Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 28 (2025), No. 4-A, pp. 1145–1151
DOI : 10.47974/JDMSC-2120

New structure of bipolar fuzzy rho-ideals in rho-algebraic semigroups

Manal Al-Labadi [†]
Department of Mathematics
Faculty of Arts and Sciences
University of Petra
Amman
Jordan

Shuker Khalil *
Ahmed Naji Hassan [§]
Department of Mathematics
College of Science
University of Basrah
Basrah
Iraq

Wasim Audeh [‡]
Department of Mathematics
Faculty of Arts and Sciences
University of Petra
Amman
Jordan

Abstract

In algebraic semigroups when we deal with classical sets every ρ-ideal does not have to be $\overline{\rho}$-ideal. As a result, it is demonstrated in this study with their expansion in some kinds of sets as fuzzy set (FS) & bipolar fuzzy set (BFS) is held. Also, a new construction of (BFS) is demonstrated in ρ-algebras. The study of novel notions such as ρ-semigroup (ρ-SG), ρ-subsemigroup (ρ-SSG), $\overline{\rho}$-ideal semigroup ($\overline{\rho}$-ISG), ρ-ideal semigroup (ρ-ISG), and other

[†] *E-mail:* manal.allabadi@uop.edu.jo
* *E-mail:* shuker.khalil@uobasrah.edu.iq (Corresponding Author)
[§] *E-mail:* ahmed.naji@uobasrah.edu.iq
[‡] *E-mail:* waudeh@uop.edu.jo

© Taru Publications

are given. Moreover, their extensions in fuzzy logic (FL) using Bipolar fuzzy are introduced. Additionally, a number of basic characteristics of our concepts are investigated and explored.

Subject Classification: *Primary 06F35, Secondary 03G25.*

Keywords: *Fuzzy sets theory, Bipolar fuzzy set, Rho-algebras, Semigroups, Ideals, Subalgebra.*

1. Introduction

A notion for Fuzzy set (FSs) is established in 1965 [1]. Subsequently, their extensions, namely fuzzy soft ([2]-[5]) and bipolar fuzzy ([6],[7]), are demonstrated and investigated for their characteristics and uses in several domains in our everyday existence. Bipolar fuzzy sets ($BFSs$) is extension of (FSs) that membership (MD) range in $[-1,1]$. Within a bipolar FSs, an element's (MD) mentions it just enough so that the element's property and (MD) [1,0] mention it in a way that implies the element's implied counter-property ([8]). Jun et al. presented classes for fuzzy ideal in $BCK/BCI-$algebras in 2009 [8]. In 2020, Khalil and Hameed [9] proposed fuzzy algebra ideas like as fuzzy $\rho-$subalgebra ($F\rho-SA$), fuzzy $\rho-$ideal ($F\rho-I$), and fuzzy $\bar{\rho}-$ideal ($F\bar{\rho}-I$). In this paper, we deal with classical sets every $\rho-$ideal does not have to be $\bar{\rho}-$ideal. As a result, it is demonstrated in this study with their expansion in (FS) and (BFS) is held. Furthermore, a new construction of (BFS) is demonstrated in $\rho-$algebras. The study of novel notions such as $\rho-$semigroup ($\rho-SG$), $\rho-$subsemigroup ($\rho-SSG$), $\bar{\rho}-$ideal semigroup($\bar{\rho}-ISG$), $\rho-$ideal semigroup ($\rho-ISG$), and other are given. Finally, to extend our results to additional classes of algebras, we looked at the conclusion of the study and the direction of future research. Fuzzy set studied is also in [12].

2 Basic Fundemntals

This section demonstrates the prerequisites for algebraic ideas.

