Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 28 (2025), No. 4-A, pp. 1067–1074
DOI : 10.47974/JDMSC-2033

On the bipolar valued fuzzy RG-sub algebras

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Abstract

This paper introduced the notions of bipolar-valued fuzzy RG-subalgebras on RG -RG-algebras and related properties. A number of theorems are posited backed by relevant examples. Afterwards, we presented a novel concept: the negative anti-fuzzy RG-subalgebra of RG-algebra. It is also studied how on SA-algebras, the bipolar valued fuzzy maps from their homomorphic images and inverse images become bipolar-valued fuzzy. Bipolar-valued fuzzy RG-subalgebras' image and inverse image are defined. Let's examine bipolar valued fuzzy by (BVF) rather.

Subject Classification: *35J88, 39B72, 58E50, 70G75, 74M15.*

Keywords: *RG-algebra, Fuzzy RG-sub algebra, Bipolar valued fuzzy RG-sub algebras, Upper level, Lower level.*

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1. Introduction

Iséki and Ima discuss BCI-algebras and BCK-algebras in [6, 7]. KU-algebras were introduced by Prabpayak [9, 10]. The Authors of [4,5], introduced the AT-algebra the notion of Bipolar Appreciated The presentation of fuzzy SA-subalgebras and fuzzy SA-ideals by Abed and Hammed [3] also provided an introduction to some of its properties. Zadeh was first introduced to the idea of fuzzy subsets in [8, 10, 11]. Zhang presented the idea of bipolar-valued fuzzy subsets in [12]. They looked at expanding fuzzy it from the interval [0,1] to [-1,0] in 1994. The idea that filters, closed ideals, and subalgebras are bipolar fuzzy, and on some filters, algebras was first introduced by Jun and colleagues in [3,5]. In this paper, we present the notions of fuzzy on RG-subalgebras. RG-ideals of RG algebras R.A.K. Omar introduced the notion of RG-- algebra in [9], and in [10], the concept of Interval of valued Fuzzy of BCI-algebras was introduced in 2001. In 2011 the notion of bipolar in [7].

2. Preliminaries

Definition 2.1: [2] An algebra (φ;*,ϑ) is designated as an RG-algebra if the subsequent axiology's are content: ∀ o, $\mathfrak{h}$,z∈ φ, (i) o*ϑ = o, (ii) o*τ = (o*z) *(τ*z), and (iii) o* $\mathfrak{h}$ = $\mathfrak{h}$ * o = ϑ → o = $\mathfrak{h}$.

Proposition 2.2: [3] In any (φ;*,ϑ), RG-*algebra* the following are true: If o, $\mathfrak{h}$, and z are in φ, then: (i) o* o= ϑ, (ii) ϑ *(ϑ * o)= o,(iii) o*(o* $\mathfrak{h}$)= $\mathfrak{h}$,(iv) o* $\mathfrak{h}$ = ϑ if and only if $\mathfrak{h}$ * o = ϑ,(v) o*ϑ=ϑ implies o=ϑ,(vi) ϑ *($\mathfrak{h}$ * o)= o* $\mathfrak{h}$.

Definition 2.3: [1] for all $\mathfrak{y}$ ∈ [0,1] & a *fuzzy* a non-empty set's subset μ φ, the *set* U(μ, $\mathfrak{y}$) ={ o∈φ | μ(o) ≥ $\mathfrak{y}$ } is known as an *upper* level cut of μ, *and* the set L(μ, $\mathfrak{y}$) ={ o∈φ | μ(o) ≤ $\mathfrak{y}$ } is *called* a lower level cut of μ.

Definition 2.4: [2] Let (φ;*,ϑ) be an RG algebra, and S be a nonempty subset of φ.*Then* S is considered an RG-sub algebra of φ if o*$\mathfrak{h}$ ∈ S for any o,$\mathfrak{h}$ ∈ S

Definition 2.4: [2] Let (φ;*,ϑ) be RG algebra, where a non-empty subset is represented by S of φ .Then S is named a RG-sub algebra of φ if o*τ∈ S for any o,τ ∈ φ.

Definition 2.5: [3] For an RG-algebra (φ;*,ϑ), a fuzzy *subset* μ of φ is a fuzzy RG-sub *algebra* of φ if ∀ o,τ∈*φ*, μ (o , $\mathfrak{h}$) ≥ min {μ (o),μ (τ)}.

