

Model operator ASA on intuitionistic fuzzy BRK-subalgebras

Alaa Salih Abed *

Noor Dhahir Abbas [†]

Department of Mathematics

Faculty of Education for Girls

University of Kufa

Najaf

Iraq

Abstract

This paper aims to announce the concept of operator $(\neg_{\alpha, \beta, \gamma, \delta, \epsilon, \tau})$ in intuitionistic fuzzy BRK-subalgebras. It states and evidences a amount of belongings. All $\psi \in \text{BRK-algebra } (\Psi; *, 0)$ intuitionistic fuzzy BRK-subalgebras gratify the subsequent $\mu_A(0) \geq \mu_A(\psi)$ and $\nu_A(0) \leq \nu_A(\psi)$ is an intuitionistic fuzzy BRK-subalgebra of If A is a BRK-subalgebra of Ψ with inf and sup properties, then Y where f is onto. Under f in Ψ , the preimage of B is an intuitionistic fuzzy. If B is an intuitionistic fuzzy BRK-subalgebra of Y with inf and sup properties, then ψ is a BRK-subalgebra of B. Let A be an intuitionistic fuzzy set and a BRK-algebra $(\Psi; *, 0)$. We shall be mentioned a fuzzy intuitionistic by Γ , BRK-algebra by BRK-g & BRK-Subalgebra by BRK-Sg and $\neg_{\alpha, \beta, \gamma, \delta, \epsilon, \tau}$ by ASA.

Subject Classification: 35J88, 39B72, 58E50, 70G75, 74M15.

Keywords: BRK algebra, Fuzzy BRK ideal, Homomorphisms of ideal, Fuzzy, BRK-subalgebra.

1. Introduction

In 1965, Zadeh announced the view of fuzzy sets [8]. Since then, this notion has been useful to other algebraic structures. A BCK-algebra is a primary class of logical algebras announced by Iseki and Tanaka [5] & has been thoroughly willful by several detectives. All BCK-algebras belong to a quasi-variety class. The intriguing question of whether the class of BCK-algebras is a variety was put forth by Iseki and Tanaka [5]. In 1985, Bhattacharya and Mukherjee offered the notion of fuzzy relations and fuzzy groups [10]. In 1989, Atanassov presented the Γ sets and willful Γ theory [6]. In 2003, Atanassov provided a new Γ model operator [7]. In 2012, R.K. Bandaru announced BRK-g,

* E-mail: alaas.abed@uokufa.edu.iq (Corresponding Author)

[†] E-mail: noord.alkhafaji@uokufa.edu.iq

BRK -Sg, and BRK-ideal of it [11]. In 2015, O. R. Elgendy Γ BRK-ideal of BRK-g [9]. In 2022, Hameed and Abraham [1] presented the clue of fuzzy RG-subalgebras of RG-algebra. Preliminaries: We now offer some definitions & initial findings that will be needed in the subsequent sections.

Definition 1.1: [1] Assume that S is a nonempty subset of Ψ 's BRK-g. Next, S is contacted a BRK-Sg of Ψ , if $\sigma * \tau \in S, \forall \sigma, \tau \in S$.

Definition 1.2: [3] Let $(\Psi, *, 0)$ & $(Y, \Delta, 0')$. A mapping $\beth : \Psi \rightarrow Y$ of a BRK -g is described as a homomorphism. if $\beth(\sigma * \tau) = \beth(\sigma) \Delta \beth(\tau), \forall \sigma, \tau \in \Psi$.