Definition 2.1: [9] Let $\delta\in\pi$. This term $(\pi,\sim,\delta)$ called $\rho-$algebra (for short, $(\rho-A)$) where $\sim:\pi\times\pi\to\pi$ satisfies: (1) $\omega\sim\omega=\delta$, (2) $\delta\sim\omega=\delta$, (3) $\omega\sim c=\delta$ and $c\sim\omega=\delta\to\omega=c$, (4) $\omega\sim c=c\sim\omega\neq\delta,\forall\omega\neq c\in\pi-\{\delta\}$. Also, let $\emptyset\neq K\subseteq\pi$, K is said to be $\rho-$subalgebra $(\rho-SA)$ of π, if $\omega\sim c\in K,\forall\omega,c\in K$. Assume $\leq$ is a relation on $(\pi,\sim,\delta)$ such that $\omega\leq c\leftrightarrow\omega\sim c=\delta$. Moreover, $(\pi,\leq)$ is a partially ordered set and $\delta\leq\omega,\forall\omega\in\pi$.

Definition 2.2: [9] Assume that $(\pi,\sim,\delta)$ is $(\rho-A)$ and $\emptyset\neq K\subseteq\pi$. K is said to be a $\rho-$ideal $(\rho-I)$ of π if such that: (1) $\omega\sim c\in K,\forall\omega,c\in K$, (2) If $\omega\sim c\in K$ and $c\in K$, then $a\in K$.

Definition 2.3: [9] Assume that $(\pi,\sim,\delta)$ is $(\rho-A)$ and $\emptyset\neq K\subseteq\pi$. We say K is a $\bar{\rho}-$ideal $(\bar{\rho}-I)$ of π if such that: (1) $\delta\in K$, (2)If $a\in K$ and $c\in\pi$, then $a\sim c\in K$.

Definition 2.4: [1] Let $\pi \neq \emptyset$. A function $\mathcal{Q}$: $W \to [0,1]$ is (FS) for W.

Definition 2.5: ([9]) Assume $\mathcal{Q}$ is a (FS) in $(\rho - A)$. $\mathcal{Q}$ is said to be a fuzzy $\rho -$ subalgebra $(\rho F - SA)$ if $\mathcal{Q}(\omega{\sim}c) \geq min\{\mathcal{Q}(\omega), \mathcal{Q}(c)\}, \forall \omega, c \in \pi$.

Definition 2.6: [7] Let $\mathcal{Q}$: $\pi \to [0,1]$ be a (FS) & $D(\mathcal{Q}, p) = \{\omega \in \pi | \mathcal{Q}(\omega) \geq p\}$, where $0 \leq \mathcal{Q}(\omega) \leq 1$ and $p \in [0,1]$.

Definition 2.7: ([10, 11]) the bipolar (BFS) is $(\mathcal{Q}^-, \mathcal{Q}^+)$, where $\mathcal{Q}^-: \pi \to [-1,0]$ & $\mathcal{Q}^+: \pi \to [0,1]$. Let $U = (a, \mathcal{Q}^-, \mathcal{Q}^+)$ be a (BFS) and $(\sigma, \partial) \in [-1,0] \times [0,1]$. We say $U_\sigma^- = \{a \in \pi | \mathcal{Q}^-(a) \leq \sigma\}$ negative $\sigma -$cut for U & $U_\partial^+ = \{a \in \pi | \mathcal{Q}^+(a) \geq \partial\}$ positive $\partial -$ cut for U.

3. New Structures for Bipolar Fuzzy Ideal.

Definition 3.1: $(\pi, \sim, \propto, \delta)$ is a $\rho -$ semigroup [for short, $(\rho - SG)$] if (1) $(\pi, \sim, \delta)$ is a $\rho -$ algebra, (2) $(\pi, \propto)$ is a semigroup (SG), (3) $\omega \propto (\vartheta \sim \tau) = (\omega \propto \vartheta) \sim (\omega \propto \tau)$ and $(\omega \sim \vartheta) \propto \tau = (\omega \propto \tau) \sim (\vartheta \propto \tau), \forall \omega, \vartheta, \tau \in \pi$.