Definition 2.6: [3] Assume (φ; *, *ϑ*) be an RG *algebra*. Doubt fuzzy RG-sub algebra of φ is defined as μ(o * $\mathfrak{h}$) ≤ max{μ(o),μ($\mathfrak{h}$)} for all o,τ ∈ φ.

3. Bipolar Valued Fuzzy RG-sub algebras of RG-algebra

Definition 3.1: An interval valued fuzzy subset $\beth = \{< \rho, \tilde{\mu}_{\beth}(\rho) > \mid \rho \in \varphi\} = \{< \rho, [\mu_{\beth}^{-}(\rho), \mu_{A}^{+}(\rho)] > \mid \rho \in X\}$ of RG -algebra (φ; * ,ϑ) is called a (BVF)RG-sub algebra of X denoted by $\beth = \{< \rho, (\tilde{\mu}_{\beth})^{\varpi}(\rho), (\tilde{\mu}_{\beth})^{\theta}(\rho) > \mid \rho \in X\} = \{< \rho, \widetilde{(\mu_{\beth}^{\varpi})}(\rho), \widetilde{(\mu_{\beth}^{\theta})}(\rho) > \mid \rho \in X\}$, $\beth^{(\varpi,\theta)} = < (\tilde{\mu}_{\beth})^{\varpi}, (\tilde{\mu}_{\beth})^{\theta} >$, if $\forall \rho, y \in X$

1) $(\tilde{\mu}_{\beth})^{\varpi}(\rho * y) \leq max\{(\tilde{\mu}_{\beth})^{\varpi}(\rho), (\tilde{\mu}_{\beth})^{\varpi}(y)\}$,
2) $(\tilde{\mu}_{\beth})^{\theta}(\rho * y) \geq min\{(\tilde{\mu}_{\beth})^{\theta}(\rho), (\tilde{\mu}_{\beth})^{\theta}(y)\}$, i.e
1) $\widetilde{(\mu_{\beth}^{\varpi})}(\rho * y) \preccurlyeq r\,max\{\widetilde{(\mu_{\beth}^{\varpi})}(\rho), \widetilde{(\mu_{\beth}^{\varpi})}(y)\}$,
2) $\widetilde{(\mu_{\beth}^{\theta})}(\rho * y) \succcurlyeq rmin\{\widetilde{(\mu_{\beth}^{\theta})}(\rho), \widetilde{(\mu_{\beth}^{\theta})}(y)\}$,
1) $(\mu_{\beth}^{-})^{\varpi}(\rho * y) \leq max\{(\mu_{\beth}^{-})^{\varpi}(\rho), (\mu_{\beth}^{-})^{\varpi}(y)\}$ and $(\mu_{\beth}^{-})^{\theta}(\rho * y) \geq min\{(\mu_{\beth}^{-})^{\theta}(\rho), (\mu_{\beth}^{-})^{\theta}(y)\}$.
2) $(\mu_{\beth}^{+})^{\varpi}(\rho * y) \leq max\{(\mu_{\beth}^{+})^{\varpi}(\rho), (\mu_{\beth}^{+})^{\varpi}(y)\}$ and $(\mu_{\beth}^{+})^{\theta}(\rho * y) \geq min\{(\mu_{\beth}^{+})^{\theta}(\rho), (\mu_{\beth}^{+})^{\theta}(y)\}$.

Remark 3.2: A (BVF)subset $\beth^{(\varpi,\theta)} = < (\tilde{\mu}_{\beth})^{\varpi}, (\tilde{\mu}_{\beth})^{\theta} >$ of RG-algebra (φ; *, ϑ), for all $\rho \in X$, thus Since $(\mu_{\beth}^{-})^{\varpi}(\rho) = (\mu_{\beth}^{\varpi})^{-}(\rho)$, $(\mu_{\beth}^{-})^{\theta}(\rho) = (\mu_{\beth}^{\theta})^{-}(\rho)$, $(\mu_{\beth}^{+})^{\varpi}(\rho) = (\mu_{\beth}^{\varpi})^{+}(\rho)$ and $(\mu_{\beth}^{+})^{\theta}(\rho) = (\mu_{\beth}^{\theta})^{+}(\rho)$, then $(\tilde{\mu}_{\beth})^{\varpi}(\rho) = [(\mu_{\beth}^{-})^{\varpi}(\rho), (\mu_{\beth}^{+})^{\varpi}(\rho)] = [(\mu_{\beth}^{\varpi})^{-}(\rho), (\mu_{\beth}^{\varpi})^{+}(\rho)] = \widetilde{(\mu_{\beth}^{\varpi})}(\rho)$ and $(\tilde{\mu}_{\beth})^{\theta}(\rho) = [(\mu_{\beth}^{-})^{\theta}(\rho), (\mu_{\beth}^{+})^{\theta}(\rho)] = [(\mu_{\beth}^{\theta})^{-}(\rho), (\mu_{\beth}^{\theta})^{+}(\rho)] = \widetilde{(\mu_{\beth}^{\theta})}(\rho)$.

