

On intuitionistic fuzzy BZ-ideals in BZ-algebras

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Abstract

The concepts of intuitionistic fuzzy BZ -ideal ($IFBZ-I$) in BZ -algebra ($BZ-A$) and intuitionistic fuzzy BZ -subalgebra ($IFBZ-SA$) will be presented and analyzed in this work. Furthermore, the operations on ($IFBZ-SA$) will be examined. In this study, we demonstrate that for any $t \in [0,1]$, if the t -cut of intuitionistic fuzzy π is a ($BZ-A$), then π is ($IFBZ-SA$) of X . Also, if π is ($IFBZ-SA$) of X , then t -cut of intuitionistic fuzzy π is a ($BZ-A$) of X , for any $t \in [0,1]$. Also, some results about the concepts of this work are shown.

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1. Introduction

In 1965, Zadeh [1] introduced the concept of a fuzzy set, which is a class of objects with a continuous range of membership grades. Atanassov [2] presented the intuitionistic fuzzy set (IFS) in 1986. Utilizing the concept of non-membership, which seems to be more accurate in terms of quantifying uncertainty, as well as providing the opportunity to accurately describe the issue based on perception and current knowledge, seems to be a more accurate way to quantify uncertainty. Subsequently, intuitionistic fuzzy sets are

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being studied by numerous mathematicians across various fields ([3],[4]). The concept of Rho-algebras is examined in fuzzy settings in 2020 [5]. The notion of intuitionistic fuzzy δ -algebra was then introduced in 2022 [6]. A class of algebra known as BZ -algebra [7] contains at least one subalgebra that like a group. Such an algebraic system lacks a minimal element yet shares the BCI/BCK -algebras' partial order. A weak BCI -algebra [8] or a BZ -algebra [7, 9] are the names given to such a system. In this paper, the notions of $(IFBZ-SA)$ and $(IFBZ-SI)$ in $(BZ-A)$ are given and discussed. Also, the operations on $(IFBZ-SA)$ are studied. In this work, we show that for any $t \in [0,1]$, if the t -cut of intuitionistic fuzzy π is a $(BZ-A)$ of X , then π is $(IFBZ-SA)$ of X . Also, if π is $(IFBZ-SA)$ of X , then t -cut of intuitionistic fuzzy π is a $(BZ-A)$ of X , for any $t \in [0,1]$. Moreover, some results about this works notions are shown.

2. Preliminaries

Definition 2.1: [7] Assume $X \neq \emptyset$ with binary operation $*$ and constant 0 . We say $(X, *, 0)$ is a BZ -algebra [for short, $(BZ-A)$] if such that: $(BZ_1) ((\sigma * \omega) * (\alpha * \omega)) * (\sigma * \alpha) = 0$, $(BZ_2) \sigma * 0 = \sigma$, $(BZ_3) \sigma * \alpha = 0$ and $\alpha * \sigma = 0$ implies that $\sigma = \alpha$, $\forall \sigma, \alpha \in X$.

Definition 2.2: [10] Assume $(X, *, 0)$ is $(BZ-A)$, $\emptyset \neq D \subseteq X$. We say D is a BZ -subalgebra [for short, $(BZ-SA)$] of X if $\sigma * \alpha \in D$, whenever $\sigma, \alpha \in D$.

Definition 2.3: [10] Assume $(X, *, 0)$ is $(BZ-A)$, $\emptyset \neq I \subseteq X$, then I is said to be a BZ -ideal [for short, $(BZ-I)$] of X if such that: $(I_1) 0 \in I$, $(I_2) \sigma * \alpha \in I$ and $\sigma \in I$ imply $\alpha \in I$, $\forall \sigma, \alpha \in X$.

Remark 2.4: [11] On $(BZ-A) (X, *, 0)$. we established a relation $\leq$ as $p \leq q \leftrightarrow p * q = 0, p, q \in X$.

Proposition 2.5: [11] Assume $(X, *, 0)$ is a $(BZ-A)$, then $(X, \leq)$ is a partially ordered set. It is intended to demonstrate that a $(BZ-A)$ satisfies the following features. $(P_1) \sigma * ((\sigma * \alpha) * \alpha) = 0$, $(P_2) \sigma * \sigma = 0$, $(P_3) \sigma * (\alpha * \omega) = \alpha * (\sigma * \omega)$, $(P_4) ((\sigma * \alpha) * \alpha) * \alpha = \sigma * \alpha$, $(P_5) (\sigma * \alpha) * 0 = (\sigma * 0) * (\alpha * 0)$, $(P_6) (\sigma * \alpha) * ((\omega * \sigma) * (\omega * \alpha)) = 0$, $(P_7) \sigma \leq \alpha$ implies that $\alpha * \omega \leq \sigma * \omega$, $(P_8) \sigma \leq \alpha \rightarrow \omega * \sigma \leq \omega * \alpha$, $\forall \sigma, \alpha, \omega \in X$.