Example 3.2: Let $\pi = \{e, f, g, h\}$ and $\omega \propto \vartheta = e, \forall \omega, \vartheta \in \pi$. Hence $(\pi, \propto)$ is a (SG). Assume $(\sim)$ is defined as table (1). Hence $(\pi, \sim, \propto, e)$ is $(\rho -$ SG). Also, let $F = \{f_i | f_i \in N; 1 \leq i \leq 4\}$ with $f_i < f_{i+1}, \forall 1 \leq i \leq 3$, and $\varepsilon \propto r = e, \forall \varepsilon, r \in F$. Hence $(F, \sim, \propto, f_1)$ is a $(\rho - SG)$. As well, determine $(\sim)$ as table (2).

Table 1
$(\pi, \sim, \propto, e)$ is a $(\rho - SG)$

$\sim$	e	f	g	h
e	e	e	e	e
f	f	e	g	g
g	f	g	e	f
h	h	g	f	e

Table 2
$(F, \sim, \propto, f_1)$ is a $(\rho - SG)$

$\sim$	f_1	f_2	f_3	f_4
f_1	f_1	f_1	f_1	f_1
f_2	f_2	f_1	f_3	f_3
f_3	f_2	f_3	f_1	f_2
f_4	f_4	f_3	f_2	f_1

Definition 3.3: Assume $\emptyset \neq \beta \subseteq \pi$ & $(\pi, \sim, \propto, , \delta)$ is $(\rho - SG)$. Then β is called a $\rho -$ subsemigroup $(\rho - SSG)$of π, if $\omega{\sim}\vartheta \in \beta$ & $\omega \propto \vartheta \in \beta, \forall \omega, \vartheta \in \beta$.

Definition 3.4: Suppose that $\emptyset \neq \beta \subseteq \pi$ and $(\pi, \sim, \propto, , \delta)$ is a $(\rho - SG)$. Then $\beta$is said to be a $\rho -$ ideal semigroup $(\rho - ISG)$ of π, if (1) β is a $(\rho - I)$ of π, (2) $\omega \propto \vartheta \in \beta$ & $\vartheta \propto \omega \in \beta, \forall \omega \in \beta \& \vartheta \in \pi$.

Definition 3.5: Let $(\pi, \sim, \propto, , \delta)$ be a $(\rho - SG)$ & $\emptyset \neq K \subseteq \pi$. K is $\bar{\rho} -$ ideal semigroup $(\bar{\rho} - ISG)$ of π, if (1) K is a $(\bar{\rho} - I)$ of π, (2) $\omega \propto \vartheta \in K$ & $\vartheta \propto \omega \in K, \forall \omega \in K \& \vartheta \in \pi$.

Example 3.6: Take $(\pi, \sim, \propto, , e)$ in Example (3.2) and $K = \{e, f, g\} \subseteq \pi = \{e, f, g, h\}$, Hence K is $(\bar{\rho} - I)$ of π. thus $\omega \propto \vartheta \in K$ & $\vartheta \propto \omega \in K$. So, K is $(\bar{\rho} - ISG)$ of π. Also, $M = \{e, f\}$ is $(\rho - ISG)$ of π. But, M is not $(\bar{\rho} - I)$ of π,

because $f \in M, h \in \pi$, but $f \propto h = g \notin M$. Hence M is not $(\bar{\rho} - ISG)$ of π. $N = \{e, g\}$ is not $(\rho - ISG)$ of π, since $e, g \in N$, but $g \propto e = f \notin N$ and hence N is not $(\rho - I)$ of π . Therefore $N = \{e, g\}$ is not $(\bar{\rho} - ISG)$ of π.

Remark 3.7: By example (3.6), that is not necessary any $(\rho - ISG)$ is $(\bar{\rho} - ISG)$.