Example 3.3: Let φ = {ϑ, 1, 2, ε} in which $*$ is described as the subsequent table :

*	ϑ	1	2	ε
ϑ	ϑ	1	2	ε
1	1	ϑ	ε	2
2	2	ε	ϑ	1
ε	ε	2	1	ϑ

Then (φ; * ,ϑ) is RG-algebra. Define a (BVF) subset $\beth^{(\varpi,\theta)} = < (\tilde{\mu}_{\beth})^{\varpi}, (\tilde{\mu}_{\beth})^{\theta} >$ of φ, where I = {ϑ, 1} is a RG -sub algebra of φ, such that: $\mu^{+}: X \to [0,1]$ and $\mu^{-}: X \to [-1,0]$ by

$$\beth_{(\rho)}^{(\varpi,\theta)} = \begin{cases} [[-0.6,-0.3],[0.3,0.9]] \text{ if } \rho \in \{\vartheta, 1\} \\ [[-0.7,-0.4],[0.2,0.6]] \text{ otherwise.} \end{cases}$$

Then $\beth_{(\rho)}^{(\varpi,\theta)} = < (\tilde{\mu}_{\beth})^{\varpi}, (\tilde{\mu}_{\beth})^{\theta} >$ of φ is a (BVF) RG -subalgebra of φ.

Proposition 3.4: If $\beth^{(\varpi,\theta)} = < (\tilde{\mu}_{\beth})^{\varpi}, (\tilde{\mu}_{\beth})^{\theta} >$ is a (BVF) RG-sub algebra of RG-algebra (φ; * ,ϑ) then $(\tilde{\mu}_{\beth})^{\varpi}(\vartheta) \leq (\tilde{\mu}_{\beth})^{\varpi}(\rho)$ and $(\tilde{\mu}_{\beth})^{\theta}(\vartheta) \geq (\tilde{\mu}_{\beth})^{\theta}(\rho)$, $\forall \rho \in \varphi$.

Proof : For all $\rho \in \varphi$ we have

$$\widetilde{(\mu_{ℶ}^{\varpi})}(\vartheta) = \widetilde{(\mu_{ℶ}^{\varpi})}(\rho * \rho) \preccurlyeq r\,max\{\widetilde{(\mu_{ℶ}^{\varpi})}(\rho), \widetilde{(\mu_{ℶ}^{\varpi})}(\rho)\}$$

$$(\tilde{\mu}_{ℶ})^{\varpi}(\vartheta) = \widetilde{(\mu_{ℶ}^{\varpi})}(\vartheta) \preccurlyeq r\,max\{\widetilde{(\mu_{ℶ}^{\varpi})}(\rho), \widetilde{(\mu_{ℶ}^{\varpi})}(\rho)\}$$

$$\leq \max\{[(\mu_{ℶ}^{\varpi})^{-}(\rho), (\mu_{ℶ}^{\varpi})^{+}(\rho)], [(\mu_{ℶ}^{\varpi})^{-}(\rho), (\mu_{ℶ}^{\varpi})^{+}(\rho)]\}$$

$$= \{[(\mu_{ℶ}^{\varpi})^{-}(\rho), (\mu_{ℶ}^{\varpi})^{+}(\rho)] = [(\mu_{ℶ}^{-})^{\varpi}(\rho), (\mu_{ℶ}^{+})^{\varpi}(\rho)] = (\tilde{\mu}_{ℶ})^{\varpi}(\rho)$$

