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Jordan permuting 3-left (resp. right) μ-centralizers on prime semi-rings

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Abstract

This article targets to offer the notions of permuting 3- left (resp. right) μ -centralizers on semiring and study this concept.

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1. Introduction

One of the most crucial notions in non-commutative algebra is permuting 3- left (resp. right) μ -centralizers. In [1] and [2], the definitions of prime semiring, semiprime semiring and 2-tortion free, respectively, were presented. For more information see [3-7]. An additive map ω: S×S×S⟶S is known as permuting if the equation $\omega(x_1,x_2,x_3) = \omega(x_{\pi(1)}, x_{\pi(2)}, x_{\pi(3)})$ is correct $\forall x_1; x_2; x_3 \in S$ .And, for each permutation $\{\pi(1); \pi(2) ; \pi(3)\}$[3]. An additive map δ : S ⟶ S such that $\delta(x) = \omega(x, x, x)$, ∀x ∈ S, where ω: S×S×S⟶S is a permuting map, is known as the trace of ω. Evidently, in case when ω: S×S×S ⟶S is a permuting map which is also 3- additive (i.e , additive in all argument), $\delta(a+b) = \delta(x) + \delta(y) + 3\omega(x,x,y) + 3\omega(x,y,y)$. If ω is permuting. Hence below are equivalent for all $a, b, n, y, n_1, y_1 \in S$.

$$\omega(ab,n,y) = \omega(a,n,y)b + a\omega(b,n,y)$$

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$$\omega(a, n + n_1, y) = \omega(a, n, y)\, n_1 + n\, \omega(a, n_1, y)$$
$$\omega(a, n, y + y_1) = \omega(a, n, y)\, y_1 + y\, \omega(a, n, y_1).$$

In this article we present the notions of permuting 3-derivation on and Jordan permuting 3-derivation on semiring S. This essay answer whether there is a connection between notions of Jordan permuting 3- left (resp. right) μ–centralizers and of Jordan triple permuting 3- left (resp. right) μ–centralizers inside a specific condition.

2. Permuting 3- left (resp. right) μ-Centralizers on Prime Semirings

Definition 2.1: If S is a semirings and d is permuting 3- left (resp. right) μ-centralizer (For shout p3l(Ar)μ-c) on S×S×S into S where μ is isomorphism on S if $\forall a, b, n, y \in$S then

$$d(ab, n, y) = d(a, n, y)\mu(b), \text{ (resp. } d(ab, n, y) = \mu(a)d(b, n, y)$$

d is called Jordan permuting 3-left (resp. right) μ-centralizer (Jp3-l(Ar)μ-c) on S×S×S into S where μ is isomorphism on S if $\forall a$, b, $n, y \in$S, then

$$d(a^2, n, y) = d(a, n, y)\, \mu(a), \text{ (resp. } d(a^2, n, y) = \mu(a)d(a, n, y))$$

d is called Jordan triple permuting 3- left (resp. right) μ-centralizers (Jtp3l(Ar)μ-c on S×S×S into S, where μ is isomorphism on S, ∀ a, b, $n, y \in$S, then

$$d(aba, n, y) = d(a, n, y)\mu(b)\mu(a),$$
$$\text{(resp. } d(aba, n, y) = \mu(a)\mu(b)d(a, n, y)$$

Example 2.2: Let A be a semiring with additive identity and S = $\left\{\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} : a \in A\right\}$ be a semiring then $d\left(\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & n \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 7a^3 \\ 0 & 0 \end{pmatrix}$, for all a,b, $n \in$S Where, $\mu\left(\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & a^2 \\ 0 & 0 \end{pmatrix}$, $\forall\, a \in$S. Then d is p3-l(Ar)μ-c on S.

Lemma 1: *Let d be Jp3-l(Ar)μ-c on semiring S×S×S into S where μ is isomorphism on S then $\forall a, b, n, y \in S$*

$$d(ab + ba, n, y) = d(a, n, y)\mu(b) + d(b, n, y)\mu(a)$$

Proof: $d((a + b)(a + b), n, y) = (d(a, n, y) + d(b, n, y))\ (\mu(a) + \mu(b)) = d(a, n, y)\mu(a) + d(a, n, y)\mu(b) + d(b, n, y)\ \mu(a) + d(b, n, y))\ (\mu(b) \ldots (1)$

Furthermore

$$d((a + b)(a + b), n, y) = d(a^2 + ab + ba + b^2, n, y)$$
$$= d(a^2, n, y) + d(b^2, n, y) + d(ab + ba, n, y)$$
$$= d(a, n, y)\mu(a) + d(b, n, y)\mu(b) + d(ab + ba, n, y) \ldots (2)$$

Comparing (1) and (2) we conclude $d(ab + ba, n, y) = d(a, n, y)\mu(b) + d(b, n, y)\mu(a)$.

