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Isomorphism of BD-algebras

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Abstract

In this paper, we introduce homomorphisms of BD-algebra and investigate its properties. Moreover, the relations between quotient BD-algebra and isomorphism also provided.

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1. Introduction

Abrahem, Hameed and Abed [1], explored fuzzy RG-ideals of RG-algebras, particularly focusing on properties such as homomorphism image and inverse image. Abed, et al. introduced multipliers within SA-algebras [2]. [3] examined novel concepts of β-multiplication Fuzzy q-ideals of RG-algebra. Jasim and Hamid [4] introduced Some Results on Antifuzzy Ideals of an RG-algebra, further explored, [4]. Jabir and Hameed introduced multipliers within $\boldsymbol{\varphi}$ -algebras [5]. Hasan and Kareem [6] and Kadham [7] introduced multipliers within KU-algebras [6-7]. Mahadi, Hameed, and Malik introduced multipliers within AT-algebras [8]. In this paper, we apply the notion of homomorphism of BD-subalgebras, ideals and BD-ideals of BD-algebras, and as a result. The purpose of this paper is to derive some straightforward consequences of the

2. Preliminaries

In this section, we introduced an algebraic structure called a BD-algebra.

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Definition 2.1: [6] An algebra $(\Upsilon, \circledast, o)$ such that for all $\iota, \varsigma \in \Upsilon$, satisfies the following properties:

(BD-1) $\iota \circledast o = \iota$,

(BD-2) $\iota \circledast \varsigma = o$ and $\varsigma \circledast \iota = o$ implies that $\iota = \varsigma$.

Proposition 2.2: [6] It is easy to show that the following properties are true for a BD-algebra. For any $\iota, \varsigma, \epsilon \in \Upsilon$:

(P-1) $o \circledast \iota = o$,

(P-2) $\iota \circledast \iota = o$,

(P-3) $(\iota \circledast \varsigma) \circledast \iota = o$,

(P-4) $(\iota \circledast (\iota \circledast \varsigma)) \circledast \varsigma = o$,

(P-5) $(\iota \circledast \varsigma) \circledast \epsilon = (\iota \circledast \varsigma) \circledast \epsilon$,

(P-6) $((\iota \circledast \varsigma) \circledast (\iota \circledast \epsilon)) \circledast (\epsilon \circledast \varsigma) = o$.

Definition 2.3: A subset S of an BD-algebra $(\Upsilon, \circledast, o)$ is called **subalgebra of** Υ if $\iota \circledast \varsigma \in S$ whenever $\iota, \varsigma \in S$.

Definition 2.4: A non-empty subset I of an BD-algebra $(\Upsilon, \circledast, o)$ is called **ideal of** Υ if for any $\iota, \varsigma \in \Upsilon$,

(1) $o \in I$,

(2) $\iota \circledast \varsigma \in I$ and $\varsigma \in I$ imply $\iota \in I$.

Definition 2.5: [6] A non-empty subset I of an BDalgebra $(\Upsilon, \circledast, o)$ is called **BDideal of** Υ if for any $\iota, \varsigma \in \Upsilon$,

(1) $o \in I$,

(2) $\iota \circledast \varsigma \in I \;\&\; \varsigma \in I \Rightarrow \iota \in I$,

(3) $\forall \iota \in I,\ \varsigma \in \Upsilon$ imply $\iota \circledast \varsigma \in I$.

Proposition 2.6:

1. Every ideal of BD-algebra $(\Upsilon, \circledast, o)$ is a BDsubalgebra of Υ **.**

2. Every BD-ideal of BD-algebra $(\Upsilon, \circledast, o)$ is a BDsubalgebra of Υ.

3. Every ideal of BZ-algebra $(\Upsilon, \circledast, o)$ is a BDideal of Υ.

3. Homomorphism of BD-algebras

Now, we will define BD-homomorphism and next we can describe properties of BD-homomorphism.

Definition 3.1: Let $(\Upsilon, \circledast, o)$ and $(\Upsilon', \circledast', o')$ be BD-algebras. A homomorphism is a map $\pi : (\Upsilon, \circledast, o) \to (\Upsilon', \circledast', o')$ satisfying $\pi(\iota \circledast \varsigma) = \pi(\iota) \circledast' \pi(\varsigma)$ for all $\iota, \varsigma \in \Upsilon$.

Definition 3.2: $\pi : (\Upsilon, \circledast, o) \to (\Upsilon', \circledast', o')$ is mapping of a BD-algebra Υ to a BD-algebraΥ' and let $I \subseteq$ X and $J \subseteq \Upsilon'$. The image of I in Υ under π is

$\pi(I) = \{\pi(\iota) \mid \iota \in I\}$ and the inverse image of J in Υ' is $\pi^{-1}(J) = \{\iota \in \Upsilon \mid \pi(\iota) \in J\}$.

Theorem 3.3: *$\pi : (\Upsilon, \circledast, o) \to (\Upsilon', \circledast', o')$ is homomorphism of a BDalgebra Υ to a BD-algebraΥ', then:*

(1) $\pi(o) = o'$.

(2) *If o is the identity in Υ, then $\pi(o) = o'$ is the identity in Υ.*

(3) *π is injective if and only if ker $\pi = \{o\}$.*

Proof: Assume that $\pi : (\Upsilon, \circledast, o) \to (\Upsilon', \circledast', o')$ is a homomorphism.