And $\widetilde{(\mu_{ℶ}^{\theta})}(\vartheta) = \widetilde{(\mu_{ℶ}^{\theta})}(\rho * \rho) \succcurlyeq r\,min\{\widetilde{(\mu_{ℶ}^{\theta})}(\rho), \widetilde{(\mu_{ℶ}^{\theta})}(\rho)\}$, $(\tilde{\mu}_{ℶ})^{\theta}(\vartheta) = \widetilde{(\mu_{ℶ}^{\theta})}(\vartheta) \succcurlyeq r\,min\{\widetilde{(\mu_{ℶ}^{\theta})}(\rho), \widetilde{(\mu_{ℶ}^{\theta})}(\rho)\} \geq \min\{[(\mu_{ℶ}^{\theta})^{-}(\rho), (\mu_{ℶ}^{\theta})^{+}(\rho)], [(\mu_{ℶ}^{\theta})^{-}(\rho), (\mu_{ℶ}^{\theta})^{+}(\rho)]\} = [(\mu_{ℶ}^{-})^{\theta}(\rho), (\mu_{ℶ}^{+})^{\theta}(\rho)] = (\tilde{\mu}_{ℶ})^{\theta}(\rho)$ $(\tilde{\mu}_{ℶ})^{\varpi}(\vartheta) \leq (\tilde{\mu}_{ℶ})^{\varpi}(\rho)$ and $(\tilde{\mu}_{ℶ})^{\theta}(\vartheta) \geq (\tilde{\mu}_{ℶ})^{\theta}(\rho)$, for all $\rho \in \varphi$.

Definition 3.5: Let φ be an $RG\text{-}algebra$, a fuzzy subset μ of φ is $called$ a doubt $fuzzy\ RG\text{-}subalgebraRG\text{-}subalgebra$ of φ if $\mu: X \to [-1,0]$ and $\forall \rho, y \in X, \mu(\rho * y) \leq max\{\mu(\rho), \mu(y)\}$

Theorem 3.6: *A (BVF)subset* $ℶ^{(\varpi,\theta)} = <(\tilde{\mu}_{ℶ})^{\varpi}, (\tilde{\mu}_{ℶ})^{\theta}>$ *of an SA-algebra* $(X; *, \vartheta)$ *is a (BVF)SA-sub algebra of* $\varphi \leftrightarrow$, $(\mu_{ℶ}^{-})^{\varpi}$ *&* $(\mu_{ℶ}^{+})^{\varpi}$ *are doubt fuzzy RG-sub algebraSA-sub algebras of* φ *and* $(\mu_{ℶ}^{-})^{\theta}$ *&* $(\mu_{ℶ}^{+})^{\theta}$ *are fuzzy RG - subalgebras of* φ.

Proof: Assuming $ℶ^{(\varpi,\theta)}$ is a (BVF)$RG\text{-}subalgebra$ of φ, then $\forall \rho$, $y \in \varphi$, we have

$$[(\mu_{ℶ}^{-})^{\varpi}(\rho * y), (\mu_{ℶ}^{+})^{\varpi}(\rho * y)] = (\tilde{\mu}_{ℶ})^{\varpi}(\rho * y)$$

$$\preccurlyeq r\,max\{(\tilde{\mu}_{ℶ})^{\varpi}(\rho), (\tilde{\mu}_{ℶ})^{\varpi}(y)\} = r\,max\{[(\mu_{ℶ}^{-})^{\varpi}(\rho), (\mu_{ℶ}^{+})^{\varpi}(\rho)], [(\mu_{ℶ}^{-})^{\varpi}(y), (\mu_{ℶ}^{+})^{\varpi}(y)]\}$$

$$= [max\{(\mu_{ℶ}^{-})^{\varpi}(\rho), (\mu_{ℶ}^{+})^{\varpi}(\rho)\}, max\{(\mu_{ℶ}^{-})^{\varpi}(y), (\mu_{ℶ}^{+})^{\varpi}(y)\}]$$

$= [max\{(\mu_{ℶ}^{-})^{\varpi}(\rho), (\mu_{ℶ}^{-})^{\varpi}(y)\}, max\{(\mu_{ℶ}^{+})^{\varpi}(\rho), (\mu_{ℶ}^{+})^{\varpi}(y)\}].$, $(\mu_{ℶ}^{-})^{\varpi}(\rho * y) \leq max\{(\mu_{ℶ}^{-})^{\varpi}(\rho), (\mu_{ℶ}^{-})^{\varpi}(y)\}$ and $(\mu_{ℶ}^{+})^{\varpi}(\rho * y) \leq max\{(\mu_{ℶ}^{+})^{\varpi}(\rho), (\mu_{ℶ}^{+})^{\varpi}(y)\}$.