Definition 3.8: Assume π and $\acute{\pi}$ are $\rho -$ semigroups. A mapping L: $(\pi, \sim, \propto, \delta)$ $\rightarrow$ $(\acute{\pi}, \acute{\sim}, \acute{\propto}, , \acute{\delta})$ is$\rho -$ semi $-$ group homomorphisms where $L(a \sim \vartheta) = L(a) \acute{\sim} L(\vartheta)$ & $L(a \propto \vartheta) = L(a) \acute{\propto} L(\vartheta), \forall a, \vartheta \in \pi$. Also, $KerL = \{a \in \pi | L(a) = \acute{\delta}\}$ & $ImL = \{L(a) \in \acute{\pi} | a \in \pi\}$ are image & kernel of L, respectively.

Definition 3.9: Let H be (FS) of π and $(\pi, \sim, \propto, , \delta)$ is a $(\rho -$ SG). Then H is said to be a fuzzy $\rho -$ semigroup $(F\rho - SG)$ of π, if every $\omega, \vartheta \in \pi$ such that (1) $H(a \sim \vartheta) \geq min\{H(a), H(\vartheta)\}$, (2) $H(a \propto \vartheta) \geq min\{H(a), H(\vartheta)\}$.

Definition 3.10: Assume H is (FS) of π & $(\pi, \sim, \propto, , \delta)$ is a $(\rho - SG)$. Then H is fuzzy $\rho -$ ideals semigroup $(F\rho - ISG)$ of π, if for any $\omega, r \in \pi$ with (1) $H(\omega \sim r) \geq min\{H(\omega), H(r)\}$, (2) $H(\omega) \geq min\{H(\omega \sim r), H(r)\}$, (3) $H(\omega \propto r) \geq min\{H(\omega), H(r)\}$.

Definition 3.11: Suppose that Q is (FS) in π and $(\pi, \sim, \propto, , \delta)$ is a $(\rho -$ SG). Q is said to be fuzzy $\bar{\rho} -$ ideal semigroup $(F\bar{\rho} - ISG)$ of π, if for any $\mu, \gamma \in \pi$ such that (1) $Q(\delta) \geq Q(\mu)$, (2) $Q(\mu \sim \gamma) \geq min\{Q(\mu), Q(\gamma)\}$, (3) $Q(\mu \propto \gamma) \geq min\{Q(\mu), Q(\gamma)\}$.

Lemma 3.12 *Suppose that Q is a $(F\rho - ISG)$ of π, then (1) $Q(\delta) \geq Q(\mu), \forall \mu \in \pi$, (2) $Q(\mu \sim \gamma) = Q(\gamma \sim \mu)$, $\forall \mu \neq \gamma \in \pi - \{\delta\}$.*

Proof: (1) If Q is a $(F\rho - ISG)$ of π, then from [Definition $(3.10) - (1)$], we obtain $Q(\mu \sim \gamma) \geq min\{Q(\mu), Q(\gamma)\}$, $\forall \mu, \gamma \in \pi$. Put $\mu = \gamma$. Then, $Q(\mu \sim \mu) \geq min\{Q(\mu), Q(\mu)\} = Q(\mu), \forall \mu \in \pi$, so we have $Q(\delta) = Q(\mu \sim \mu) \geq Q(\mu), \forall \mu \in \pi$. (2) Assume Q is a $(F\rho - ISG)$ of π. By [Definition $(2.1) - (4)$], we get $\mu \sim \gamma = \gamma \sim \mu \neq \delta, \forall \mu \neq \gamma \in \pi - \{\delta\}$.So $Q(\mu \sim \gamma) = Q(\gamma \sim \mu), \forall \mu \neq \gamma \in \pi - \{\delta\}$.

Remark 3.13: If Q is a $(F\bar{\rho} - ISG)$ of π, then $Q(\mu \sim \gamma) = Q(\gamma \sim \mu), \forall \mu \neq \gamma \in \pi - \{\delta\}$.

