - \partial_{32}\,\partial_{31}(u) - \partial_{21}^{(2)}\,\partial_{32}(t) - \partial_{21}\,\partial_{31}(t)$=0. So $\mathrm{Im}(\mathcal{v}_2) \subseteq \ker(\mathcal{v}_1)$

Theorem 2.7: *The following series is complex*

$$\mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\mathcal{v}_3} \begin{matrix} \mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix} \xrightarrow{\mathcal{v}_2} \begin{matrix} \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_{13}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \end{matrix}$$
$$\xrightarrow{\mathcal{v}_1} \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \xrightarrow{\acute{d}_{(12,7,4)}(\mathcal{F})} \mathcal{K}_{(12,7,4)}(\mathcal{F}) \longrightarrow 0$$

Proof: The polarization map is injective [1], so $\mathcal{v}_3$ is injective. From propositions (2.5) and (2.6) thus the series is a complex.

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