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Pseudo weakly completely prime ideal in near ring

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Abstract

The aims for this article is to study some important result related to the concept for pseudo w-completely prime ideal in a near-ring η. We determine a relationship between pseudo w-completely prime ideal with other sorts for ideals in a non-zero symmetric near-ring η. Also, we give some properties for pseudo w-completely prime ideal & proved some propositions. We include examples whenever necessary.

Subject Classification: *11R44.*

Keywords: *Completely prime ideal, Pseudo w-completely prime ideal, Prime ideal, Weakly prime ideal.*

1. Introduction

A near ring is considered by Pilz [1] who is the founder for the near-ring theory. Our goal is to present the concept for pseudo w-comp-prime ideals, pseudo w-comp-2-prime ideals & pseudo w-comp-3-prime ideals for near-rings η (not necessarily commutative) & link between them. Also, we explain the properties for these concepts. We determined the relationships among these concepts & determine the relationships among these concepts & other kinds for ideals for a non-symmetric η. we proved that the pseudo w-comp-prime ideals in the zero-symmetric weakly commutative η is IFP under a specific condition. Furthermore, we proved that the intersection for two pseudo w-comp-prime ideals is not pseudo w-comp-prime ideal, while the last is satisfying if the required ideals are one contained in the other.

Here we are listing some preliminary definitions for Algebra that we need.

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(Pilz[1]) A triple (η,+,.) is said to be near-rings (right - near-rings) when (η,+) be group (no necessarily abelian), (η,.) semi group & $\forall\, a, n, m \in \eta$:

$(a + n).m = a.m + n.m$, & a near-ring is left if the distribution from the left is satisfied so where both left & right distributions satisfies then it is called near-ring.

(Pilz [1]) Let ł be normal sub - groups for (η,+) if łη⊆ł so its right ideal, & an ideal is left if $\alpha(\mu + \omega) - xy \in$ ł $\forall\, \mu, \omega \in \eta, \forall \alpha \in$ ł & is said to be an ideal if it is both right & left ideal.

(Pilz[1]) the near-rings η is weakly commutative where $xyz = xzy$, $\forall\, xyz \in \eta$.

(Groenewald [2]) An ideal ł for η is called com. prime ideal if $u, v \in \eta$, $u.v \in$ ł implies that $u \in$ ł or $v \in$ ł.

(Birkenmeier [3, 4]) An ideals ł for η be prime where α, β be ideals for near-rings η & $\alpha.\beta \subseteq$ ł , either $\alpha \subseteq$ ł or $\beta \subseteq$ ł.

(&erson & Smith[4]) A proper ideal ł for η is w-prime (w.p.) ideal if A,B ideals for a near-ring η implies that $\emptyset \neq AB \subseteq$ ł then $A \subseteq$ łor $B \subseteq$ ł.

(Beddani & Messirdi[5]) A proper ideal ł for η be 2-prime (2-p.) ideal if whenever $a, b \in \eta$ & $a.b \in$ ł then either $a^2 \in$ ł or $b^2 \in$ ł.

(Groenewald[2]) an ideal ł for η named 3-prime(3-p.) ideals when xηy ⊆ ł then $x \in$ ł or $y \in$ ł.

(Bell[6]) A symmetric near-ring is the non-trivial near-ring is if $x.0 = 0$, $\forall x \in \eta$.

(Babushri, [7]) An ideal ł for η have the insertions for factor property (IFP) if $u, v \in \eta \ni uv \in$ ł then $ufv \in$ ł $\forall\, f \in \eta$.

(Satyanarayana[8]) An element n belongs to η is called an idempotent if $n^2 = n$. Also n is called to be nilpotent element if $\exists k \in Z^+ \ni n^k = 0$.

& we can generalized the ps.w- com.p. ideal, ps.w-com.2-p. ideal & ps.w-com.3-p. ideal for η in other ideals [9] or in algebraic graph[10-13] or ciphers[14]

Properties for ps.w-com. p. ideal with other ideals

This section define ps.w-com. p. ideal on η with presenting various characteristics & relations:

Definition 1: An ideal ł for η is called ps.w-com. p. ideal if $\forall\, \gamma, \delta \in \eta$, 0=$\gamma.\delta \in$ ł then either $\gamma \in$ ł or $\delta \in$ ł,

Proposition 2: An ideal ł = {0} for a non-zero divisor in η is ps.w-com. p. ideal.