Definition 2.6: [10] Assume $X \neq \emptyset$. A fuzzy set (FS) μ of X is a mapping $\mu: X \rightarrow [0,1]$.

Definition 2.7: [10] Let $X \neq \emptyset$, and μ be a (FS) of X , for any $t \in [0,1]$ the set $U(\mu, t) = \{p \in X \mid \mu(p) \geq t\}$ is said to be an upper t -level cut of μ . Also, we say a lower t -level cut of μ for the set $L(\mu, t) = \{p \in X \mid \mu(p) \leq t\}$.

Proposition 3.8: [12] Assume $(X, *, 0)$ is a $(BZ-A)$ and μ be (FS) of X . Then the upper t -level cut and the lower t -level cut of μ are BZ -subalgebras $(BZ-SAs)$, for any $t \in [0,1]$ if and only if μ is $(BZ-SA)$.

Definition 2.9: [2] A set $H = \{< \sigma, (\zeta_H(\sigma), \vartheta_H(\sigma)) > \mid \sigma \in X\}$ is an intuitionistic fuzzy set (IFS) over X , where $\zeta_H(\sigma): X \rightarrow [0,1]$, $\vartheta_H(\sigma): X \rightarrow [0,1]$ with $0 \leq \zeta_H(\sigma) + \vartheta_H(\sigma) \leq 1$, $\forall \sigma \in X$.

Some Operations on (IFSs) 2.10 [2] Suppose that $H = \{< \sigma, (\zeta_H(\sigma), \vartheta_H(\sigma)) > | \sigma \in X\}$ and $D = \{< \sigma, (\zeta_D(\sigma), \vartheta_D(\sigma)) > | \sigma \in X\}$ are two (IFSs) of X . We came to the following conclusions: (1) $H \subseteq D$ if and only if $\zeta_H(\sigma) \leq \zeta_D(\sigma)$ and $\vartheta_H(\sigma) \geq \vartheta_D(\sigma), \forall \sigma \in X$, (2) $H = D$ if and only if $H \subseteq D$ and $D \subseteq H$, (3) $H \cap D = \{< \sigma, (\min\{\zeta_H(\sigma), \zeta_D(\sigma)\}, \max\{\vartheta_H(\sigma), \vartheta_D(\sigma)\}) > | \sigma \in X\}$, (4) $H \cup D = \{< \sigma, (\max\{\zeta_H(\sigma), \zeta_D(\sigma)\}, \min\{\vartheta_H(\sigma), \vartheta_D(\sigma)\}) > | \sigma \in X\}$, (5) $H^c = \{< p, (\vartheta_H(\sigma), \zeta_H(\sigma)) > | \sigma \in X\}$, (6) $H - D = \{< \sigma, (\min\{\zeta_H(\sigma), \vartheta_D(\sigma)\}, \max\{\vartheta_H(\sigma), \zeta_D(\sigma)\}) > | \sigma \in X\}$.

Definition 2.11: [2] Assume $H = \{< p, (\zeta_H(p), \vartheta_H(p)) > | p \in X\}$ is (IFS) of X and $t \in [0, 1]$ then the set $H_t = \{p \in X; \zeta_H(p) \geq t \text{ \& } \vartheta_H(p) \leq t\}$ is said to be t -cut of (IFS) H , [briefly, $(t\text{-cut-}H)$].

3. Intuitionistic Fuzzy BZ-Ideals.

Definition 3.1: Assume $(X, *, 0)$ is a $(BZ - A)$ and $\pi = \{< \rho, (\zeta_\pi(\rho), \vartheta_\pi(\rho)) > | \rho \in X\}$ be (IFS) of X . π is an intuitionistic fuzzy BZ -subalgebra ($IFBZ - SA$) of X if such that: (1) $\zeta_\pi(\rho * \gamma) \geq \min\{\zeta_\pi(\rho), \zeta_\pi(\gamma)\}$, (2) $\vartheta_\pi(\rho * \gamma) \leq \max\{\zeta_\pi(\rho), \zeta_\pi(\gamma)\}, \forall \rho, \gamma \in X$.