Also, $[(\mu_{ℶ}^{-})^{\theta}(\rho * y), (\mu_{ℶ}^{+})^{\theta}(\rho * y)] = (\tilde{\mu}_{ℶ})^{\theta}(\rho * y) \succcurlyeq r\,min\{(\tilde{\mu}_{ℶ})^{\theta}(\rho), (\tilde{\mu}_{ℶ})^{\theta}(y)\}$

$$= r\,min\{[(\mu_{ℶ}^{-})^{\theta}(\rho), (\mu_{ℶ}^{+})^{\theta}(\rho)], [(\mu_{ℶ}^{-})^{\theta}(y), (\mu_{ℶ}^{+})^{\theta}(y)]\}$$

$$= [min\{(\mu_{ℶ}^{-})^{\theta}(\rho), (\mu_{ℶ}^{+})^{\theta}(\rho)\}, min\{(\mu_{ℶ}^{-})^{\theta}(y), (\mu_{ℶ}^{+})^{\theta}(y)\}]$$

$$= [min\{(\mu_{ℶ}^{-})^{\theta}(\rho), (\mu_{ℶ}^{-})^{\theta}(y)\}, min\{(\mu_{ℶ}^{+})^{\theta}(\rho), (\mu_{ℶ}^{+})^{\theta}(y)\}]$$

$(\mu_{ℶ}^{-})^{\theta}(\rho * y) \geq min\{(\mu_{ℶ}^{-})^{\theta}(\rho), (\mu_{ℶ}^{-})^{\theta}(y)\}$ and

$(\mu_{ℶ}^{+})^{\theta}(\rho * y) \geq min\{(\mu_{ℶ}^{+})^{\theta}(\rho), (\mu_{ℶ}^{+})^{\theta}(y)\}$.

Hence, we get that $(\mu_{ב}^{-})^{\varpi}$ & $(\mu_{ב}^{+})^{\varpi}$ are doubt fuzzy RG-$subalgebra\ RG$-$subalgebras$ of φ and $(\mu_{ב}^{-})^{\theta}$ & $(\mu_{ב}^{+})^{\theta}$ are fuzzy RG-$subalgebras$ of φ. Conversely, if $(\mu_{ב}^{-})^{\varpi}$& $(\mu_{ב}^{+})^{\varpi}$are doubt fuzzy RG-sub algebraSA-$subalgebras\ of\ X$ and $(\mu_{ב}^{-})^{\theta}$, $(\mu_{ב}^{+})^{\theta}$ are fuzzy RG -sub algebras of φ,$\forall\rho,\ y\in\varphi$.

Observe $(\tilde{\mu}_{ב})^{\varpi}(\rho*y)=[(\mu_{ב}^{-})^{\varpi}(\rho*y),(\mu_{ב}^{+})^{\varpi}(\rho*y)]$

$$\leq[max\{(\mu_{ב}^{-})^{\varpi}(\rho),(\mu_{ב}^{-})^{\varpi}(y)\},max\{(\mu_{ב}^{+})^{\varpi}(\rho),(\mu_{ב}^{+})^{\varpi}(y)\}]$$
$$=r\,max\{[(\mu_{ב}^{-})^{\varpi}(\rho),(\mu_{ב}^{+})^{\varpi}(\rho)],[(\mu_{ב}^{-})^{\varpi}(y),(\mu_{ב}^{+})^{\varpi}(y)]\}$$
$$=r\,max\{(\tilde{\mu}_{ב})^{\varpi}(\rho),(\tilde{\mu}_{ב})^{\varpi}(y)\}.$$ And , $(\tilde{\mu}_{ב})^{\theta}(\rho)(\rho*y)$
$$=[(\mu_{ב}^{-})^{\theta}(\rho*y),(\mu_{ב}^{+})^{\theta}(\rho*y)]$$
$$\geq[min\{(\mu_{ב}^{-})^{\theta}(\rho),(\mu_{ב}^{-})^{\theta}(y)\},min\{(\mu_{ב}^{+})^{\theta}(\rho),(\mu_{ב}^{+})^{\theta}(y)\}]$$
$$=r\,min\{[(\mu_{ב}^{-})^{\theta}(\rho),(\mu_{ב}^{+})^{\theta}(\rho)],[(\mu_{ב}^{-})^{\theta}(y),(\mu_{ב}^{+})^{\theta}(y)]\}$$
$$=r\,min\{(\tilde{\mu}_{ב})^{\theta}(\rho),(\tilde{\mu}_{ב})^{\theta}(y)\}.$$

Consequently, we can draw a conclusion. that $ב^{(\varpi,\theta)}$ is a (BVF)RG-sub algebra of φ.