Definition 1.3: [4] For any Γ set $\aleph = \{(o, \Theta_{\aleph}(o), \nu_{\aleph}(o)) | o \in \Psi\}$ of X & $\alpha, \beta, \gamma, \varpi \in [0,1]$ & $\max(\alpha, \beta) + \gamma + \varpi \leq 1, \min(\alpha, \beta) + \gamma + \varpi \geq 0$, then $(\varepsilon = 0 = \tau)$ the operator ASA: $\text{IFS}(\Psi) \rightarrow \text{IFS}(\Psi)$ is defined as

$$\text{ASA}(\aleph) = \{(o, \alpha \Theta_{\aleph}(o) + \gamma, \beta \nu_{\aleph}(o) + \varpi) | o \in \Psi\}.$$

Definition 1.4: [3] For any Γ set $\aleph = \{(o, \Theta_{\aleph}(o), \nu_{\aleph}(o)) | o \in \Psi\}$ of X & $\varpi, \beta, \gamma, \tau, \alpha \in [0,1]$ & $\max(\alpha - \varepsilon, \beta - \tau) + \gamma + \varpi \leq 1, \min(\alpha - \varepsilon, \beta - \tau) + \gamma + \varpi \geq 0 \rightarrow$ the operator $\neg_{\varpi, \beta, \gamma, \tau, \alpha}: \text{IFS}(\Psi) \rightarrow \text{IFS}(\Psi)$ is labelled as $\text{ASA}(\aleph) = \{(o, \alpha \Theta_{\aleph}(o) - \varepsilon \nu_{\aleph}(o) + \gamma, \beta \nu_{\aleph}(o) - \tau \mu_{\aleph}(o) + \varpi) | o \in \Psi\}$.

Remark 1.5: [3] If A & B be two Γ sets of X & Y consequently & $f: \Psi \rightarrow Y$ be a mapping, where $\alpha, \beta, \gamma, \varpi, \varepsilon, \tau \in [0,1]$ then $\beth^{-1}(\text{ASA}(B)) = \text{ASA}(\beth^{-1}(B))$ & $\beth(\text{ASA}(\aleph)) \subseteq \text{ASA}(\beth(\aleph))$.

Definition 1.6: [2] An Γ set $\aleph = \{(o, \Theta_{\aleph}(o), \nu_{\aleph}(o))$ is named a Γ subalgebra of Ψ if it, $\forall \sigma, \tau \in \Psi$ 1) $\Theta_{\aleph}(\sigma * \tau) \geq \min\{\Theta_{\aleph}(\sigma), \Theta_{\aleph}(\tau)\}$, 2) $\nu_{\aleph}(\sigma * \tau) \leq \max\{\nu_{\aleph}(\sigma), \nu_{\aleph}(\tau)\}$.

Definition 1.7: [2] For any set $\aleph = \{(o, \Theta_{\aleph}(o), \nu_{\aleph}(o)) | o \in \Psi\}$ of the set Ψ represented by (IFS) & $\alpha \in [0,1]$ the operator $\dagger: \text{IFS}(\Psi) \rightarrow \text{IFS}(\Psi)$, $\boxplus: \text{IFS}(\Psi) \rightarrow \text{IFS}(\Psi)$ are defined as $\dagger \aleph = \{(o, \Theta_{\aleph}(o), 1 - \Theta_{\aleph}(o)) | o \in \Psi\}$, $\boxplus \aleph = \{(o, 1 - \nu_{\aleph}(o), \nu_{\aleph}(o)) | o \in \Psi\}$.

2. Model Operator $\neg_{\varpi, \beta, \gamma, \tau, \alpha}$ (ASA) on Intuitionistic Fuzzy BRK - subalgebras

This section aimed to offer the idea an operator $\neg_{\varpi, \beta, \gamma, \tau, \alpha}$ in Γ BRK -Sg & Γ BRK - ideals of BRK-g, numerous belongings & theorems are detailed & evidenced

Theorem 2.1: Let $\aleph = \{(o, \Theta_{\aleph}(o), \nu_{\aleph}(o)) | o \in \Psi\}$ be an Γ set of a BRK-g $(X; *, 0)$. A is an Γ BRK -Sg of Ψ if & only if $\dagger \aleph$ & $\boxplus \aleph$ are Γ BRK-Sg of X [12, 13].