Proof: Let $\rho, \tau \in \eta$, $\ni\ \rho, \tau \in$ ł.

Since = {0} then $\rho.\tau$ =0 .

Since η has no zero divisor then either that ρ =0 or τ =0, then either $\rho \in$ ł or $\tau \in$ ł. So that ł is ps.w-com. p. ideal .

Example 3: If η ={0,δ, ε ,σ } a near-rings with two operation (+) & (.) define in Table 1. Let ł = {0} be ideal for η.

Table 1
" Multiplication & addition table for η "

+	0	δ	ε	σ		.	0	δ	ε	σ
0	0	δ	ε	σ		0	0	0	0	0
δ	δ	0	σ	ε		δ	δ	δ	δ	δ
ε	ε	σ	0	δ		ε	ε	ε	ε	ε
σ	σ	ε	δ	0		σ	σ	σ	σ	σ

We can see that ł = {0} is ps.w-com. p. ideal.

Proposition 4: Let ł ≠{0} is com.p. ideal for η then is w-p. ideal.

Proof: Suppose that ł is not w- p. ideal.

There are β, δ ideals ∋ ∅ ≠ βδ ⊆ ł then β ⊈ ł & δ ⊈ ł
Since βδ ≠ ∅ , there is t ∈ β & s ∈ δ ∋ t. s ∈ βδ ⊆ ł .
So t. s ∈ ł then t ∉ ł & s ∉ ł (since β ⊈ ł & δ ⊈ ł)
Then ł is not com.p. ideal. Contradiction.
Hence ł is w-p. ideal.

Remark 5: The converse for proposition (4) is not true.

Example 6: If η ={0, 1, 2, 3, α, γ, ε, δ} near-ring with two operation (+) & (.) define in Table 2. Let $ł_1$ = {0, ε} & $ł_2$ = {0, 2 ε, δ} are ideals for η .

Table 2
"Multiplication & addition table for η"

+	0	1	2	3	α	γ	ε	δ		.	0	1	2	3	α	γ	ε	δ
0	0	1	2	3	α	γ	ε	δ		0	0	0	0	0	0	0	0	0
1	1	2	3	0	δ	ε	α	γ		1	0	1	2	3	α	γ	ε	δ
2	2	3	0	1	γ	α	δ	ε		2	0	2	0	2	0	2	0	2
3	3	0	1	2	ε	δ	γ	α		3	0	3	2	1	γ	α	ε	δ
α	α	δ	γ	ε	2	0	1	3		α	0	α	2	γ	α	γ	ε	δ
γ	γ	ε	α	δ	0	2	3	1		γ	0	γ	2	α	γ	α	ε	δ
ε	ε	α	δ	γ	1	3	0	2		ε	0	ε	0	ε	0	ε	0	ε
δ	δ	γ	ε	α	3	1	2	0		δ	0	δ	0	δ	2	2	0	0

The com.p. ideal $ł_2$={0,2 ε, δ} is w-p. ideal, while $ł_1$={0, ε} is w-p. ideal but not com.p. ideal.

Remark 7: An ideal ł ≠ {0} for a commutative η is com. p. ideal if & only if ł is w-p. ideal.

Proposition 8: Let $ł \neq \{0\}$ be an ideal for a non-symmetric η then is com.p. ideal if & only if is ps.w- com. p.ideal.

Proof: Let ł a com.p. ideal to prove ł is ps.w-com.p ideal.

Let $\alpha, \beta \in \eta \ni \alpha, \beta \in ł$, the profor in two cases:

Case 1: If η has zero-divisor.

So, $0 = \alpha.\beta \in ł$ & $\alpha \neq 0, \beta \neq 0$ since ł is com.p.ideal then $\alpha \in ł$ or $\beta \in ł$.
So, ł be ps.w-com. p.ideal.