Example 3.2 Assume $X = \{0, \partial, \theta, \delta\}$ with binary operation $(*)$ defined as table (1) and define (IFS) $H = \{< \rho, (\zeta_H(\rho), \vartheta_H(\rho)) > | \rho \in X\}$ of X by table (2):

Table 1
 $(X, *, 0)$ is $(BZ - A)$

$*$	0	∂	θ	δ
0	0	∂	θ	δ
∂	∂	0	δ	θ
θ	θ	δ	0	∂
δ	δ	θ	∂	0

Table 2
 H is $(IFBZ - SA)$ of X .

ρ	0	∂	θ	δ
$\zeta_H(\rho)$	0.6	0.5	0.4	0.4
$\vartheta_H(\rho)$	0.1	0.2	0.3	0.3

Then $(X, *, 0)$ is $(BZ - A)$. Also, H is $(IFBZ - SA)$ of X .

Definition 3.3: Assume $(X, *, 0)$ is a $(BZ - A)$ and $H = \{< \partial, (\zeta_H(\partial), \vartheta_H(\partial)) > | \partial \in X\}$ be (IFS) of X . H is intuitionistic fuzzy BZ -ideal ($IFBZ - I$) of X if satisfies: (1) $\zeta_H(0) \geq \zeta_H(\partial)$ and $\vartheta_H(0) \leq \vartheta_H(\partial)$, (2) $\zeta_H(\partial * \delta) \geq \min\{\zeta_H(\partial), \zeta_H(\delta)\}$ & $\vartheta_H(\partial)(\partial * \delta) \leq \max\{\vartheta_H(\partial), \vartheta_H(\delta)\}, \forall \partial, \delta \in X$.

Example 3.4: Assume $X = \{0, \rho, \theta, \delta, t\}$ with binary operation $(*)$ defined as table (3) and define (IFS) $U = \{< \partial, (\zeta_U(\partial), \vartheta_U(\partial)) > | \partial \in X\}$ of X by table (4).

Table 3
 $(X, *, 0)$ is $(BZ - A)$

*	0	ρ	θ	δ	t
0	0	0	0	0	0
ρ	ρ	0	ρ	0	0
θ	θ	θ	0	0	0
δ	δ	δ	ρ	0	0
t	t	δ	t	δ	0

Table 4
 U is $(IFBZ - I)$ of X .

∂	0	ρ	θ	δ	t
$\zeta_U(\partial)$	0.8	0.3	0.5	0.3	0.6
$\vartheta_U(\partial)$	0.1	0.7	0.4	0.7	0.2

Then $(X, *, 0)$ is $(BZ - A)$. Also, U is $(IFBZ - I)$ of X .

Lemma 3.5: Assume K and G are two (IFSSs) in a nonempty set X , if $K - G$ is $(IFBZ - I)$ of X then $K \cap G^c$ is $(IFBZ - I)$ of X also.

Proof: Suppose that $K = \{< \delta, (\zeta_K(\delta), \vartheta_K(\delta)) > | \delta \in X\}$ and $G = \{< \delta, (\zeta_G(\delta), \vartheta_G(\delta)) > | \delta \in X\}$ are two (IFSSs) of X , let $K - G$ be $(IFBZ - I)$ of X . Then, we get (1) $\zeta_{K-G}(0) \geq \zeta_{K-G}(\delta) \& \vartheta_{K-G}(0) \leq \vartheta_H(\delta)$, (2) $\zeta_{K-G}(\delta * \theta) \geq \min\{\zeta_{K-G}(\delta), \zeta_{K-G}(\theta)\} \& \vartheta_{K-G}(\delta * \theta) \leq \max\{\vartheta_{K-G}(\delta), \vartheta_{K-G}(\theta)\}$. However, $G^c = \{< \delta, (\vartheta_K(\delta), \zeta_G(\delta)) > | \delta \in X\}$. This implies that $K \cap G^c = \{< \delta, (\min\{\zeta_K(\delta), \vartheta_G(\delta)\}, \max\{\vartheta_K(\delta), \zeta_G(\delta)\}) > | \delta \in X\} = K - G$. Hence, we get (1) $\zeta_{K \cap G^c}(0) = \zeta_{K-G}(0) \geq \zeta_{K-G}(\delta) = \zeta_{K \cap G^c}(\delta) \& \vartheta_{K \cap G^c}(0) = \vartheta_{K-G}(0) \leq \vartheta_H(\delta) = \vartheta_{K \cap G^c}(\delta)$, and (2) $\zeta_{K \cap G^c}(\delta * \theta) = \zeta_{K-G}(\delta * \theta) \geq \min\{\zeta_{K-G}(\delta), \zeta_{K-G}(\theta)\} = \min\{\zeta_{K \cap G^c}(\delta), \zeta_{K \cap G^c}(\theta)\}$ and $\vartheta_{K \cap G^c}(\delta * \theta) = \vartheta_{K-G}(\delta * \theta) \leq \max\{\vartheta_{K-G}(\delta), \vartheta_{K-G}(\theta)\} = \max\{\vartheta_{K \cap G^c}(\delta), \vartheta_{K \cap G^c}(\theta)\}$. Then $K \cap G^c$ is $(IFBZ - I)$ of X also.