Proof: If $\aleph = \{(o, \theta_{\aleph}(o), v_{\aleph}(o)) | o \in \Psi\}$ is an Γ BRK -Sg of X , then $\theta_{\aleph}$ & $\bar{v}_{\aleph}$ are fuzzy BRK -Sg & $v_{\aleph}$ & $\bar{\theta}_{\aleph}$ are fuzzy subalgebras. Hence, $\dagger \aleph$ & $\boxplus \aleph$ are Γ BRK -Sg. Conversely, let $\dagger \aleph = \{(o, \theta_{\aleph}(o), 1 - \theta_{\aleph}(o)) | o \in \Psi\}$ & $\boxplus \aleph = \{(o, 1 - v_{\aleph}(o), v_{\aleph}(o)) | o \in \Psi\}$ are Γ BRK -Sg, then the fuzzy subset $\theta_{\aleph}$ & $\bar{v}_{\aleph}$ are fuzzy BRK -Sg of X & $v_{\aleph}$ & $\bar{\theta}_{\aleph}$ are fuzzy BRK -Sg of X . Hence $\aleph = \{(o, \theta_{\aleph}(o), v_{\aleph}(o)) | o \in \Psi\}$ Γ BRK -subalgebra Sg.

Lemma 2.2: If Γ set $\aleph = \{(o, \theta_{\aleph}(o), v_{\aleph}(o)) | o \in \Psi\}$ is an Γ BRK -Sg of X , then

$$ASA(\aleph) = \{(o, \alpha \theta_{\aleph}(o) + \varepsilon \bar{v}_{\aleph}(o) + \gamma - \varepsilon, \beta v_{\aleph}(o) + \tau \bar{\theta}_{\aleph}(o) + \varpi - \tau) | o \in \Psi\}.$$

Proof: Since $ASA(\aleph) = \{(o, \alpha \theta_{\aleph}(o) - \varepsilon v_{\aleph}(o) + \gamma, \beta v_{\aleph}(o) - \tau \theta_{\aleph}(o) + \varpi) | o \in \Psi\}$, & $\bar{v}_{\aleph} = 1 - v_{\aleph}$, $\bar{\theta}_{\aleph} = 1 - \theta_{\aleph}$, thus $-\varepsilon v_{\aleph}(o) = -\varepsilon(1 - \bar{v}_{\aleph})(o) = \bar{v}_{\aleph}(o) - \varepsilon$ & $-\tau \theta_{\aleph}(o) = -\tau(1 - \bar{\theta}_{\aleph})(o) = \bar{\theta}_{\aleph}(o) - \tau$. Hence $ASA(\aleph) = \{(o, \alpha \theta_{\aleph}(o) + \varepsilon \bar{v}_{\aleph}(o) + \gamma - \varepsilon, \beta v_{\aleph}(o) + \tau \bar{\theta}_{\aleph}(o) + \varpi - \tau) | o \in \Psi\}$.

Proposition 2.3: If A & B be two Γ subsets of BRK-g Ψ & BRK-g Y separately & $f: (\Psi; *, 0) \rightarrow (Y; *, 0)$ be a mapping, now

- 1) $\beth(ASA(\aleph)) \subseteq ASA(\beth(\aleph))$
- 2) $\beth^{-1}(ASAB) = ASA(\beth^{-1}(B))$

Proof:

- 1) Let $o \in X$, then

$$\beth(ASA(\aleph))(x) = (x, \mu(\beth(ASA(\aleph)))(x), v(\beth(ASA(\aleph)))(x))$$

$$\text{Also, } \beth(ASA(\aleph))(x) = (\mu(\beth(ASA(\aleph)))(x), (v(\beth(ASA(\aleph)))(x))) \\ = (\theta_{ASA(\aleph)}(\beth(x)), (v(ASA(\aleph))(\beth(x)))).$$

$$\text{Now, } \mu(\beth(ASA(\aleph)))(x) = \sup_{t \in \beth(x)} \mu(ASA(\aleph))(t)$$

$$= \sup_{t \in f(x)} (\alpha \theta_{\aleph}(t) - \varepsilon v_{\aleph}(t) + \gamma) = \\ \alpha \sup_{t \in f(x)} \theta_{\aleph}(t) - \varepsilon \sup_{t \in f(x)} v_{\aleph}(t) + \gamma$$

$$\leq \alpha \sup_{t \in Y} \theta_{\beth(A)}(t) - \varepsilon \sup_{t \in Y} v_{\beth(A)}(t) + \gamma$$

$$= \alpha \theta_{\beth(A)}(f(x)) - \varepsilon v_{\beth(A)}(\beth(x)) + \gamma, \beth(\mu(\aleph)) \subseteq \mu(\beth(A)).$$

Likewise

$$v(\beth(ASA(\aleph)))(x) = \inf_{t \in f(x)} v(ASA(\aleph))(t)$$

$$= \inf_{t \in f(x)} (\beta v(t) - \tau \theta_{\aleph}(t) + \delta)$$

$$= \beta \inf_{t \in \beth(x)} v_{\aleph}(t) - \tau \inf_{t \in \beth(x)} \theta_{\aleph}(t) + \delta$$

$$\leq \beta \inf_{t \in Y} v_{\beth(A)}(t) - \tau \inf_{t \in Y} \theta_{\beth(A)}(t) + \delta$$

$$= \beta v_{\beth(A)}(f(x)) - \tau \theta_{\beth(A)}(f(x)) + \delta, \beth(v(\aleph)) \subseteq v(\beth(\aleph))$$

$$\text{Thus } f(ASA(\aleph)) \subseteq ASA(f(\aleph)).$$

$$\begin{aligned}
2) \quad \beth^{-1}(\text{ASA } B)(y) &= (y, \theta(\beth^{-1}(\text{ASA } B)(y)), (v(\beth^{-1}(\text{ASA } B)(y)))) \\
\text{Moreover,} \\
\mathfrak{Z}^{-1}(\text{ASA } (A))(y) &= (\mu(\beth^{-1}(\text{ASA } (B))(y)), (v(\beth^{-1}(\text{ASA } (B))(y))) \\
&= (\theta \text{ASA } (B)) * (f^{-1}(y)), v(\text{ASA } (B), (f^{-1}(y))) \\
\text{So, } \theta \text{ASA } (B))(\mathfrak{Z}^{-1}(y)) &= \alpha \theta_B(\mathfrak{Z}^{-1}(y)) - \varepsilon v_B(\mathfrak{Z}^{-1}(y)) + \gamma \\
&= \alpha \theta_B(\mathfrak{Z}^{-1}(y)) - \varepsilon v_B(\mathfrak{Z}^{-1}(y)) + \gamma = \\
&\quad \alpha \theta_{\mathfrak{Z}^{-1}(B)}(y) - \varepsilon v_{\mathfrak{Z}^{-1}(B)}(y) + \gamma \\
&= \theta \text{ASA } (\mathfrak{Z}^{-1}(B))(y), \& \\
v \text{ASA } (B))(\mathfrak{Z}^{-1}(y)) &= \beta v_B(\mathfrak{Z}^{-1}(y)) - \tau \mu_B(\mathfrak{Z}^{-1}(y)) + \delta \\
&= \beta v_B(\mathfrak{Z}^{-1}(y)) - \tau \mu_B(\mathfrak{Z}(y)) + \delta = \beta v_{\mathfrak{Z}^{-1}(B)}(y) - \tau \mu_{\mathfrak{Z}^{-1}(B)}(y) + \delta \\
&= v \text{ASA } (\mathfrak{Z}^{-1}(B))(y). \text{ Hence, } \mathfrak{Z}^{-1}(\text{ASA } (B)) = \text{ASA } (\mathfrak{Z}^{-1}(B)).
\end{aligned}$$

Proposition 2.4: If an Γ set $\aleph = \{(o, \theta_{\aleph}(o), v_{\aleph}(o)) | o \in \Psi\}$ is an Γ BRK -Sg of Ψ , $\rightarrow \text{ASA}(A)$ is also an Γ Z-Sg of Ψ .