Definition 2.2: Suppose S is a semiring and d is Jp3-l(Ar)μ-c on S×S×S into S, we define a^b_{ny} on S by $a^b_{ny} = d(ab, n, y)$-$d(a,n,y)\mu(b)$, for all$a, b, n, y \in$S. Below, we offer some properties of a^b_{ny} .

Lemma 2: *Let d be Jp3-l(Ar)μ-c on Abelean additive semiring S×S×S into S then* $\forall a, b, n, y, t \in S$

1) $a^b_{ny} =$ - b^a_{ny}

2) $a^{b+t}_{ny} = a^b_{ny} + a^t_{ny}$

3) $(a+t)^b_{ny} = a^b_{ny} + t^b_{ny}$

Proof:

1) $d(ab + ba, n, y) = d(a, n, y)\mu(b) + d(b, n, y)\mu(a)$
$d(ab, n, y) + d(ba, n, y) = d(a, n, y)\mu(b) + d(b, n, y)\mu(a)$
$d(ab, n, y) - d(a, n, y)\mu(b) = -d(ba, n, y) + d(b, n, y)\mu(a)$
$a^b_{ny} =$ - b^a_{ny}

2) $a^{b+t}_{ny} = d(a(b+t), n, y) - d(a, n, y)\mu(b+t)$
$= d(ab + at, n, y) - d(a, n, y)\mu(b+t)$
$= d(ab, n, y) + d(at, n, y) - d(a, n, y)\mu(b) - d(a, n, y)\mu(t)$
$= d(ab, n, y) - d(a, n, y)\mu(b) + d(at, n, y) -$
$d(a, n, y)\mu(t) = a^b_{ny} + a^t_{ny}$

2) $(a+t)^b_{ny} = d(ab, n, y) + d(tb, n, y) - d(a, n, y)\mu(b) - d(t, n, y)\mu(b)$
$= d(ab, n, y) - d(a, n, y)\mu(b) + d(tb, n, y) - d(t, n, y)\mu(b) = a^b_{ny} + a^t_{ny}$

Remark 2.3: Let S be semiring with additive identity and d be p3-l(Ar)μ-c on S×S×S into S if and only if $a^b_{ny} = 0, \forall\ a, b, n, y \in$S.

3. The Main Results

Lemma 3: *let d be Jp3-l(Ar)μ-* c *on semiprime semiring with additive identity and inverse S×S×S into S then* $\forall a, b, n, y, t, r \in R$ *then* $a^b_{ny}\ \mu(r)[\mu(a), \mu(b)] = 0$

Proof: Let $w = abrba + barab$

$$d(w, n, y) = d(abrba + barab, n, y)$$

$$= d(a, n, y)\mu(b)\mu(r)\mu(b)\mu(a) + d(b, n, y)\mu(a)\mu(r)\mu(a)\mu(b) \quad \ldots(1)$$

Furthermore,

$$= d((ab)r(ba) + (ba)r(ab), n, y)$$

$$= d(ab, n, y)\mu(r)\mu(b)\mu(a) + d(ba, n, y)\mu(r)\mu(a)\mu(b) \quad \ldots (2)$$

Comparing (1) and (2) we conclude

$$0 = (d(ab, n, y) - d(a, n, y)\mu(b))\,\mu(r)\mu(b)\mu(a) + (d(ba, n, y) - d(b, n, y)\mu(a))\mu(r)\mu(a)\mu(b) = a^b_{ny}\ \mu(r)[\mu(a), \mu(b)]$$

Lemma 4: *Let d be Jp3-l(Ar)μ-c on prime semiring with additive identity and inverse S×S×S into S then $\forall a, b, n, y, r, p, q \in S$*

$$a^b_{ny}\ \mu(r)\ [\mu(p), \mu(q)]\ = 0$$

Proof: Replacing a by $a + p$ in Lemma 3 we get

$$(a + p)^b_{ny}\ \mu(r)[\mu(a + p), \mu(b)]\ = 0$$

By lemma 3 we get

$$a^b_{ny}\,\mu(r)[\mu(p), \mu(b)] + p^b_{ny}\ \mu(r)[\mu(a), \mu(b)]\ = 0$$

Therefore, 0 = - $a^b_{ny}\ \mu(r)[\mu(p), \mu(b)]\ \mu(r)\ p^b_{ny}\ \mu(r)[\mu(a), \mu(b)]$