Lemma 3.6: Assume H and D are two (IFSSs) in a nonempty set X , if $H - D$ is $(IFBZ - I)$ of X then $D^c - H^c$ is $(IFBZ - I)$ of X also.

Proof: Assume $H = \{< \theta, (\zeta_H(\theta), \vartheta_H(\theta)) > | \theta \in X\}$ and $D = \{< \theta, (\zeta_D(\theta), \vartheta_D(\theta)) > | \theta \in X\}$ are two (IFSSs) of X , let $H - D$ be $(IFBZ - I)$ of X . Then, we get (1) $\zeta_{H-D}(0) \geq \zeta_{H-D}(\theta) \& \vartheta_{H-D}(0) \leq \vartheta_H(\theta)$, (2) $\zeta_{H-D}(\theta * \partial) \geq \min\{\zeta_{H-D}(\theta), \zeta_{H-D}(\partial)\} \& \vartheta_{H-D}(\theta * \partial) \leq \max\{\vartheta_{H-D}(\theta), \vartheta_{H-D}(\partial)\}$. We get $H^c = \{< \theta, (\vartheta_H(\theta), \zeta_H(\theta)) > | \theta \in X\}$ and $D^c = \{< \theta, (\vartheta_D(\theta), \zeta_D(\theta)) > | \theta \in X\}$, it implies that $D^c - H^c = \{< \theta, (\min\{\zeta_H(\theta), \vartheta_D(\theta)\}, \max\{\vartheta_H(\theta), \zeta_D(\theta)\}) > | \theta \in X\} = H - D$. Hence, we get (1) $\zeta_{D^c-H^c}(0) = \zeta_{H-D}(0) \geq \zeta_{H-D}(\theta) = \zeta_{D^c-H^c}(\theta) \& \vartheta_{D^c-H^c}(0) = \vartheta_{H-D}(0) \leq \vartheta_H(\theta) = \vartheta_{D^c-H^c}(\theta)$, and (2) $\zeta_{D^c-H^c}(\theta * \partial) = \zeta_{H-D}(\theta * \partial) \geq \min\{\zeta_{H-D}(\theta), \zeta_{H-D}(\partial)\} = \min\{\zeta_{D^c-H^c}(\theta), \zeta_{D^c-H^c}(\partial)\} \& \vartheta_{D^c-H^c}(\theta * \partial) = \vartheta_{H-D}(\theta * \partial) \leq \max\{\vartheta_{H-D}(\theta), \vartheta_{H-D}(\partial)\} = \max\{\vartheta_{D^c-H^c}(\theta), \vartheta_{D^c-H^c}(\partial)\}$. Then $D^c - H^c$ is $(IFBZ - I)$ of X .

Proposition 3.7: Assume $(X, *, 0)$ is a $(BZ - A)$ & $K = \{< \theta, (\zeta_K(\theta), \vartheta_H(\theta)) > | \theta \in X\}$ be (IFS) of X . If K is an $(IFBZ - I)$ of X , then K is an $(IFBZ - SA)$.

Proof: The proof is obvious from [Definition (3.3)-(2)].

Lemma 3.8: Assume K and D are two (IFSSs) in a nonempty set X , if $K - D$ is $(IFBZ - SA)$ of X then; (1) $K \cap D^c$ is $(IFBZ - SA)$ of X , (2) $D^c - K^c$ is $(IFBZ - SA)$ of X .

Proof: In a similar manner to the second part of Lemmas (2.5) and (2.6) proofs, we consider that (1) and (2), respectively, are valid.