Proof: For now $\text{ASA}(\aleph) = \{(o, \alpha \theta_{\aleph}(o) + \varepsilon \bar{v}_{\aleph}(o) + \gamma - \varepsilon, \beta v_{\aleph}(o) + \tau \bar{\theta}_{\aleph}(o) + \varpi - \tau) | o \in \Psi\}$, by Lemma 2.5. then $\forall x, y \in X, \theta_{\neg \varpi, \beta, \gamma, \tau, \alpha}(\aleph)(o * \sigma) = \alpha \theta_{\aleph}(o * \sigma) + \varepsilon \bar{v}_{\aleph}(o * \sigma) + \gamma - \varepsilon \geq \alpha (\min\{\theta_A(o), \theta_A(\sigma)\}) + \varepsilon \min\{\bar{v}_{\aleph}(o), \bar{v}_{\aleph}(\sigma)\} + \gamma - \varepsilon$

$$\begin{aligned}
&= (\min\{\alpha \theta_A(o), \alpha \theta_A(\sigma)\}) + \min\{\varepsilon \bar{v}_{\aleph}(o), \varepsilon \bar{v}_{\aleph}(\sigma)\} + \gamma - \varepsilon \\
&= \min\{\alpha \theta_A(o) + \varepsilon \bar{v}_{\aleph}(o) + \gamma - \varepsilon, \alpha \theta_A(\sigma) + \varepsilon \bar{v}_{\aleph}(\sigma) + \gamma - \varepsilon\} \\
&= \min\{\theta_{\text{ASA}}(\aleph)(x), \theta_{\text{ASA}}(\aleph)(y)\}.
\end{aligned}$$

$$\begin{aligned}
\theta_{\text{ASA}}(\aleph) \text{ is BRK -Sg of } \Psi. v_{\text{ASA}}(\aleph)(o * \sigma) &= \beta v_{\aleph}(o * \sigma) + \\
\tau \bar{\theta}_{\aleph}(o * \sigma) + \varpi - \tau
\end{aligned}$$

$$\begin{aligned}
&\leq \beta(\max\{v_A(o), v_A(\sigma)\}) + \tau \max\{\bar{\theta}_{\aleph}(o), \bar{\theta}_{\aleph}(\sigma)\} + \varpi - \tau \\
&= \max\{\beta v_{\aleph}(o), \beta v_{\aleph}(\sigma)\} + \max\{\tau \bar{\theta}_{\aleph}(o), \tau \bar{\theta}_{\aleph}(\sigma)\} + \varpi - \tau \\
&= \max\{\beta v_{\aleph}(o) + \tau \bar{\theta}_{\aleph}(o) + \varpi - \tau, \beta v_{\aleph}(\sigma) + \tau \bar{\theta}_{\aleph}(\sigma) + \varpi - \tau\} \\
&= \max\{v_{\text{ASA}}(\aleph)(x), v_{\text{ASA}}(\aleph)(y)\}.
\end{aligned}$$

$v_{\text{ASA}}(\aleph)$ is a doubt BRK -Sg of Ψ . Hence $\text{ASA}(\aleph)$ is an Γ BRK -Sg of Ψ .

Remark 2.6: According to the following example, Proposition (2.4) is not true in reverse as shown in Table 1.

