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New resultes for T-prime fuzzy modules

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Abstract

In this paper, A topic for **T**-*fuzzy prime submodule* was studied, which is one for the types for partial fuzzy models. This work was divided into three sections, each for which has a study that begins by giving an introduction to the topic & then giving the basic definitions & then explaining the details for the topic for **T**-*fuzzy prime submodule* supported by valuable & important examples & observations from What is important is to consolidate that concept. The final section deals with finding the relationship between this concept **T**-*fuzzy prime submodule* with some other concepts such as the concept multiplication fuzzy module & also the concept for maximal fuzzy ideal. This section has also been enhanced with many non-examples, observations, & properties that prove that relationship. Finally, many examples & observations were given that demonstrate & serve to demonstrate & prove this concept. This is considered a reinforcement & confirmation for those properties & observations that were addressed in the body for this research, citing many sources that are related to this concept for ours.

Subject Classification: *03B52, 03E72.*

Keywords: *Prime Ғ-module, Ƭ-prime Ғ-module, Primary Ғ-module, Faithful Ғ-module, Multiplication Ғ-module.*

1. Introduction

In [21] Zadah we introduce the concept fuzzy set.Saad Salim & other authors in [1], we interduce the concept for Ғ- prime *ule*(*modules*). Here we will study the concept for Ƭ-Ғ-prime *modules* [2]. This concept was explained & its relationship with other corresponding concepts was explained.

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We reinforce our work by providing many examples & characteristics, the most important for which are: $\mathcal{X}$ is Ţ-F-prime $module$ iff $\mathcal{X}_t$ is Ţ-prime $submodule$**[3, 4].** This feature helped us a lot in the process for linking concepts & properties & moving from the normal state to the ambiguous state. As well as other properties. $\mathcal{X}$ is Ţ-F-prime $module$ iff X_t is Ţ-prime $submodule$**[**7]**.** As well as other important results that were mentioned in this research :Every F-prime $module$ is Ţ- prime $module$, $\mathcal{X}$ is Ţ-F-prime $module$[8], then $ann_R\mathcal{X}$ is Ţ-F-prime $i$deal for R [9] ,& in section three we study the the relationships between Ţ- F prime $modules$ with another F-$modules$.Giving some examples & notes that illustrate this relationship .This important for these properties is: Let $\mathcal{X}$ be $m$ultiplication F-$module$ then $\mathcal{X}$ is Ţ-F-prime $module$ iff $ann_R\mathcal{X}$ is Ţ-F prime $i$deal for R. As well as many found in the search. The aim for this research is to study the class for fuzzy modules for the type (T) over a wider area by applying it to other types for fuzzy modules & finding new results that have not been discussed previously.

2. T-prime fuzzy modules

T-prime module is defined as a module over ring if (0) T-prime submodule where T-prime submodule is satisfied the following ,if r∈R ,x∈M ,rx∈N implies x∈N or r^2∈[N:M].

We present the notion for Ţ-F-prime $module$, which is a generalization for F-prime $module$. Specifically, a F- $module$ $\mathcal{X}$ is considered Ţ-F-prime $module$ iff (0_t) is a Ţ-F-prime $submodule$ for $\mathcal{X}$. For example, if $\mathcal{X}$ is a Ţ-$\mathcal{X}$-prime $module$, then for any non-zero $p$roper F- $module$ $\mathcal{K}$ for $\mathcal{X}$, $ann_R\mathcal{X}$ is a Ţ-F-$prime$ $i$deal.

Definition 2.1: Let $\mathcal{X}$ be a F-$module$ for an R- $module$ $\mathcal{M}$. $\mathcal{X}$ is said to Ţ-F-prime $module$ if (O_t) is a Ţ-F-prime $module$ for $\mathcal{X}$ that is $\{r_\ell: r_\ell^2\ x_t^2 \subseteq O_t$ for $x_t^2 \subseteq \mathcal{X}$ & r_ℓ be a F-$s$ingleton for R. $e$quvialeity: A ring R is said to be Ţ-F-prime ring if (O_t) is Ţ-F-prime $i$deal for R.