Proposition 3.9: Assume $(X, *, 0)$ is a $(BZ - A)$ & $H = \{< \rho, (\zeta_H(\rho), \vartheta_H(\rho)) > | \rho \in X\}$ be (IFS) of X . If H is a $(IFBZ - SA)$ of X , then $(t - \text{cut}-H) H_t$ is a $(BZ - SA)$ of X , for any $t \in [0, 1]$.

Proof: Assume that $H = \{< \rho, (\zeta_H(\rho), \vartheta_H(\rho)) > | \rho \in X\}$ is an $(IFBZ - SA)$ of X , and let $\rho, \theta \in X$ with $\rho \in H_t$ and $\theta \in H_t$, where $H_t = \{\rho \in X; \zeta_H(\rho) \geq t \& \vartheta_H(\rho) \leq t\}$ then $\zeta_H(\rho) \geq t$, $\zeta_H(\theta) \geq t$, $\vartheta_H(\rho) \leq t$ and $\vartheta_H(\theta) \leq t$. Since H is a $(IFBZ - SA)$, it follows that $\zeta_H(\rho * \theta) \geq \min\{\zeta_H(\rho), \zeta_H(\theta)\} \geq t$ and $\vartheta_H(\rho * \theta) \leq \max$

$\{\vartheta_H(\rho), \vartheta_H(\theta)\} \leq t$, thus $\zeta_H(\rho * \theta) \geq t$ and $\vartheta_H(\rho * \theta) \leq t$ and this implies that $(\rho * q) \in H_t$. Hence $(t - \text{cut} - H) \ H_t$ is a $(BZ - SA)$ of X , for any $t \in [0, 1]$.

Proposition 3.10: Assume $(X, *, 0)$ is a $(BZ - A)$ & $H = \{< \rho, (\zeta_H(\rho), \vartheta_H(\rho)) > \mid \rho \in X\}$ be (IFS) of X . If for any $t \in [0, 1]$, the $(t - \text{cut} - H) \ H_t$ is a $(BZ - SA)$ of X , then H is a $(IFBZ - SA)$ of X .

Proof We need to show that $\zeta_H(\rho * \theta) \geq \min\{\zeta_H(\rho), \zeta_H(\theta)\}$ and $\vartheta_H(\rho * \theta) \leq \max\{\vartheta_H(\rho), \vartheta_H(\theta)\}$, for any $\rho, \theta \in X$. Assume that there exist $\rho', \theta' \in X$ such that $\zeta_H(\rho' * \theta') < \min\{\zeta_H(\rho'), \zeta_H(\theta')\}$ or $\vartheta_H(\rho' * \theta') > \max\{\vartheta_H(\rho'), \vartheta_H(\theta')\}$. If $\zeta_H(\rho' * \theta') < \min\{\zeta_H(\rho'), \zeta_H(\theta')\}$, putting $t' = (\zeta_H(\rho' * \theta') + \min\{\zeta_H(\rho'), \zeta_H(\theta')\})/2$, thus $0 \leq t' < \min\{\zeta_H(\rho'), \zeta_H(\theta')\} \leq 1$, hence $\zeta_H(\rho') > t'$ and $\zeta_H(\theta') > t'$, which imply that $\rho' \in U(\zeta_H, t') = \{\rho \in X \mid \zeta_H(\rho) \geq t'\}$ & $\theta' \in U(\zeta_H, t') = \{\rho \in X \mid \zeta_H(\rho) \geq t'\}$, since $U(\zeta_H, t')$ is a $(BZ - SA)$, it follows that $\rho' * \theta' \in U(\zeta_H, t')$, and that $\zeta_H(\rho' * \theta') \geq t'$, but this is a contradiction. Also, if $\vartheta_H(\rho' * \theta') > \max\{\vartheta_H(\rho'), \vartheta_H(\theta')\}$, putting $t' = (\vartheta_H(\rho' * \theta') + \max\{\vartheta_H(\rho'), \vartheta_H(\theta')\})/2$, then $0 \leq t' > \max\{\vartheta_H(\rho'), \vartheta_H(\theta')\} \leq 1$, hence $\vartheta_H(\rho') < t'$ and $\vartheta_H(\theta') < t'$, which imply that $\rho' \in L(\vartheta_H, t') = \{\rho \in X \mid \vartheta_H(\rho) \leq t'\}$ and $\theta' \in L(\vartheta_H, t') = \{\rho \in X \mid \vartheta_H(\rho) \leq t'\}$, since $U(\vartheta_H, t')$ is a $(BZ - SA)$, it follows that $\rho' * \theta' \in L(\vartheta_H, t')$, and that $\vartheta_H(\rho * q') \leq t'$, but this is a contradiction. Then H is a $(IFBZ - SA)$ of X .