Example 2.7: Assume $\Psi = \{0, 1, 2\}$, where the following table 2 describes *

Table 1

*	0	1	2
0	0	2	2
1	1	0	0
2	2	0	0

Table 2

Ψ	0	1	2	3
$\theta_{\aleph}$	0.6	0.6	0.2	0.4
$v_{\aleph}$	0.7	0.2	0.3	0.2
Ψ	0	1	2	3
θ_{ASA}	0.48	0.42	0.34	0.36
v_{ASA}	0.12	0.22	0.22	0.16

$\aleph = \{(o, \theta_{\aleph}(o), v_{\aleph}(o)) | o \in \Psi\}$ is not an Γ BRK -Sg, since $\theta_{\aleph}$ are not fuzzy BRK -Sg of Ψ , since $(\theta_{\aleph})(1 * 3) = (\theta_{\aleph})(2) = 0.2 \not\geq \min \{ \theta_{\aleph}(1), \theta_{\aleph}(3) \} = 0.4$. If we take $\alpha = .2, \beta = .2, \gamma = .3, \varpi = .2, \varepsilon = 0, \tau = .2$, $ASA(\aleph)$ be an Γ BRK -Sg of X .

Proposition 2.8: If $\aleph$ is an Γ BRK -Sg of Ψ , at the present

$$\theta_{ASA}(\aleph)(0) \geq \theta_{ASA}(\aleph)(x) \text{ \& } v_{ASA}(\aleph)(0) \leq v_{ASA}(\aleph)(x), \forall x \in X.$$

Proof: Since $\aleph$ is an Γ BRK -Sg of X , $ASA(\aleph)$ is also an Γ BRK -Sg of X . Currently, for any $x \in X$, $\theta_{ASA}(\aleph)(0) = \theta_{ASA}(\aleph)(o * o) \geq \min \{ \theta_{ASA}(\aleph)(x), \theta_{ASA}(\aleph)(o) \} = \theta_{ASA}(\aleph)(x)$ & $v_{ASA}(\aleph)(0) = v_{ASA}(\aleph)(o * o) \leq \max \{ v_{ASA}(\aleph)(x), v_{\neg \varpi, \beta, \gamma, \tau, \alpha}(\aleph)(o) \} = v_{ASA}(\aleph)(x)$.

Proposition 2.9: If $\aleph = \{(o, \theta_{\aleph}(o), v_{\aleph}(o)) | o \in \Psi\}$ & $B = \{(o, \theta_B(o), v_B(o)) | o \in \Psi\}$ are Γ BRK -Sg of $\Psi \rightarrow$ the intersection on intuitionistic of A & B , $ASA(\aleph \cap B)$ is Γ BRK -Sg of X .

Proof: Suppose $o, \sigma \in \Psi$ by using

$$\theta_{ASA}(\aleph \cap B)(o * \sigma) = \alpha \theta_{\aleph \cap B}(o * \sigma) - \varepsilon v_{\aleph \cup B}(o * \sigma) + \gamma.$$

$$\begin{aligned} & \theta_{ASA}(\aleph \cap B)(o * \sigma) = \alpha \theta_{\aleph \cap B}(o * \sigma) + \varepsilon \bar{v}_{\aleph \cap B}(o * \sigma) + \gamma - \varepsilon \\ & \geq \min \{ \alpha \theta_{\aleph \cap B}(o), \alpha \theta_{\aleph \cap B}(\sigma) \} + \min \{ \varepsilon \bar{v}_{\aleph \cap B}(o), \varepsilon \bar{v}_{\aleph \cap B}(\sigma) \} + \gamma - \varepsilon \\ & = \min \{ \alpha \theta_{\aleph \cap B}(o) + \varepsilon \bar{v}_{\aleph \cap B}(o) + \gamma - \varepsilon, \alpha \theta_{\aleph \cap B}(\sigma) + \varepsilon \bar{v}_{\aleph \cap B}(\sigma) + \gamma - \varepsilon \} \\ & \geq \min \{ \theta_{ASA}(\aleph \cap B)(o), \theta_{ASA}(\aleph \cap B)(\sigma) \} \end{aligned}$$