Case 2: If η has no-zero divisor

$0 = \alpha.\beta \in ł$ then either α=0 or β=0
If $\alpha = 0$ then $0 = 0.\beta \in ł$
So $\alpha \in ł \ni \beta.0 \neq 0$ (non- symmetric η)
Then ł is ps.w-com. p. ideal.
Conversely, let ł a ps. w-com. p. ideal to prove that ł is com.p. ideal.
Let ł is not com.p. ideal & $\alpha, \beta \in N$ s.t$\alpha.\beta ł \in$ therefore $\alpha \notin ł$ & $\beta \notin ł$

Case 1: If η have zero divisor so $0 = \alpha.\beta \in ł$ &$\alpha \neq 0$ &$\beta \neq 0$

Hence ł not ps.w-com.p. ideal (since $\alpha \notin ł$ & $\beta \notin ł$)
That is contradiction.
Then ł is com.p.ideal.

Case 2: If η has no-zero divisor

$0 = \alpha.\beta \in ł$ then either $\alpha = 0$ or $\beta = 0$, but $\alpha \notin ł$ & $\beta \notin ł$ (by hypothesis)
So, ł is not ps.w-com. p.ideal. Contradiction.
Hence ł is com. p .ideal. □

Example 9: In example (3) as $ł_1$={0,a} , $ł_2$={0,b} are com.p. ideals if & only if they are ps.w-com. p. ideals for a non-symmetric for η has no-zero divisor.

Definition 10: An ideal ł is said ps.w-com. 2-p. ideal if 0= u.v $\in ł$ then either $u^2 \in ł$ or $v^2 \in ł, \forall u, v \in \eta$.

Proposition 11: If $ł \neq \{0\}$ is ps.w-com. p. ideal containing nilpotent or idempotent elements for η then is ps.w-com 2-p. ideal.

Proof: Let $a, b \in \eta \ni 0 = a.b \in ł$,

Since ł is ps.w-com. p. ideal then $a \in ł$ or $b \in ł$ if $a \in ł$

Case 1: If a=0 then $0^2 = 0 \in ł$, so $a^2 \in ł$

Case 2: If $a \neq 0$ $a^2 = a \in ł$

Subcase 1: If (a) nilpotent then $a^2 = 0 \in ł$ & then $a^2 \in ł$

Subcase 2: If (a) idempotent then $a^2 = a \in ł$ & then $a^2 \in ł$ same manner if $b \in ł$ then ł is ps.w-com. p.ideal .

Remark 12: The converse for proposition (11) is not true .

Example 13: In example (6) as $ł_1$={0 ε,}& $ł_2$={0,2,ε δ, } are ideals for η. We can see that $ł_2$ is ps.w-com.p. ideal then it is ps.w-com.2-p.ideal. While $ł_1$ is ps.w-com. 2-p. ideal but not ps.w-com.p. ideal (since $\delta.\delta = 0 \in ł_1$ but $\delta \notin ł_1$ while $\delta^2 = 0 \in ł_1$).

Proposition 14: The ideal $ł \neq \{0\}$ is ps.w-com. 2-p.ideal for a non-symmetric η if & only if is ps.w-com. p.ideal whenever containing idempotent or nilpotent elements.

Proof: Suppose that ł is not ps.w-com. p. ideal let $a, b \in \eta$

Case 1: If N has zero divisor that is $0 = a.b \in ł$, $a \neq 0$, $b \neq 0$, proved in two cases :

Subcase 1: If a , b idempotent elements that is $a^2 = a$, $b^2 = b$ since $a \notin ł$ & $b \notin ł$ then $a^2 \notin ł$ & $b^2 \notin ł$.

So ł is not pseudo w-completely 2-prime ideal. Contradiction.

Subcase 2: If a idempotent element & b nilpotent element such that $a.b = 0 \neq b.a$, that is $a^2 = a$, $b^2 = 0$ since $a \notin ł$ & $b \notin ł$, $0 \in ł$ then $a^2 \notin ł$ & $b^2 \notin ł$.

So ł is not pseudo w-completely 2-prime ideal. Contradiction.
Hence ł is pseudo w-completely prime ideal.

Case 2: If N have no-zero divisor that is $0 = a.b \in ł$ & $a = 0$ or $b = 0$ if $a = 0$ & $0 \in ł$ then $a \in ł$ but $a \notin ł$ (by hypothesis) so $b \in ł$ but $b \notin ł$ (by hypothesis), that is contradiction.

Hence ł is pseudo w-completely prime ideal.
Conversely, same as profor case (2) in proposition (11).