By primness of S we get

$$a^b_{ny}\ \mu(r)[\mu(p), \mu(b)]\ = 0 \qquad \ldots(1)$$

Replace b by $b + q$ in Lemma 3

$$a^{(b+q)}_{ny}\ \mu(r)[\mu(a), \mu(b + q)] = 0$$

$$a^b_{ny}\ \mu(r)\ [\mu(a), \mu(b)]\ +\ a^b_{ny}\ \mu(r)\ [\mu(a), \mu(q)]\ +\ a^q_{ny}\ \mu(r)\ [\mu(a), \mu(b)]\ +\ a^q_{ny}\ \mu(r)\ [\mu(a), \mu(q)]\ = 0$$

Therefore, 0 = - $a^b_{ny}\ \mu(r)\ [\mu(a), \mu(q)]\ \mu(r)\ a^q_{ny}\ \mu(r)\ [\mu(a), \mu(b)]$

Since S is primness we get

$$a^b_{ny}\ \mu(r)\ [\mu(a), \mu(q)]\ = 0 \qquad \ldots(2)$$

Now, $a^b_{ny}\ \mu(r)\ [\mu(a + p), \mu(b + q)]\ = 0$

$a^b_{ny}\ \mu(r)\ [\mu(a), \mu(b)]$ +$a^b_{ny}\ \mu(r)\ [\mu(a), \mu(q)]$ + $a^b_{ny}\ \mu(r)\ [\mu(p), \mu(b)]$ + $a^b_{ny}\ \mu(r)\ [\mu(p), \mu(q)] = 0$. By Lemma 3 and (1), (2) we obtain

$$a^b_{ny}\ \mu(r)\ [\mu(p), \mu(q)]\ = 0$$

Theorem 5: *Every Jp3-l(Ar)μ-c on 2-torsion free prime semiring with additive identity and inverse S×S×S into S is p3-l(Ar)μ-c on S×S×S into S.*

Proof: Let d be Jordan permuting 3- left (resp. right) μ-centralizers on 2- torsion free semiring S then , by Lemma 4 and since primness of S we get either a^b_{ny} =0, $\forall\ a, b, n, y \in$S or $[\mu(p), \mu(q)]\ = 0, \forall\ p, q \in S$.

If $[\mu(p), \mu(q)]\ \neq\ 0\ \forall\ p, q\ \in S$. Then a^b_{ny} = 0, $\forall a, b, n, y \in$S. we get by Remark 2.3 d is permuting 3- left (resp. right) μ-centralizers on S.

If $[\mu(p), \mu(q)] = 0, \forall p, q \in S$ then S is multiplication commutative and hence by Lemma 1 we get $d(2ab, n, y)$ =2 $d(a, n, y)$μ(b). S is 2- torsion free we get d is permuting 3- left (resp. right) μ-centralizers on S.

Proposition 6: Every Jp3-l(Ar)μ-c on 2-torsion free semiring S×S×S into S is Jtp3-l(Ar)μ-c on S×S×S into S.

Proof: Replace ab+ba for b in Lemma 1 we conclude

$d(a(ab+ba)+(ab+ba)a,n,y) = d(a,n,y)\mu(ab+ba) + d(ab+ba,n,y)\mu(a) = d(a,n,y)\mu(ab) + d(a,n,y)\mu(ba) + d(a,n,y)\mu(b)\mu(a) + d(b,n,y)\mu(a)\mu(a)$ …(1)

Otherwise

$d(a(ab+ba)+(ab+ba)a,n,y) = d(aab+aba+aba+baa,n,y)$

$= d(aab,n,y) + d(baa,n,y) + d(2aba)$

$= d(a,n,y)\mu(ab) + d(b,n,y)\mu(aa) + d(2aba,n,y)$ … (2)

Comparing (1) and (2) we get, $2\,d(aba,n,y) = 2\,d(a,n,y)\mu(b)\mu(a)$

Since S is 2-torsion free we get, $d(aba,n,y) = d(a,n,y)\mu(b)\mu(a)$

Thus, S is Jtp3-l(Ar)μ-c on S.

Corollary 7: *Every p3-l(Ar)μ- c on 2-torsion free semiring with additive identity with inverse $S \times S \times S$ into S is Jtp3-l(Ar)μ- c on $S \times S \times S$ into S.*

Proof: Let d p3-l(Ar)μ-c on 2-torsion free semiring S×S×S into S then by definition 2.1 then d is Jp3-l(Ar)μ-c on 2-torsion free semiring S×S×S into S and by Proposition 6 we have d is Jp3-l(Ar)μ-c on S×S×S into S.

Conclusions

This essay led to offer the concepts of permuting 3- left (resp. right) μ -centralizers on semiring and study this concept.

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