$\theta_{ASA}(\aleph \cap B)$ is fuzzy BRK-Sg of X . Currently,

$$v_{ASA}(\aleph \cap B)(o) = \beta v_{\aleph \cup B}(o) - \tau \theta_{\aleph \cap B}(o) + \varpi.$$

$$\begin{aligned} v_{ASA}(\aleph \cup B)(o * \sigma) &= \beta v_{\aleph \cup B}(o * \sigma) + \tau \bar{\theta}_{\aleph \cup B}(o * \sigma) + \varpi - \tau \\ &\leq \beta (\max \{ v_{\aleph \cup B}(o), v_{\aleph \cup B}(\sigma) \}) + \tau \max \{ \bar{\theta}_{\aleph \cup B}(o), \bar{\theta}_{\aleph \cup B}(\sigma) \} + \varpi - \tau \\ &\leq \max \{ v_{ASA}(\aleph \cup B)(o), v_{ASA}(\aleph \cup B)(\sigma) \} \end{aligned}$$

$v_{ASA}(\aleph \cup B)$ is a doubt fuzzy BRK -Sg of Ψ . Hence the intersection on Γ of $\aleph$ & $B \neg \varpi, \beta, \gamma, \tau, \alpha(\aleph \cap B)$ be an Γ BRK -Sg of X .

Proposition 2.10: If $\aleph = \{(o, \Theta_{\aleph}(o), v_{\aleph}(o)) \mid o \in \Psi\}$ & $B = \{o, \Theta_B(o), v_B(o) \mid o \in \Psi\}$ are Γ BRK-Sg of $\Psi \rightarrow$ the union on Γ of B & $\aleph$, $ASA(\aleph \cup B)$ is Γ BRK-Sg of Ψ , where $v_{\aleph_i}$ is chain (Artesian) & $\Theta_{\aleph_i}$ chain (Noetherian).

Proof: Suppose $o, \sigma \in \Psi$

$$\begin{aligned}
 \Theta_{ASA}(\aleph \cup B)(o * \sigma) &= \alpha \Theta_{\aleph \cup B}(o * \sigma) - \varepsilon v_{\aleph \cap B}(o * \sigma) + \gamma. \\
 \Theta_{ASA}(\aleph \cup B)(o * \sigma) &= \alpha \Theta_{\aleph \cup B}(o * \sigma) + \varepsilon \bar{v}_{\aleph \cup B}(o * \sigma) + \gamma - \varepsilon \\
 &\geq \alpha (\max\{\Theta_{\aleph \cup B}(o), \Theta_{\aleph \cup B}(\sigma)\}) + \varepsilon \max\{\bar{v}_{\aleph \cup B}(o), \bar{v}_{\aleph \cup B}(\sigma)\} + \gamma - \varepsilon \\
 &\geq \max\{\alpha \Theta_{\aleph \cup B}(o), \alpha \Theta_{\aleph \cup B}(\sigma)\} + \max\{\varepsilon \bar{v}_{\aleph \cup B}(o), \varepsilon \bar{v}_{\aleph \cup B}(\sigma)\} + \gamma - \varepsilon \\
 \Theta_{ASA}(\aleph \cup B) &\text{ is fuzzy BRK-Sg of } \Psi. \text{ Now,} \\
 v_{ASA}(\aleph \cap B)(o) &= \beta v_{\aleph \cup B}(o) - \tau \Theta_{\aleph \cap B}(o) + \varpi. \\
 v_{ASA}(\aleph \cap B)(o * \sigma) &= \beta v_{\aleph \cap B}(o * \sigma) + \tau \bar{\Theta}_{\aleph \cap B}(o * \sigma) + \varpi - \tau \\
 &\leq \beta (\min\{v_{\aleph \cap B}(o), v_{\aleph \cap B}(\sigma)\}) + \tau \min\{\bar{\Theta}_{\aleph \cap B}(o), \bar{\Theta}_{\aleph \cap B}(\sigma)\} + \varpi - \tau \\
 &\leq \min\{v_{ASA}(\aleph \cap B)(o), v_{ASA}(\aleph \cap B)(\sigma)\} \\
 v_{ASA}(\aleph \cap B) &\text{ is a doubt fuzzy BRK-Sg of } X. \text{ Hence the union on } \Gamma \text{ of } A \text{ \& } B \\
 ASA(\aleph \cup B) &\text{ be an } \Gamma \text{ BRK-Sg of } \Psi.
 \end{aligned}$$