Proposition 3.11: If $(X, *, 0)$ is a $(BZ - A)$, then any $H = \{< \rho, (\zeta_H(\rho), \vartheta_H(\rho)) > \mid \rho \in X\}$ $(IFBZ - SA)$ of X such that $\zeta_H(0) \geq \zeta_H(\rho)$ and $\vartheta_H(0) \leq \vartheta_H(\rho)$.

Proof: From [Proposition (2.5)-(P₂)], we have $p * p = 0$, then $\zeta_H(0) = \zeta_H(\rho * \rho) \geq \min\{\zeta_H(\rho) * \zeta_H(\rho)\} = \zeta_H(\rho)$. Also, $\vartheta_H(0) = \vartheta_H(\rho * \rho) \leq \max\{\vartheta_H(\rho) * \vartheta_H(\rho)\} = \vartheta_H(\rho)$. Hence, $\zeta_H(0) \geq \zeta_H(\rho)$ and $\vartheta_H(0) \leq \vartheta_H(\rho)$.

Proposition 3.12: Assume $H = \{< \rho, (\zeta_H(\rho), \vartheta_H(\rho)) > \mid \rho \in X\}$ is (IFS) of X , there exists as sequence $\{\rho_n\}$ in X such that $\zeta_H(\rho_n) = 1$ & $\vartheta_H(\rho_n) = 0$ then $\zeta_H(0) = 1$ & $\vartheta_H(0) = 0$.

Proof: From [proposition (3.11)] we get $\zeta_H(0) \geq \zeta_H(\rho)$, for all $\rho \in X$, therefore $\zeta_H(0) \geq \zeta_H(\rho_n)$ for every positive integer n . Let $\zeta_H(\rho_n) = 1$. Hence $1 \geq \zeta_H(0) \geq \zeta_H(\rho_n) = 1$. Then $\zeta_H(0) = 1$. Also, from [proposition (3.11)] we obtain that $\vartheta_H(0) \leq \vartheta_H(\rho)$, $\forall \rho \in X$. Thus $\vartheta_H(0) \leq \vartheta_H(\rho_n)$, for every positive integer n . Now, $0 \leq \vartheta_H(0) \leq \vartheta_H(\rho) = 0$. Hence $\vartheta_H(0) = 0$.

Proposition 3.13: Assume $H = \{< p, (\zeta_H(\partial), \vartheta_H(\partial)) > \mid \partial \in X\}$ is $(IFBZ - SA)$ of X . Then for all $\partial \in X$, $\zeta_H(0 * \partial) \geq \zeta_H(\partial)$ and $\vartheta_H(\partial * 0) \leq \vartheta_H(\partial)$.

Proof: For all $\partial \in X$, $\zeta_H(0 * \partial) \geq \min\{\zeta_H(0), \zeta_H(\partial)\} = \min\{\zeta_H(\partial * \partial), \zeta_H(\partial)\} \geq \min\{\min\{\zeta_H(\partial), \zeta_H(\partial)\}, \zeta_H(\partial)\} = \zeta_H(\partial)$. Also, $\vartheta_H(0 * \partial) \leq \max\{\vartheta_H(0), \vartheta_H(\partial)\} = \max\{\vartheta_H(\partial * \partial), \vartheta_H(\partial)\} \leq \max\{\max\{\vartheta_H(\partial), \vartheta_H(\partial)\}, \vartheta_H(\partial)\} = \vartheta_H(\partial)$. Hence, $\zeta_H(0 * \partial) \geq \zeta_H(\partial)$ and $\vartheta_H(\partial * 0) \leq \vartheta_H(\partial)$, $\forall \partial \in X$.

4. Conclusion

In this study, classes of (IFS) in $(BZ - A)$ are introduced, including $(IFBZ - SA)$ and $(IFBZ - I)$ of X . Next, the operations on intuitionistic fuzzy sets of $(BZ - A)$ are examined. Some interesting results are given in this work. In future work, some other

operations on intuitionistic fuzzy sets of $(BZ - A)$, like addition $\oplus$, multiplication $\otimes$, symmetric difference Δ , Cartesian product $\times$, and others all of them will be investigated and discussed their relations with these classes of (IFS) in $(BZ - A)$.

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