Definition 3.7: Let $(X\,;*,\vartheta)$ be an RG-algeba. A (BVF) subset

$ב^{(\varpi,\theta)}=<(\tilde{\mu}_{ב})^{\varpi},(\tilde{\mu}_{ב})^{\theta}>$ of , for all $\tilde{\mathfrak{h}}=[\mathfrak{h}_1,\ \mathfrak{h}_2]\in$ D[0, 1], the set $\widetilde{U}\,(ב^{(\varpi,\theta)};\tilde{\mathfrak{h}})$ is a level set of X such that: $\widetilde{U}\left(ב^{(\varpi,\theta)};\tilde{\mathfrak{h}}\right)=\{\rho\in X\mid\tilde{\mu}_{ב}(\rho)\geqslant\mathfrak{h}\}=\{\rho\in X\mid[(\tilde{\mu}_{ב})^{\varpi}(\rho),(\tilde{\mu}_{ב})^{\theta}(\rho)]\geqslant[\mathfrak{h}_1,\mathfrak{h}_2]\}$

$=\{\rho\in X\mid(\tilde{\mu}_{ב})^{\varpi}(\rho)\leq\mathfrak{h}_1,\ (\tilde{\mu}_{ב})^{\theta}(\rho)\geq\ \mathfrak{h}_2\}$.

Proposition 3.8: Let Xbe an SA-algebra. A (BVF) subset $ב^{(\varpi,\theta)}=<(\tilde{\mu}_{ב})^{\varpi},(\tilde{\mu}_{ב})^{\theta}>$ of X. If $ב^{(\varpi,\theta)}$ is a (BVF) RG-$subalgebra$ of φ, then for any $\tilde{\mathfrak{h}}=[\mathfrak{h}_1,\ \mathfrak{h}_2]\in$ D[0, 1], the set $\widetilde{U}\,(ב^{(\varpi,\theta)};\tilde{\mathfrak{h}})$ is a RG-sub algebra of X .

Proof. Assuming $ב^{(\varpi,\theta)}$ is a (BVF)RG-sub algebra of X and let $\tilde{t}=[t_1,\ t_2]\in$ D[0, 1] where $\widetilde{U}\,(ב^{(\varpi,\theta)};\tilde{t})\neq\emptyset$, & let $\rho\,,y\ \in X$ such that $\rho\,,y\ \in\widetilde{U}\,(ב^{(\varpi,\theta)};\tilde{t})$, then $(\tilde{\mu}_{ב})^{\varpi}(\rho)\leq t_1$, $(\tilde{\mu}_{ב})^{\varpi}(y)\leq t_1$ $(\tilde{\mu}_{ב})^{\theta}(\rho)\geq t_2$ and $(\tilde{\mu}_{ב})^{\theta}(y)\geq t_2$. Since $ב^{(\varpi,\theta)}$ is a (BVF)RG -sub algebra of φ, we get 1) $(\tilde{\mu}_{ב})^{\varpi}\,(\rho*y)\leq max\{(\tilde{\mu}_{ב})^{\varpi}(\rho),(\tilde{\mu}_{ב})^{\varpi}(y)\}\leq t_1$, 2) $(\tilde{\mu}_{ב})^{\theta}\,(\rho*y)\ \geq min\{(\tilde{\mu}_{ב})^{\theta}(\rho),(\tilde{\mu}_{ב})^{\theta}(y)\}\geq t_2$, Therefore, $\rho*y\ \in\widetilde{U}\,(ב^{(N,P)};\tilde{\mathfrak{y}})$, then $\widetilde{U}\,(ב^{(N,P)};\tilde{\mathfrak{y}})$ is a RG-sub algebra of φ.

Proposition 3.9: Let $(X\,;*,\vartheta)$ be an $RG\ \text{-}algebra$. A (BVF) subset $ב^{(\varpi,\theta)}=<(\tilde{\mu}_{ב})^{\varpi},(\tilde{\mu}_{ב})^{\theta}>$ of . If $\forall\ \tilde{\mathfrak{y}}=[\mathfrak{h}_1,\ \mathfrak{h}_2]\in$ D[0, 1], the set $\widetilde{U}\,(ב^{(\varpi,\theta)};\tilde{\mathfrak{h}})$ is a RG-$subalgebra$ of X, then $ב^{(\varpi,\theta)}$ is a (BVF)RG-$subalgebra$ of φ.