Definition 3.14: If Q is (FS) of π & $(\pi, \sim, \propto, , \delta)$ is a $(\rho -$ SG). Q be bipolar fuzzy $\rho -$ sub $-$ semi $-$ groups $(BF\rho -$ SG) for π, where (1) $Q^+(\varepsilon \sim c) \geq min\{Q^+(\varepsilon), Q^+(c)\}$ & $Q^-(\varepsilon \sim c) \leq max\{Q^-(\varepsilon), Q^-(c)\}$, (2) $Q^+(\varepsilon \propto c) \geq min\{Q^+(\varepsilon), Q^+(c)\}$ & $Q^-(\varepsilon \propto c) \leq max\{Q^-(\varepsilon), Q^-(c)\}$. $\forall \varepsilon, c \in \pi$.

Example 3.15: Take $(\pi, \sim, \propto, , e)$ in Example (3.2). Then $(\pi, \sim, \propto, , e)$ is a $(\rho - SG)$, and let $V = (a, Q^+, Q^-)$ be defined as V= {(e, −0.7,0.8), (f, −0.6, 0.6), (g, −0.4,0.5), (h,−0.3,0.2)}, then V is a $(BF\rho -$ SG) of π.

Definition 3.16 Suppose that $\mathcal{Q}$ is (FS) of π and $(\pi, \sim, \propto, , \delta)$ is a $(\rho - \text{SG})$. Then the (FS) $\mathcal{Q}$ is bipolar fuzzy $\rho -$ ideal semi $-$ groups $(BF\rho - \text{ISG})$ of π, if

(1) $\mathcal{Q}^+(\varepsilon \sim \vartheta) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(\vartheta)\}$ & $\mathcal{Q}^-(\varepsilon \sim \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\vartheta)\}$,

(2) $\mathcal{Q}^+(\varepsilon) \geq min\{\mathcal{Q}^+(\varepsilon \sim c), \mathcal{Q}^+(\vartheta)\}$&$\mathcal{Q}^-(\varepsilon) \leq max\{\mathcal{Q}^-(\varepsilon \sim \vartheta), \mathcal{Q}^-(\vartheta)\}$,

(3) $\mathcal{Q}^+(\varepsilon \propto \vartheta) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(\vartheta)\}$ & $\mathcal{Q}^-(\varepsilon \propto \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\vartheta)\}$, $\forall \varepsilon, \vartheta \in \pi$.

Definition 3.17: Suppose that $\mathcal{Q}$ is a (FS) of π and $(\pi, \sim, \propto, , \delta)$ is a $(\rho - SG)$. Then the (FS) $\mathcal{Q}$ be bipolar fuzzy $\bar{\rho} -$ ideal sem $-$ igroups $(BF\bar{\rho} - \text{ISG})$ of π, if (1) $\mathcal{Q}^+(\delta) \geq \mathcal{Q}^+(\varepsilon)$ & $\mathcal{Q}^-(\delta) \leq \mathcal{Q}^-(\varepsilon)$, (2) $\mathcal{Q}^+(\varepsilon \sim \vartheta) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(\vartheta)\}$ & $\mathcal{Q}^-(\varepsilon \sim \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\vartheta)\}$, (3) $\mathcal{Q}^+(\varepsilon \propto c) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(\vartheta)\}$ & $\mathcal{Q}^-(\varepsilon \propto \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\vartheta)\}$, $\forall \varepsilon, \vartheta \in \pi$. اكتب المعادلة هنا.

Proposition 3.18: Let $(\pi, \sim, \propto, , \delta)$ be a $(\rho - SG)$ and $U = (\varepsilon, \mathcal{Q}^-, \mathcal{Q}^+)$ be a $(BF\rho - \text{ISG})$ of π. Then $\mathcal{Q}^+(\delta) \geq \mathcal{Q}^+(\varepsilon)$ and $\mathcal{Q}^-(\delta) \leq \mathcal{Q}^-(\varepsilon)$.