Proposition 2.2: [2,12,15] $\mathcal{X}$ is Ţ-F-prime $module$ iff $\mathcal{X}_t$ is Ţ-prime $submodule$**.**

Proof: Define: $\mathcal{X}$: $\mathcal{M} \longrightarrow$ [0,1] as follows $\mathcal{X}(x) = \begin{cases} 1 & \text{if } x \in M \\ 0 & \text{otherwise} \end{cases}$ & define (0_1) $(\mathcal{Y}) = \begin{cases} 1 & \text{if } \mathcal{Y} = 0 \\ 0 & \text{otherwise} \end{cases}$

It is clear that $(0_1)_t \leq 0_t$ & $(0_1)_t = 0_t$& $\mathcal{X}_t$=$\mathcal{M}$, but (0_1) is Ţ-F-$submodule$

$\Leftarrow$ $(0_1)_t$ is Ţ-$submodule$ for $\mathcal{X}_t$. That is (0_t) is Ţ-prime $submodule$for $\mathcal{M}$.

Examples & Remarks (2.3):

1. Every Ғ-$prime\ module$ isҬ-Ғ-prime $module$.

Proof: Let $\mathcal{X}$ be $prime$, so (0_t) is the Ғ-prime $submodule$. Thus by (Remark & Examples (2.3) (1) [2]) $(0_t\ )$ is Ҭ-Ғ-prime $submodule$. Hence $\mathcal{X}$ is a Ҭ-Ғ-prime $module$.by the proposition (2.1).

2. The converse of (1) is not true. For example:
Let $\mathcal{N}$=$\mathcal{Z}_4$, $\mathcal{M}$=$\mathcal{Z}$, $\mathcal{A}$: $\mathcal{M} \to$ [0,1] as follows
$\mathcal{A}(x)$=$\begin{cases}1 \text{ if } x \in \mathcal{N} \\ 0 \text{ oterwise}\end{cases}$ $\forall t \in (0,1]$, $define$ $\mathcal{B}$: $\mathcal{M} \to$ [0,1] as follows $\mathcal{B}(x)$=$\begin{cases} t \text{ if } x \in \mathcal{M} \\ 0 \text{ otherwise}\end{cases}$. clearly $\mathcal{A}$ aҒ-$module$ & $\mathcal{A}_t = \mathcal{N}$
$\mathcal{A}_t = \mathcal{Z}_4$ a Ҭ-prime $module$ of $\mathcal{Z}$. [2,11,13,17]. $\mathcal{A}_t = \mathcal{N}$ is Ҭ-prime $module$,so $\mathcal{A}$ Ҭ-Ғ-prime $module$ by (2.2) & $\mathcal{Z}$ Ҭ-$prime\ module$ so $\mathcal{Z}$ is not $prime\ module$. Thus $\mathcal{A}_t$= $\mathcal{N}$ not prime $module$ & $\mathcal{Z}_4$ not Ғ- prime $module$. Since (0_t) is a Ҭ-Ғ-prime $module$. $therefore$ $\mathcal{X}$ $is\ not$ Ғ-prime $module$ [8,14,16,18].

3. Every $s$imple Ғ-$module$ is Ҭ-Ғ-prime $module$.

Proof: let $\mathcal{X}$ be a, Ғ-$module$, $\mathcal{X}$ is called a $s$imple Ғ-$module$ if $\mathcal{X}$ has no $p$roper Ғ-$submodule$ [9, 22-23]. Since every $s$imple is Ғ-prime $module$ [6,19,20]. So by (1) we set the result.

4. The converse for (3) is not true for example $\mathcal{Z}$ as $\mathcal{Z}$-$module$ is a Ҭ-Ғ-prime $module$, which is not $s$imple.

5. Every Ҭ-Ғ-prime $module$ is a Ғ-$p$rimary $module$, where a Ғ-$module$ $\mathcal{X}$ is $p$rimary if (0_t) is the Ғ-$p$rimary $submodule$ for $\mathcal{X}$.

Proof: Let $\mathcal{X}$ is a Ҭ-Ғ-prime $module$, since $\mathcal{X}$ is Ҭ-Ғ-prime $submodule$. & by (Remarks a example (2.1), (4)), so (0_t) is a Ғ-$p$rimary $submodule$. Hence $\mathcal{X}$ is a Ғ-$p$rimary $module$.

6. The converse of (5) is false for example;
Let $\mathcal{X}$=$\mathcal{Z}$, R=$\mathcal{Z}$ & $\mathcal{W}$=$\mathcal{Z}_8$, Define $\mathcal{A}$:$\mathcal{X} \to$ [0,1] As follow:

$$\mathcal{A}(x)=\begin{cases}1\ if\ x \in \mathcal{W} \\ 0\ otherwise\end{cases}$$