Proof: Assume that $\widetilde{U}\,(ב^{(\varpi,\theta)};\tilde{\mathfrak{h}})$ is a RG-sub algebra of φ and let $\rho\,,y\ \in X$ such that

1) $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho * y) > max\{\ (\tilde{\mu}_{ℷ})^{\varpi}(\rho), (\tilde{\mu}_{ℷ}(y)\}$, Consider $\alpha = 1\ /\ 2\ \{\ (\tilde{\mu}_{ℷ})^{\varpi}\ (\rho + y)\ +\ max\{(\tilde{\mu}_{ℷ})^{\varpi}(\rho), (\tilde{\mu}_{ℷ})^{\varpi}\ (\ y)\}\ \}$ and $\beta = 1/2\ \{\ (\tilde{\mu}_{ℷ})^{\varpi}\ (\rho * y)\ +\ max\{(\tilde{\mu}_{ℷ})^{\varpi}(\rho), (\tilde{\mu}_{ℷ})^{\varpi}\ (\ y)\}\ \}$

We have $\alpha, \beta \in\ [0,1]$, $(\tilde{\mu}_{ℷ})^{\varpi}$ (ρ∗y) >α> max $\{(\tilde{\mu}_{ℷ})^{\varpi}$ (ρ),$(\tilde{\mu}_{ℷ})^{\varpi}$ (y)}.

It follows that $\rho, y \in \widetilde{U}\ (ℷ^{(\varpi,\theta)}; \tilde{\mathfrak{h}})$ and (ρ+y) $\notin\ \widetilde{U}\ (ℷ^{(\varpi,\theta)}; \tilde{\mathfrak{h}})$. There is inconsistency here. Hence, $(\tilde{\mu}_{ℷ})^{\varpi}(\rho * y) \leq$ max$\{(\tilde{\mu}_{ℷ})^{\varpi}(\rho), (\tilde{\mu}_{ℷ})^{\varpi}(y)\} \leq \mathfrak{h}$

Similarly, $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho * y)\ \geq min\{\ (\tilde{\mu}_{ℷ})^{\theta}(\rho), (\tilde{\mu}_{ℷ})^{\theta}(y)\} \geq \mathfrak{h}_2$,

Therefore $ℷ^{(\varpi,\theta)}$ is a (BVF)RG-sub algebra of X. ⌂

Theorem 3.10: *Any RG-sub algebra of RG-algebra$(X\ ;*, \vartheta)$ can be acknowledged as the upper [$\mathfrak{h}_{1}$, $\mathfrak{h}_{2}$]-Level of some bipolar valued RG -sub algebra of X.*

Proof: Let I be a RG-sub algebra of X and $ℷ^{(\varpi,\theta)} = <\ (\tilde{\mu}_{ℷ})^{\varpi}, (\tilde{\mu}_{ℷ})^{\theta}\ >$ be (BVF) subset on X defined by $\tilde{\mu}_{ℷ}(\rho) = \begin{cases} [\alpha_1, \alpha_2], \text{if } \rho \in I \\ [0,0], \text{otherwise} \end{cases}$.

$\forall[\alpha_1, \alpha_2] \in D[0,1]$, we consider the ensuing instances instance 1) If ρ, $y \in I$, then $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho) \leq \alpha_1$, $(\tilde{\mu}_{ℷ})^{\varpi}\ (y) \leq \alpha_1$and $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho) \geq \alpha_2\ and\ (\tilde{\mu}_{ℷ})^{\theta}\ (y) \geq \alpha_2$, thus

1) $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho * y)\ \leq max\{\ (\tilde{\mu}_{ℷ})^{\varpi}(\rho), (\tilde{\mu}_{ℷ})^{\varpi}(y)\} \leq \alpha_1$,
2) $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho * y)\ \geq min\{\ (\tilde{\mu}_{ℷ})^{\theta}(\rho), (\tilde{\mu}_{ℷ})^{\theta}(y)\} \geq \alpha_2$,