Proof: If $U = (\varepsilon, \mathcal{Q}^-, \mathcal{Q}^+)$ is $(BF\rho - \text{ISG})$. So $\mathcal{Q}^+(\varepsilon \sim \vartheta) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(c)\}$ & $\mathcal{Q}^-(\varepsilon \sim \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\vartheta)\}$ [from Definition $(3.16) - (1)$], put $\varepsilon = \vartheta$. $\mathcal{Q}^+(\varepsilon \sim \varepsilon) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(\varepsilon)\} = \mathcal{Q}^+(\varepsilon)$ & $\mathcal{Q}^-(\varepsilon \sim \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\varepsilon)\} = \mathcal{Q}^-(\varepsilon)$. But, $\varepsilon \sim \varepsilon = \delta, \forall \varepsilon \in \pi$. Then $\mathcal{Q}^+(\delta) \geq \mathcal{Q}^+(\varepsilon)$ and $\mathcal{Q}^-(\delta) \leq \mathcal{Q}^-(\varepsilon)$.

Lemma 3.19: *Let $(\pi, \sim, \propto, , \delta)$ be a $(\rho - SG)$ and $U = (\varepsilon, \mathcal{Q}^-, \mathcal{Q}^+)$ be a $(BF\rho - ISG)$ of π. Then U is ($BF\bar{\rho} - ISG$) of π.*

Proof: $\mathcal{Q}^+(\delta) \geq \mathcal{Q}^+(\varepsilon)$ and $\mathcal{Q}^-(\delta) \leq \mathcal{Q}^-(\varepsilon)$ [by Proposition (3.18). Also $\mathcal{Q}^+(\varepsilon \sim \vartheta) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(\vartheta)\}$ & $\mathcal{Q}^-(a \sim \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\vartheta)\}$ [by Definition $(3.16) - (1)$] and hence we get $\mathcal{Q}^+(\varepsilon \propto \vartheta) \geq min\{\mathcal{Q}^+(\varepsilon), \mathcal{Q}^+(\vartheta)\}$& $\mathcal{Q}^-(\varepsilon \propto \vartheta) \leq max\{\mathcal{Q}^-(\varepsilon), \mathcal{Q}^-(\vartheta)\}$, $\forall \varepsilon, \vartheta \in \pi$. Then U is $(BF\bar{\rho} - \text{ISG})$ of π.

Remarks 3.20: (1) Lemma (3.19) leads us to consider that any $(BF\rho - \text{ISG})$ is $(BF\bar{\rho} - \text{ISG})$but in classical sets, this is generally not the case, as demonstrated by Example (3.6). (2) We demonstrate in Example (3.21) that not every $(BF\bar{\rho} - \text{ISG})$ of π is $(BF\bar{\rho} - \text{ISG})$ of π in general.

Example 3.21: Let $(F, \sim, \propto, f_1)$ be in example (3.2). Then $(\pi, \sim, \propto, f_1)$ is $(\rho - \text{SG})$. Let $U = (\varepsilon, \mathcal{Q}^-, \mathcal{Q}^+)$ be defined as $U = \{(f_i, -\left(\frac{1}{f_i{}^2}\right), \left(\frac{1}{f_i{}^2}\right)) | 1 \leq i \leq 4\}$. Then U is a $(BF\bar{\rho} - \text{ISG})$ of π, but, it is not $(BF\rho - \text{ISG})$ of π, since $\mathcal{Q}^+(f_3) < min\{\mathcal{Q}^+(f_3 \sim f_1), \mathcal{Q}^+(f_1)\}$.

Proposition 3.22: Let $(\pi, \sim, \propto, , \delta)$ be $(\rho - \text{SG})$ & $U = (\varepsilon, \mathcal{Q}^-, \mathcal{Q}^+)$ be $(BF\rho - \text{ISG})$ of π. If $r < \varepsilon$, then $\mathcal{Q}^+(\varepsilon) \geq \mathcal{Q}^+(r)$ and $\mathcal{Q}^-(\varepsilon) \leq \mathcal{Q}^-(r)$.