Example 15: In example (2.3) as $ł_1$={0,δ} & $ł_2$={0,ε} are ideals for non-symmetric η with no zero-divisor. Ideals $ł_1$ & $ł_2$ are ps.w-com. p. if & only if they are ps.w-com.2-p.ideals.

Proposition 16: Let $ł \neq \{0\}$ ideals for non-symmetric η so ł be com.p. ideal iff ł is ps. w-com. 2-p. ideal.

Proof: From propositions (8) & (11).

Proposition 17: Let $ł \neq \{0\}$ be a w-p. ideal for a commutative η then is ps.w-comp. 2-p.ideal.

Proof: From remark (7) & proposition (16).

Proposition 18: Let $ł \neq \{0\}$ be a com.p. ideal for η then is prime ideal.

Remark 19: The converse for proposition (18) is true if η is non-symmetric.

Proposition 20: Let $ł \neq \{0\}$ is prime ideals for η so is w-p. ideal.

Proof: Let β & α be an ideals for η such that $\emptyset \neq \beta \,.\, \alpha \subseteq ł$, since $ł$ is prime ideal then either $\beta \subseteq ł$ or $\alpha \subseteq ł$.

Hence $ł$ is w-p. ideal.

Remark 21: The converse for proposition (20) is not true. But it is true if η is non-symmetric.

Example 22: In example (3) the ideals $I_1=\{0,\delta\}$ & $I_2=\{0,\varepsilon\}$ are prime if & only if they are w-p. ideals for a non-symmetric for η.

Proposition 23: Let $ł = \{0, a\}$ be a prime ideal containing idempotent or nilpotent elements for η then is 2-p. ideal.

Proof: Let $\tau, \rho \in \eta \ \ni \ \tau.\rho \in ł$, let β & μ be an ideals for η such that $\beta.\mu \subseteq ł$ & $\tau \in \beta\,, \rho \in \mu$. Then $\tau.\rho \in \beta.\mu \subseteq ł$

Case 1: If $0 = \tau.\rho \in ł$ then either $0 = \tau \in ł$ or $0 = \rho \in ł$ since $ł$ a prime ideal containing nilpotent elements such that either $\beta \subseteq ł$or $\mu \subseteq ł$, then either $\tau^2 = 0 \in ł$ or $\rho^2 = 0 \in ł$

Case 2: If $0 \neq \tau.\rho \in ł$. Then either $\tau \in ł$ or $\rho \in ł$, since $ł$ a prime ideal containing idempotent elements such that either $\beta \subseteq ł$ or $\mu \subseteq ł$, then either $\tau^2 = \tau \in I$ or $\rho^2 = \rho \in ł$.

Hence $ł$ is 2-p. ideal.

Remark 24: The converse for proposition (23) is not true.

Example 25: In example (6) as $I_1=\{0,\varepsilon\}$,$I_2=\{0,2,\varepsilon, \delta\}$. The prime ideal I_2 is 2-p. ideal while I_1 is 2-p. ideal but not a prime ideal.

Proposition 26: Let $ł = \{0, a\}$ is ideal for non-symmetric η such that $a.x \in ł, \forall\, x \in \eta$ & $a \in ł$ then $ł$ is prime ideal iff is 3-p. ideal.

Proof: Assume $\dot{R}$,$\dot{S}$ ideals for $\eta \ni \dot{R}.\dot{S} \subseteq ł$, where $a \in \dot{R}\,, b \in \dot{S}$ so $a.b \in \dot{R}.\dot{S} \subseteq ł$ then $a.b \in ł$

Case 1: If $0 = a.b \in ł$, we have two states:

Subcase 1: If η has zero-divisor $0 = a.b \in ł$, $a \neq 0$, $b \neq 0$, either $\dot{R} \subseteq ł$ or $\dot{S} \subseteq ł$ (since ł p. ideal) then $a \in ł$ or $b \in ł$.

Subcase 2: If η has non-zero-divisor $0 = a.b \in ł$, either $a = 0$ or $b = 0$

If $a = 0$ then $0.b = 0 \in ł$, so $a = 0 \in ł$ (since η is non-symmetric then $b \neq 0$)

Hence ł is 3-prime ideal.

Case 2: If $0 \neq a.b \in ł$, since $a.x \in ł \ \forall x \in \eta \ \& \ a \in ł$ then $a.x = 0 \in ł$ or $a.x = a \in ł$, so $a \in ł$.