& $\mathcal{B}$:$\mathcal{X} \to$ [0,1] as follows $\mathcal{B}(x)$=$\begin{cases} t\ if\ x \in \mathcal{W} \\ 0\ otherwise\end{cases}$ $\forall\ t \in (0,1]$ clearly $\mathcal{A}$ & $\mathcal{B}$ Ғ- $module$ & $\mathcal{A}_t$=$\mathcal{X}$, $\mathcal{B}_t = \mathcal{W}$. But $\mathcal{W}$ is $p$rimary since $2 \cdot 4 \subseteq \mathcal{W}$ & $4=2^2 \nsubseteq [\mathcal{Z}_8, \mathcal{Z}] = \mathcal{Z}_8$, so $\mathcal{W}$ not Ҭ-$p$rimary $module$. Thus $\mathcal{A}_t$ is not Ҭ-prime $module$. clearly $\mathcal{A}$ & $\mathcal{B}$ Ғ-$module$. Hence $\mathcal{B}$ not Ҭ-Ғ-prime $module$. Because (0_t) is not a Ҭ-Ғ-prime $submodule$ because $2.4=0 \subseteq (0_t)$, but $2^2=4 \nsubseteq [(0_t):_{\mathcal{Z}} \mathcal{Z}_8]= 8\mathcal{Z}$.

7. The $\mathcal{Z}$-*module* $\mathcal{Z}_6$ is not ϒ-F-prime *module* (which does not be *p*rimary [2,4] since (0_t) is not a ϒ-F-prime *submodule* because 3.2 = 6⊆ (6), but $2 \subseteq (0_t)$ & $3^2 = 9 \subseteq [(0_t) :_{\mathcal{Z}} \mathcal{Z}_6] = 6\mathcal{Z}$.

Proposition 2.4: Let $\mathcal{X}$ be a F-*module*. So we have an equivalent statements:-

1) $\mathcal{X}$ ϒ-F-prime *module*.

2) $a_t^2 \subseteq ann\mathcal{X}, a_t$ be F-singleton for R, iff $a_t^2 \subseteq ann\, c_\ell$, for each c≠ 0_t, $c_\ell \subseteq \mathcal{X}$. $a_t^2 \subseteq ann\mathcal{X}$, a_t is F-singleton for R, iff $a_t^2 \subseteq ann\, \mathcal{K}$, for each non-zero F-*submodule* $\mathcal{K}$ for $\mathcal{X}$.

Proposition 2.5: $\mathcal{X}$ is ϒ-F-prime *module*, then $ann_R\mathcal{X}$ is ϒ-F- prime *i*deal for R.

The converse for Proposition (2.5) is false as the following example shows.

Example 2.6: Consider the $\mathcal{Z}$-*module* $\mathcal{Z}_8$ is not a ϒ-F-prime *module* since 2.4= $0 \subseteq (0_t)$, 4⊈(0_t) but $2^2 = 4 \nsubseteq ann\ \mathcal{Z}_8 = 8\mathcal{Z}$. On the other h& for every non zero *submodule* $\mathcal{K}$ for $\mathcal{Z}_8$, $ann_{\mathcal{Z}}\ \mathcal{K}$ is a ϒ-F-prime *i*deal.

Corollary 2.7: *Assume* $\mathcal{X}$ *a* ϒ-F-*prime module, then* $ann_R\mathcal{K}$ *is the* ϒ-F-*prime ideal for R. for any non-zero submodule* $\mathcal{K}$ *for* $\mathcal{X}$.

Proof: Assume $a_\ell\ b_s \subseteq ann_R\mathcal{K}$, where a_ℓ,b_s are be a F-singleton for R, Suppose $b_s^2 \subseteq ann_R\mathcal{K}$, hence $b_s^2 x_t \neq 0_t$ for some $x_t \subseteq \mathcal{K}$, & $a_\ell b_s \subseteq ann_R\, \mathcal{X}$ implies $a_\ell b_s x_t = 0_t$. On the other h&, $\mathcal{X}$ is ϒ-F-prime *module*, so (0_t) is a ϒ-F-prime *submodule* for $\mathcal{X}$ & therefore $a_\ell^2 \subseteq ann_R\mathcal{X}$ & by proposition (2.4) then $a_\ell^2 \subseteq ann_R\mathcal{K}$. thus $ann_R\mathcal{K}$ is a ϒ-F-prime *i*deal in R.

In the following example we show that the converse for (2.2.7). is not true.