Proof: If $U = (\varepsilon, \mathcal{Q}^-, \mathcal{Q}^+)$ is a $(BF\rho - \text{ISG})$, so $\mathcal{Q}^+(\varepsilon) \geq min\{\mathcal{Q}^+(\varepsilon \sim r), \mathcal{Q}^+(r)\}$ & $\mathcal{Q}^-(\varepsilon) \leq max\{\mathcal{Q}^-(\varepsilon \sim r), \mathcal{Q}^-(r)\}$ [by Definition $(3.16) - (2)$]. at $r < \varepsilon$, we

get $\varepsilon\sim r = \delta$. So, $\mathcal{Q}^+(\varepsilon) \geq min\{\mathcal{Q}^+(\varepsilon\sim r), \mathcal{Q}^+(r)\} = min\{\mathcal{Q}^+(\delta), \mathcal{Q}^+(r)\}$ & $\mathcal{Q}^-(\varepsilon) \leq max\{\mathcal{Q}^-(\varepsilon\sim r), \mathcal{Q}^-(\mathrm{r})\} = max\{\mathcal{Q}^-(\delta), \mathcal{Q}^-(r)\}$. By [Proposition (3.18)] we get $min\{\mathcal{Q}^+(\delta),\ \mathcal{Q}^+(r)\} = \mathcal{Q}^+(r)$ and $max = \{\mathcal{Q}^-(\delta), \mathcal{Q}^-(r)\} = \mathcal{Q}^-(r)$. Hence $\mathcal{Q}^+(\varepsilon) \geq \mathcal{Q}^+(r)$ and $\mathcal{Q}^-(\varepsilon) \leq \mathcal{Q}^-(r)$.

Proposition 3.23: Let $(\pi, \sim, \propto, , \delta)$ be a $(\rho - \mathrm{SG})$ and $U = (\varepsilon, \mathcal{Q}^-, \mathcal{Q}^+)$ be a $(BF\rho - \mathrm{SG})$ of π. If $s < \varepsilon\sim r$, then $\mathcal{Q}^+(\varepsilon\sim r) \geq \mathcal{Q}^+(s)$ and $\mathcal{Q}^-(\varepsilon\sim r) \leq \mathcal{Q}^-(s)$.

Proof: The proof is obvious from [Definition $(3.16) - (2)$ & Proposition (3.18)]

Remarks 3.24: (1) if $(\pi, \sim, \propto, , \delta)$ be $(\rho - \mathrm{SG})$ & $U = (a, \mathcal{Q}^-, \mathcal{Q}^+)$ be $(BF\rho - \mathrm{ISG})$ of π. Then U is a $(BF\rho - \mathrm{SG})$ of π [From Definition $(3.16) - (1), (3)$] (2) Assume that $(\pi, \sim, \propto, , \delta)$ is a $(\rho - \mathrm{SG})$ with $U = (a, \mathcal{Q}^-, \mathcal{Q}^+)$ is a $(BF\bar{\rho} - \mathrm{ISG})$ of π. Then U is a $(BF\rho - \mathrm{SG})$ of π [From Definition $(3.17) - (2), (3)$]

Proposition 3.25: Let $(\pi, \sim, \propto, , \delta)$ be a $(\rho - \mathrm{SG})$ and $U = (a, \mathcal{Q}^-, \mathcal{Q}^+)$ be a $(BF\rho - \mathrm{ISG})$ of π. Then (1) $E_1 = \{a \in \pi | \mathcal{Q}^+(a) = \mathcal{Q}^+(\delta)\}$ is $(\rho - \mathrm{ISG})$ of π, (2) $E_2 = \{a \in \pi | \mathcal{Q}^-(a) = \mathcal{Q}^-(\delta)\}$ is $(\rho - \mathrm{ISG})$ of π.

Proof: The proof is obvious from [Proposition (3.18)] & [Definition (3.16)].