Then ł is 3-p. ideal.

Conversely, the profor in the same manner .

Proposition 27: Let $ł \neq \{0\}$ ideal for non-symmetric η so ł be prime ideal iff is ps. w- com. p. ideal.

Proof: From Proposition (18) , Remark (19) & Proposition (8) the proof complete.

Example 28: In example (3) the ideals are prime if & only if they are ps.w-com. p. ideals for a non- symmetric for η .

Definition 29: An ideal ł is ps.w-com. 3-p.ideal if $0 = a.\eta.b \subseteq ł$ so $a \in ł$ or $b \in ł, \forall$ a and $b \in \eta$.

Proposition 30: Letł $\neq \{0\}$ be a ps.w-com. p. ideal for weakly commutative η then is ps.w- com. 3-p. ideal.

Proof: Assume $a, b \in \eta \ \ni \ 0 = a.\eta.b \subseteq ł$, so $0 = a.b.\eta \subseteq ł$ (since η weakly commutative near-ring) since $0.a = 0 \ \forall \ a \in \eta \ \& \ \eta \neq \emptyset$ therefore $a.b = 0$, as ł is ps.w-com. p. ideal then $a \in ł$ or $b \in ł$.□

Remark 31: The converse for proposition (30) is not true. But it is true if η is non-symmetric.

Remark 32: If $ł \neq \{0\}$ ideal for non-symmetric η so ł be ps.w-com. 3-p. ideal iff ł is ps.w-com. 2-p. ideal.

Proof: The proof is satisfied from proposition (30), Remark (31), & proposition (14).

Proposition 33: Let $ł \neq \{0\}$ be an ideal for a non-symmetric η then ł is 3-p. ideal iff is ps.w-com. 3-p. ideal.

Proof: From propositions (26),(27), (30) & Remark (31) the proof complete.

Proposition 34: Let $ł \neq \{0\}$ is c-p. ideal for commutative η then is ps. w-com. 3-p. ideal.

Proof: Let $a, b \in \eta \ \ni \ 0 = a.\eta.b \subseteq ł$, since $ł$ is c-prime ideal then $a.b \in ł$,

Now, if $0 = a.b \in ł$, so $0 = a.b = a.\eta.b$,

Case 1: If a = b = 0 then $0.0 = 0 \in ł$ then either $a = 0 \in ł$ or $b \in ł$

Case 2: Since η is commutative so

Subcase 1: If a=0 , $b \neq 0$ then $0.b = 0 \in ł$ then $a = 0 \in ł$

Subcase 2: If $a \neq 0$, b=0 then $a.0 = 0 \in ł$ then $b = 0 \in ł$

Case 3: If $a \neq 0, b \neq 0$, $a.b = 0 \in ł$ (zero divisor) then either $a \in ł$ or $b \in ł$ ($ł$ is com.p. ideal)

Hence $ł$ is ps. w-com. 3-p. ideal.

Proposition 35: Let $ł_1$ & $ł_2$ are ps.w-com. p. ideals for non-symmetric η with no zero-divisor, where $ł_1 \cap ł_2 = \{0\}$, then $ł_1 \cap ł_2$ is ps. w-com. p. ideal.

Proof: Since $ł_1 \cap ł_2 = \{0\}$, so from Proposition (2) then $ł_1 \cap ł_2$ is ps. w-com. p. ideal.

Proposition 36: Let $I_1 \neq \{0\}$ & $I_2 \neq \{0\}$ are ps.w-com. p. ideals for η if $ł_1 \subseteq ł_2$ or $ł_2 \subseteq ł_1$ then $ł_1 \cap ł_2$ is ps. w-com. p. ideal.

Proof: Let $a, b \in \eta \ \ni \ 0 = a.b \in ł_1 \cap ł_2$

Case 1: If $ł_1 \subseteq ł_2$ then $ł_1 \cap ł_2 = ł_1$ so $0 = a.b \in I_1 = ł_1 \cap ł_2$ since $ł_1$ is ps. w-com. p. ideal then either $a \in ł_1 \cap ł_2$ or $b \in ł_1 \cap ł_2$

Case 2: If $ł_2 \subseteq ł_1$ then $ł_1 \cap ł_2 = ł_2$ so $0 = a.b \in ł_2 = ł_1 \cap ł_2$ since $ł_2$ is ps. w-com. p. ideal then either $a \in ł_1 \cap ł_2$ or $b \in ł_1 \cap ł_2$ hence $ł_1 \cap ł_2$ is ps.w-com. p. ideal.