Example 2.8: We will give the converse for corollary (2.2.7) which is false, for example: let $\mathcal{X}$ be the $\mathcal{Z}$-*module* $\mathcal{Z} \oplus \mathcal{Z}_9$, define $\mathcal{X}$:$\mathcal{X} \to [0,1]$ as follow

$$\mathcal{A}(\mathcal{X}) = \begin{cases} 1 \text{ if } x \in \mathcal{Z} \oplus \mathcal{Z}_9 \\ 0 \quad \text{otherwise} \end{cases}$$

It is clear that$(0,0)_t$ is not ϒ-F-prime *submodule*. but $ann_{\mathcal{Z}}\mathcal{X} = (0_t)$ is F-prime *i*deal.Thus it is ϒ-F-prime *i*deal in $\mathcal{Z}$, however. by (remark & example (1) [2]), $\mathcal{Z} \oplus \mathcal{Z}_9$ is not *p*rimary, hence is not a ϒ-F-prime *module*.

3. The relationships between $\mathcal{T}$-$\mathcal{F}$-prime *module* with another$\mathcal{F}$- *modules*

Proposition 3.1: Let $\mathcal{X}$ be *m*ultiplication F-*module* then $\mathcal{X}$ is ϒ-F-prime *module* iff $ann_R\mathcal{X}$ is ϒ-F-prime *i*deal for R.

Proof: We get the outcome from corollary (2.1.) & definition (2.1).

Corollary 3.2: *Let $\mathcal{X}$ will be cyclic* Ғ-$module$ *& then* $ann_R\mathcal{X}$ *a* Ƭ-Ғ-*prime ideal iff* $\mathcal{X}$ *be* Ƭ-Ғ-*prime module.*

Proof: Since $\mathcal{X}$ is a cyclic Ғ-$module$, then $\mathcal{X}=(x_t)$ for some $x_t \subseteq \mathcal{X}$.

Such that $r_t m_l \subseteq 0_t$ let $r_t$be a Ғ-singleton for R, let $m_l \subseteq \mathcal{X}$, $m_l \not\subseteq 0_t$,

Since $m_l \subseteq \mathcal{X} = (x_t)$, then $m_l= c_i\, x_t \subseteq \mathcal{X}$ for some $c_i$be a Ғ-singleton for R. then $r_t c_i m_l \subseteq 0_t$ which implies that $r_t c_i \subseteq ann\ \mathcal{X}$, but $ann\mathcal{X}$ be Ƭ-Ғ-prime $i$deal for R. so either $r_t \subseteq ann\mathcal{X}$ or $c_i \subseteq ann\mathcal{X}$, implies $m_l \subseteq 0_t$

Hence $r_t \subseteq ann\mathcal{X} =[0_t{:}\mathcal{X}]$. thus $0_t$is a Ƭ-Ғ-prime $submodule$ & $\mathcal{X}$ is a Ƭ-Ғ- prime $module$ by proposition (2.2).

Proposition 3.3:

1- If $\mathcal{X}$ be Ƭ-Ғ-prime $module$ for R, so R is not necessary be Ƭ-Ғ-prime ring. For instance, the $\mathcal{Z}_6$-$module$ $\mathcal{Z}_2$ is Ƭ-Ғ-prime, but the ring $\mathcal{Z}_6$ is not Ƭ-Ғ-prime ring.

2- If R is Ƭ-Ғ-prime ring, then a Ғ-$module$ $\mathcal{X}$ is not necessary be Ƭ-Ғ-prime $module$, as the following example; the $\mathcal{Z}$-$module$ $\mathcal{Z}_6$is not Ƭ-Ғ-prime $module$, but $\mathcal{Z}$ is a Ƭ-Ғ-prime ring.

Remark 3.4: Let $\mathcal{X}$ be a $f$aithful Ƭ-Ғ-prime $module$, so R be Ƭ-Ғ- prime ring.

Proof: Since $\mathcal{X}$ is Ƭ-Ғ-prime $module$, $ann_R\mathcal{X}$ is Ƭ-Ғ- prime $i$deal for R from(2.6). but $\mathcal{X}$ $f$aithful Ƭ-Ғ-prime $module$, so R beƬ-Ғ- prime ring.

Proposition 3.5: If $\mathcal{X}$ $m$ultiplication Ғ-$f$aithful Ғ-$module$. If R be Ƭ-Ғ-prime ring so $\mathcal{X}$ be Ƭ-Ғ-prime $module$.

Proof: Since R be Ƭ-Ғ-prime ring so (0_t) be the Ƭ-Ғ-prime $i$deal for R, but $\mathcal{X}$ *is a* $f$aithful Ғ-$module$, hence $ann_R\mathcal{X}=(0_t)$. Therefore,$ann_R\mathcal{X}$ is Ƭ-Ғ-prime $i$deal. On the other h&, $\mathcal{X}$ is $m$ultiplication Ғ-$module$. Then by proposition (3.1). $\mathcal{X}$ isƬ-Ғ-prime $module$.