3. Conclusion In this research, we concluded the subsequent :

In Γ BRK-Sg the notion of operator ASA is introduced in this paper. It all validates several characteristics. The following inequalities are satisfied by Γ BRK-Sg with $\psi \in \text{BRK-g}(\Psi; *, 0)$: where $\mu_A(0) \geq \mu_A(\psi)$ and $v_A(0) \leq v_A(\psi)$ A is an Γ BRK-Sg of Ψ with sup and inf properties, and it gives the BRK-Sg, where f is onto. If B is an Γ BRK-subalgebra of Y with inf and sup properties, then an Γ BRK-Sg of Ψ is the preimage of B under f in Ψ . Let A be an Γ set and Ψ be a BRK-g.

References

- [1] A. T. Hameed and S. M. Abraham, "Model operator $\Box \alpha, \beta, \gamma, \delta, \varepsilon, \tau$ on Intuitionistic Fuzzy RG-algebras," *Int. J. Eng. Inf. Syst.* (IJEAIS), vol. 7, no. 1, pp. 66–67 (Jan. 2023).
- [2] D. Dubois and H. Prade, *Fuzzy Sets and Systems*. London, U.K.: Academic Press Inc. (1980).
- [3] G. Cuvalcioglu, "The Extension of Modal Operators' Diagram With Last Operators," *Notes Intuitionistic Fuzzy Sets*, vol. 19, no. 3, pp. 56–61 (2013).
- [4] K. Dencheva, "Extension of Intuitionistic Fuzzy Modal Operators $\dagger$ & $\boxplus$," in *Proc. Second Int. IEEE Symp. Intelligent Systems*, Varna, Bulgaria, pp. 21–22 (2004).
- [5] K. Iséki and S. Tanaka, "Ideal Theory of BCK-algebras," *Math. Japon.*, vol. 21, pp. 351–366 (1976).

- [6] K. T. Atanassov, "On the New Intuitionistic Fuzzy Operator $X(a,b,c,d,e) f$," *Notes Intuitionistic Fuzzy Sets*, vol. 16, no. 2, pp. 35–38, 2010.
- [7] K. T. Atanassov, *On Intuitionistic Fuzzy Sets Theory*. Berlin, Germany: Springer-Verlag (2012).
- [8] L. A. Zadeh, "Fuzzy Sets," *Inform. Control*, vol. 8, pp. 338–353 (1965).
- [9] O. R. Elgendy, "Fuzzy BRK-ideal of BRK-algebras," *Br. J. Math. Comput. Sci.*, vol. 10, no. 6, pp. 1–9 (2015).
- [10] P. Bhattacharye and N. P. Mukherjee, "Fuzzy Relations and Fuzzy Group," *Inf. Sci.*, vol. 36, pp. 267–282 (1985).
- [11] R. K. Bandaru, "On BRK-Algebra," *Int. J. Math. Math. Sci.*, vol. 2012, pp. 1–6 (2012).
- [12] S. Sharma and A. J. Obaid, "Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox," *J. Interdiscip. Math.*, vol. 23, no. 4, pp. 843–849 (2020).
- [13] S. A. Al-Ameedee and A. J. Obaid, "New results and application of differential quasi subordinations for higher-order derivatives of meromorphic multivalent functions," *J. Interdiscip. Math.*, pp. 1–8 (2020), doi: 10.47974/JIM-1483.

Received July, 2024