Example 37: If η ={0,1,2,3,4,5,6,7} is near-rings that have two operation(+) & (.) define in Tables 3. Let $ł_1$={0,2,4,6} & $ł_2$={0,2,3,4,6} are ideals for η.

Table 3
"Multiplication & addition table for η"

+	0	1	2	3	4	5	6	7		.	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7		0	0	0	0	0	0	0	0	0
1	1	0	3	2	5	4	7	6		1	0	2	4	2	0	2	4	6
2	2	3	0	1	6	7	4	5		2	0	4	0	4	0	4	0	4
3	3	2	1	0	7	6	5	4		3	0	6	4	6	0	6	4	2

Contd...

4	4	5	6	7	0	1	2	3		4	0	0	0	0	0	0	0
5	5	4	7	6	1	0	3	2		5	0	2	4	2	0	2	4	6
6	6	7	4	5	2	3	0	1		6	0	4	0	4	0	4	0	4
7	7	6	5	4	3	2	1	0		7	0	6	4	6	0	6	4	2

We can see that $ł_1, ł_2$ are ps.w-com. p. ideals where $ł_1 \subseteq ł_2$ then $ł_1 \cap ł_2 = ł_1$ is ps.w- com. p. ideal.

Proposition 38: Let $ł \neq \{0\}$ be a ps.w-com. p. ideal for a symmetric weakly commutative η. Then for any $\alpha, \mu \in ł$, $0 = \alpha\mu \in ł$ implies $0 = \mu\alpha \in ł$.

Proof: Let $\alpha\mu = 0\ \forall\ \alpha, \mu \in ł$ & let $\rho, \tau \in \eta$ such that $\alpha\rho = \alpha$ & $\mu = \tau\mu$,so$\mu\alpha = \tau\mu\rho\alpha = \tau\rho\mu\alpha = \tau\rho\alpha\mu = \tau\rho 0 = 0 \in ł$.□

Proposition 39: Let $ł \neq \{0\}$ be ps.w-com. p. ideal for a zero-symmetric weakly commutative η if $\eta i \subseteq ł$ then $0 = \eta i\eta \subseteq ł\ \forall\ i \in .\ ł$

Proof: Since $\eta i \subseteq ł$, there are two cases:

Case 1: If η i=0 & $i\eta \subseteq ł$ then $0 = \eta i\eta = 0 \subseteq \eta ł$

Case 2: If $\eta\ i \neq 0$ let $ai \in \eta i\ , \forall a \in \eta, i \in ł$ then $ai \in ł$ (since $\eta i \subseteq ł$) since ł is ps.w-com. p. ideal then $0 = aix \in \eta i\eta\ , \forall x \in \eta$, so $0 = axi \in \eta i \subseteq ł$ (Proposition 38) & then $\eta i\eta \subseteq \eta i \subseteq ł$. Hence $0 = \eta i\eta \subseteq ł$.□

Corollary 40: *Let* $ł \neq \{0\}$ *be a ps.w-com. p. ideal for zero- symmetric* η *if* $\eta\ i \subseteq ł$ *so* ł *be IFP* $\forall\ i \in ł$.

Proof: Directly from proposition (39).□

Example 41: In example (37) as I_l={0,2,4,6} an ideal for η . We can see that $0 = \eta i\eta \subseteq ł_1\ , \forall\ i \in ł_1$ & for any $a, b \in ł_1$, if $0 = ab \in ł_1$ implies $0 = ba \in ł_1$.

Proposition 42: Let $ł \neq \{0\}$ is ps.w-com. p. ideal for zero-symmetric weakly commutative η if $\{0\} \neq \eta I \subseteq ł$ then $łη = \eta ł$.

Proof: To prove $łη = \eta ł$, we must prove $łη \subseteq \eta ł$ & $\eta ł \subseteq łη$.