Proposition 3.6: Let $\mathcal{X}$ be Ғ-$module$ for R & $\mathcal{A}$ be Ғ-$i$deal for R, which is contained in $ann_R\mathcal{X}$. then $\mathcal{X}$ is Ƭ-Ғ-$module$ iff $\mathcal{X}$ is Ƭ-Ғ-prime$R/_{\mathcal{A}}$ $module$.

Proof: If $\mathcal{X}$ is Ƭ-Ғ-prime $module$, to show that $\mathcal{X}$ is Ƭ-Ғ- prime $R/_{\mathcal{A}}$-$module$; that is to prove (0_t) is Ƭ-Ғ-prime $R/_{\mathcal{A}}$-$submodule$. Let $(r_\ell+\mathcal{A})\ m_t = 0_t$ for $r_\ell + \mathcal{A} \subseteq R/_{\mathcal{A}}$, $m_t \subseteq \mathcal{X}$ & suppose $m_t \neq 0_t$, then $(r_\ell+\mathcal{A})\ m_t= r_\ell m_t= 0_t$. But (0_t) is a Ƭ-Ғ-prime $submodule$ & $m_t \neq 0_t$, this implies $r_\ell^2 \subseteq ann_R\mathcal{X}$. hence,$r_\ell^2\ m_t= (r_\ell^2 + \mathcal{A})m_t = (r_\ell+\mathcal{A})^2 m_t =0_t$ for all $m_t \subseteq \mathcal{X}$. Therefore, $(r_\ell + \mathcal{A})^2 \subseteq ann_{R/_{\mathcal{A}}}\mathcal{X}$. thus, (0_t) is Ƭ-Ғ-prime$R/_{\mathcal{A}}$– $submodule$ & $\mathcal{X}$ is a Ƭ-Ғ-prime $R/_{\mathcal{A}}$- $module$.

Conversely: Let $r_\ell m_t \subseteq (0_t)$, where r_ℓ be a F-singleton for R, & $m_t \subseteq \mathcal{X}$, & assume that $m_t \neq 0_t$ thus $r_\ell m_t$ = (R + $\mathcal{A}$)m_t= 0_t , so (r_ℓ+ $\mathcal{A}$) $m_t \subseteq (0_t)$. But (0_t) is a Ƭ-F-prime $^{R}/_{\mathcal{A}}$ *-submodule* & $m_t \neq 0_t$, this implies (r_ℓ+ $\mathcal{A}$)^2 $\subseteq ann_R\mathcal{X}$. but ($r_\ell$+ $\mathcal{A}$)^2= r_ℓ^2+ $\mathcal{A}$ = r_ℓ^2. i.e. $r_\ell^2 \subseteq ann_R\mathcal{X}$. hence, $\mathcal{X}$ is Ƭ-F-prime *module*.

Proposition 3.7: Let $\mathcal{X}_1$ & $\mathcal{X}_2$ be two F-*modules* for $\mathcal{X}$ & let $\mathcal{X}$= $\mathcal{X}_1 \oplus \mathcal{X}_2$. If $\mathcal{X}$ is Ƭ-F-prime *module*, then $\mathcal{X}_1$& $\mathcal{X}_2$ are Ƭ-F-prime *module*.

Proof: Since $\mathcal{X}$=$\mathcal{X}_1 \oplus \mathcal{X}_2$ is Ƭ-F- prime *module* so that $(0_t, 0_t)$ is Ƭ-f-prime *submodule* for F-*module* $\mathcal{X}$, but $(0_t, 0_t)$ = $(0_t) \oplus (0_t)$ by [5,10] $(0_t)_{\mathcal{X}_1}$is Ƭ-F-prime *submodule* for $\mathcal{X}_1$ & $(0_t)_{\mathcal{X}_2}$is Ƭ-F-prime *submodule* for $\mathcal{X}_2$ therefore, $\mathcal{X}_1$ & $\mathcal{X}_t$ are Ƭ-F-prime *module* by Definition (2.1).

4. Conclusion

Through this research, a type for fuzzy module was studied, which is (T). This concept was applied to other concepts, & many new results were derived that had never been proven by a researcher. We proved

1- $\mathcal{X}$ is Ƭ-F-prime *module*, then $ann_R\mathcal{X}$ is Ƭ-F- prime *i*deal for R.

2- Let $\mathcal{X}$ be F-*module* for R & $\mathcal{A}$ be F-*i*deal for R, which is contained in $ann_R\mathcal{X}$. then $\mathcal{X}$ is Ƭ-F-*module* iff $\mathcal{X}$ is Ƭ-F-prime $^{R}/_{\mathcal{A}}$ *module*.

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