instance 2) If $\rho\ \in I$ and $y \notin I, \rightarrow$ $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho) \leq \alpha_1$, $(\tilde{\mu}_{ℷ})^{\varpi}\ (y) \leq \vartheta$ and $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho) \geq \alpha_2\ and\ (\tilde{\mu}_{ℷ})^{\theta}\ (y) \geq \vartheta$, thus 1)$(\tilde{\mu}_{ℷ})^{\varpi}$ (ρ ∗ $y)\ \leq max\{\ (\tilde{\mu}_{ℷ})^{\varpi}(\rho)$, $(\tilde{\mu}_{ℷ})^{\varpi}(y)\} \leq \alpha_1$ 2) $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho * y)\ \geq min\{\ (\tilde{\mu}_{ℷ})^{\theta}(\rho), (\tilde{\mu}_{ℷ})^{\theta}(y)\} \geq \vartheta$, instance 3) If $\rho \notin I \wedge y \in I, \rightarrow$ $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho) \leq \vartheta$, $(\tilde{\mu}_{ℷ})^{\varpi}\ (y) \leq \alpha_1$& $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho) \geq \vartheta\ and\ (\tilde{\mu}_{ℷ})^{\theta}\ (y) \geq \alpha_2$, thus

1) $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho * y)\ \leq max\{\ (\tilde{\mu}_{ℷ})^{\varpi}(\rho), (\tilde{\mu}_{ℷ})^{\varpi}(y)\} \leq \alpha_1$,
2) $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho * y)\ \geq min\{\ (\tilde{\mu}_{ℷ})^{\theta}(\rho), (\tilde{\mu}_{ℷ})^{\theta}(y)\} \geq \vartheta$,

instance 4) If $\rho \notin I\ \wedge\ \ y \notin I, \rightarrow$ $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho) \leq \vartheta$, $(\tilde{\mu}_{ℷ})^{\varpi}\ (y) \leq \vartheta$ & $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho) \geq \vartheta$ & $(\tilde{\mu}_{ℷ})^{\theta}\ (y) \geq \vartheta$, thus

1) $(\tilde{\mu}_{ℷ})^{\varpi}\ (\rho * y)\ \leq max\{\ (\tilde{\mu}_{ℷ})^{\varpi}(\rho), (\tilde{\mu}_{ℷ})^{\varpi}(y)\} \leq \vartheta$,
2) $(\tilde{\mu}_{ℷ})^{\theta}\ (\rho * y)\ \geq min\{\ (\tilde{\mu}_{ℷ})^{\theta}(\rho), (\tilde{\mu}_{ℷ})^{\theta}(y)\} \geq \vartheta$,

Therefore, $ℷ^{(\varpi,\theta)}$ is a bipolar valued fuzzyRG-sub algebra of X.⌂

Corollary 3.11: *Let $(X\ ;*, \vartheta)$ be a RG-algebra, Ɏ form a portion of φ and let $ℷ^{(\varpi,\theta)} = <\ (\tilde{\mu}_{ℷ})^{\varpi}, (\tilde{\mu}_{ℷ})^{\theta} >$ be a (BVF) subset on φ defined by :*

$$\tilde{\mu}_A(\rho) = \begin{cases}[\alpha_1, \alpha_2] \; if \; \rho \in \text{Ұ} \\ [0,0] \; otherwise\end{cases}.$$

Where $\alpha_1, \alpha_2 \in (0, 1]$ with $\alpha_1 < \alpha_2$. If $\beth^{(\varpi,\theta)}$ is a bipolar valued fuzzy RG-sub algebra of φ, → Ұ is a RG-sub algebra of φ.

Proof: Since that $\beth^{(\varpi,\theta)}$ is a (BVF)RG-sub algebra of φ. Let ρ, $y \in$ Ұ, → by Definition (3.1),

1) $(\tilde{\mu}_\beth)^{\varpi}(\rho * y) \leq max\{(\tilde{\mu}_\beth)^{\varpi}(\rho), (\tilde{\mu}_\beth)^{\varpi}(y)\} \leq \alpha_1$,
2) $(\tilde{\mu}_\beth)^{\theta}(\rho * y) \geq min\{(\tilde{\mu}_\beth)^{\theta}(\rho), (\tilde{\mu}_\beth)^{\theta}(y)\} \geq \alpha_2$,

This implies that $\rho * y \in$ Ұ. Hence Ұ is a RG-sub algebra of φ. ⌂

Conclusions

This piece of writing uses the ideas of bipolar valued fuzzy RG-subalgebras on RG -algebras and their related properties. A number of theorems are put forth and backed up by relevant instances. We then introduced a novel concept: the negative anti-fuzzy RG-subalgebra of the RG-algebra. Additionally, the process by which bipolar valued fuzzy on SA-algebras becomes its inverse and homomorphic images is studied. We describe the bipolarly valued fuzzy image and its inverse image. RG-subalgebras. Rather, we will examine bipolar valued fuzzy by (BVF).

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Received July, 2024