So let $a \in \eta$, $i \in\ łη \subseteq \eta ł \ni ia \in łη$ since $łη \subseteq ł$ (right ideal) then $ia \in ł$ if $ia = 0 \in ł$ then $0 \in \eta ł$ (since $\eta ł \subseteq\ ł$) & then $łη \subseteq \eta ł$, if $ia \neq 0 \in\ ł$, & since ł is ps. w-com. p. ideal $\exists\ x \in \eta\ \ni 0 = iax \in ł$, so $0 = iax \in\ łη$ then $0 = iax \in \eta ł$ (since $\eta ł \subseteq\ ł$) , so $łη \subseteq \eta ł$.

Let $a \in \eta$, $i \in ł\ \ni ai \in \eta ł$ $ai \in ł$ (since $\eta ł \subseteq ł$) if $ai = 0 \in ł$ then $0 \in łη$ (since $łη \subseteq ł$ as right ideal) , & then $\eta ł \subseteq łη$, if $ai \neq 0 \in ł$, & since ł is ps.w-com. p. ideal then $\exists\ x \in \eta\ \ni 0 = aix \in ł$ since η weakly commutative, then $0 = aix = axi \in ł$, so $0 = axi \in \eta\ ł$, then $0 = axi \in łη$ (since $łη \subseteq ł$ as right ideal) so $\eta ł \subseteq łη$. & hence $łη = \eta ł$.

Conclusion

We concluded that the ps. w-com. p. ideal has a symmetric relation with most other types for ideals in non-symmetric η. In addition, a left proper ideal ł equivalent to a right proper ideal in zero-symmetric weakly commutative η if $\eta I \subseteq I$.

References

[1] G. Pilz, Near-Rings: The Theory & Its Applications, vol. 23, North-Holl& Math. Stud. Amsterdam, The Netherlands: North-Holl& (1983).

[2] N. J. Groenewald, "The completely prime radical in near-rings," *Acta Math. Hung.*, vol. 51, no. 3-4, pp. 301–305 (1988).

[3] G. Birkenmeier, H. Heatherly, and E. Lee, "Prime ideals in near-rings," *Results Math.*, vol. 24, pp. 27–48 (1993).

[4] D. D. Anderson and E. Smith, "Weakly prime ideals," *Houston J. Math.*, vol. 29, no. 4, pp. 831–840 (2003).

[5] C. Beddani and W. Messirdi, "2-prime ideals & their applications," *J. Algebra Appl.*, vol. 15, no. 3, p. 1650051 (2016).

[6] H. E. Bell, "Near-rings in which each element is a power for itself," *Bull. Aust. Math. Soc.*, vol. 2, no. 3, pp. 363–368 (1970).

[7] K. B. Srinivas, B. Jagadeesha, K. S. Prasad, and J. Suresh, "Different prime graphs for a nearring with respect to an ideal," in Nearrings, Nearfields & Related Topics, Singapore: World Scientific, pp. 185–203 (2017).

[8] B. Satyanarayana and K. S. Prasad, Near Rings, Fuzzy Ideals, & Graph Theory. New York, NY, USA: CRC Press (2013).

[9] A. M. Darweesh, R. H. Hasan, and S. M. Kadham, "Results for fourth-order differential superordination & subordination for univalent functions defined by integral operator," *Math. Model. Eng. Problems*, vol. 11, no. 2 (2024).

[10] A. A. Al-Swidi, "Uniquely colorable completely equiprime graph for a near ring," in AIP Conf. Proc. (2021).

[11] A. S. N. Jabir, K. S. Maki, M. M. Ahmed, and N. K. Abbass, "Encryption & enhance medical image using hybrid transform (Ã-module & partial fuzzy Ȟ-transform)," *J. Discrete Math. Sci. Cryptogr.,* vol. 26, no. 7, pp. 1903–1910 (2023), doi: 10.47974/JDMSC-1683.

[12] T. M. Al-Shami, A. Mhemdi, A. M. Abd El-latif, and F. A. A. Shaheen, "Finite sfort-open sets: Characterizations, operators & continuity," *AIMS Math.,* vol. 9, no. 4, pp. 10363–10385 (2024).

[13] J. Jose, B. George, and R. K. Thumbakara, "Sfort graphs: A comprehensive survey," *New Math. Nat. Comput.,* pp. 1–52 (2024).

[14] S. Anwar, M. Azeem, and M. K. Jamil, "Topological numbers for fuzzy sfort graphs & their applications in globalizing the world by mutual trade," *Appl. Sfort Comput.,* vol. 159, p. 111642 (2024).

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