Theorem 3.26: *Let* $(\pi, \sim, \propto, , \delta)$ *be a* $(\rho - SG)$ *and* $U = (a, \mathcal{Q}^-, \mathcal{Q}^+)$ *be a* $(BF\rho - ISG)$ *of* π. *(1) If* $U_\sigma^- = \{a \in \pi | \mathcal{Q}^-(a) \leq \sigma\} \neq \emptyset, \forall \sigma \in [-1,0]$, *then* $(U_\sigma^-, \sim, \delta)$ *is* $(\rho - I)$ *of* π. *(2) If* $U_\partial^+ = \{a \in \pi | \mathcal{Q}^+(a) \geq \partial\} \neq \emptyset, \forall \partial \in [0,1]$, *then* $(U_\partial^+, \sim, \delta)$ *is* $(\rho - I)$ *of* π.

Proof: The proof for (1) &(2) are obvious from [Definition (3.16)-(1),(2)].

4. Conclusion

Using (BFS), we are already investigating new classes of ideals in Rho-algebraic semi groups. In addition, this work introduces and discusses novel concepts in (FL). In further work, we will describe additional applications that investigate algebraic logic problems using new (FL) technology.

References

[1] L. A. Zadeh, "Fuzzy Sets," *Information and Control,* vol. 8, pp. 338–353 (1965).

[2] P. Maji, A. Roy, and R. Biswas, "Fuzzy soft sets," *Journal of Fuzzy Mathematics,* vol. 9, no. 3, pp. 589–602 (2001).

[3] S. Mahmood, "Decision Making Using New Category of Similarity Measures and Study Their Applications in Medical Diagnosis

Problems," *Afrika Matematika*, vol. 32, pp. 865–878 (2021). DOI: 10.1007/s13370-020-00866-2.

[4] A. A. Atshan and S. Khalil, "On fuzzy normal RHO-filters and usual RHO-filters of RHO-algebras," *AIP Conference Proceedings*, vol. 2834, no. 1, pp. 080036 (2023).

[5] S. Mahmood and A. Hassan, "Applications of Fuzzy Soft ρ-Ideals in ρ-Algebras," *Fuzzy Information and Engineering*, vol. 10, no. 4, pp. 467–475 (2018).

[6] S. Bashir, R. Mazhar, H. Abbas, and M. Shabir, "Regular ternary semirings in terms of bipolar fuzzy ideals," *Computational and Applied Mathematics*, vol. 39, no. 4, pp. 1–18 (2020).

[7] Y. B. Jun, H. S. Kim, and K. J. Lee, "Bipolar fuzzy translation in BCK/BCI-algebra," *Journal of the Chungcheong Mathematical Society,* vol. 22, no. 3, pp. 399–408 (2009).

[8] G. Muhiuddin, "Bipolar Fuzzy KU-Subalgebras/Ideals of KU-Algebras," *Annals of Fuzzy Mathematics and Informatics,* vol. 8, no. 3, pp. 409–418 (2014).

[9] S. Khalil and F. Hameed, "Applications of Fuzzy ρ-Ideals in ρ-Algebras," *Soft Computing,* vol. 24, pp. 13997–14004 (2020).

[10] K. M. Lee, "Bipolar-Valued Fuzzy Sets and Their Operations," in Proceedings of International Conference on Intelligent Technologies, Bangkok, Thailand, pp. 307–312 (2000).

[11] T. Meghwar, S. Khan Awan, M. Tariq, M. Suleman, and A. A. Shaikh, "Substitutional Based Gauss-Seidel Method for Solving System of Linear Algebraic Equations," *Babylonian Journal of Mathematics*, pp. 1–12 (2024).

[12] S. H. Asaad, I. S. Ahmed, and A. S. Mohammed, "Generalization Fuzzy Completely Intra G-ideals in G-Semi Groups," *Journal of Physics: Conference Series,* vol. 1664, no. 1 (2020). DOI: 10.1088/1742-6596/1664/1/012028.

Received December, 2024