(1) Since $o \circledast o = o$, then $\pi(o) = \pi(o \circledast o) = \pi(o) \circledast' \pi(o) = o'$.

(2) Assume that o is the identity in Υ and o' is the identity in Υ'.

From BD-2, $o' \circledast' \pi(o) = o'$ and $o' \circledast' \pi(o) = \big(\pi(o) \circledast' \pi(o)\big) \circledast' \pi(o) = \pi(o \circledast o) \circledast' \pi(o) = \pi(o) \circledast' \pi(o) = o'$. We get that $\pi(o) = o'$. This show that $\pi(o)$ is the identity in Υ'.

(3) Suppose that π is injective and $\iota \in \ker \pi$. It follows that $\pi(\iota) = o'$. Since $\pi(o) = o'$, so $\pi(o) = \pi(\iota)$. By assumption, $\iota = o$. Thus $\ker \pi = \{o\}$.

Conversely, suppose that $\ker \pi = \{o\}$. Let $\iota, \varsigma \in \Upsilon$ be such that $\pi(\iota) = \pi(\varsigma)$. Then we get that $\pi(\iota \circledast \varsigma) = \pi(\iota) \circledast' \pi(\varsigma) = o'$ and $\pi(\varsigma \circledast \iota) = \pi(\varsigma) \circledast' \pi(\iota) = o'$, thus $\iota \circledast \varsigma,\ \varsigma \circledast \iota \in \ker \pi$, this means that $\iota \circledast \varsigma = o = \varsigma \circledast \iota$. From BD-3, $\iota = \varsigma$, and shows that π is injective.

Theorem 3.4: *Let $\pi : (\Upsilon, \circledast, o) \to (\Upsilon', \circledast', o')$ be a homomorphism. Then:*

(1) *I is a BD-subalgebra of Υ, $\Rightarrow \pi(I)$ is a BD-subalgebra of Υ'.*

(2) *I is an ideal of Υ, $\Rightarrow \pi(I)$ is an ideal of Υ'.*

(3) *I is an BDideal of Υ, $\Rightarrow \pi(I)$ is an BDideal of Υ'.*

(4) *J is a BDsubalgebra of Υ', $\Rightarrow \pi^{-1}(J)$ is a BDsubalgebra of Υ.*

(5) *J is an ideal in Υ', $\Rightarrow \pi^{-1}(J)$ is an ideal in Υ.*

(6) *J is an BDideal in Υ', $\Rightarrow \pi^{-1}(J)$ is an BDideal in Υ.*

Proof: Assume that $\pi : (\Upsilon, \circledast, o) \to (\Upsilon', \circledast', o')$ is a homomorphism.

(1) Let I be a BD-subalgebra of Υ and $\iota, \varsigma \in \pi(I)$. Then there exist $a, b \in I$ such that $\iota = \pi(a)$ and $\varsigma = \pi(b)$. Since $\iota \circledast' \varsigma = \pi(a) \circledast' \pi(b) = \pi(a \circledast b) \in \pi(I)$. Thus $\pi(I)$ is a BD-subalgebra of Υ'.

(2) Let I be an ideal of Υ. We see that $o \in I$, and by Theorem (3.3(1)), $o' = \pi(o) \in \pi(I)$, so $o' \in \pi(I)$. Now, assume that $\pi(\iota) \circledast' \pi(\varsigma) \in \pi(I)$ and $\pi(\iota) \in \pi(I)$, it follows that $\pi(\iota \circledast \varsigma) \in \pi(I)$, so $\iota \circledast \varsigma,\ \iota \in I$. Since I is an ideal of Υ, $x \in I$, it follows that $\pi(\iota) \in \pi(I)$. Hence $\pi(I)$ is an ideal of Υ'.

(3) Let I be an BD-ideal of Υ. We see that $o \in I$, and by Theorem (3.3(1)), $o' = \pi(o) \in \pi(I)$, so $o' \in \pi(I)$.

Let $\pi(\iota) *' \pi(\varsigma) \in \pi(I)$ and $\pi(\iota) \in \pi(I)$, it follows that $\pi(\iota * \varsigma) \in \pi(I)$, so $\iota * \varsigma$, $\iota \in I$. Since I is an ideal of Υ, $x \in I$, it follows that $\pi(\iota) \in \pi(I)$.

$\forall \iota \in I$, $\varsigma \in \Upsilon$, $\iota *' \varsigma = \pi(a) *' \pi(b) = \pi(a * b) \in \pi(I)$. Thus $\pi(I)$ is an BD-ideal of Υ'.

(4) Let J be a BD-subalgebra of Υ' and $\iota, \varsigma \in \pi^{-1}(J)$. Then $\pi(\iota) = c$ and $\pi(\varsigma) = d$ for some $c, d \in J$. Thus $\pi(\iota * \varsigma) = \pi(\iota) *' \pi(\varsigma) = c *' d \in J$, as J is a BD-subalgebra. Hence $\iota *' \varsigma \in \pi^{-1}(J)$. Thus $\pi^{-1}(J)$ is a BD-subalgebra of Υ.

(5) Let J be an ideal in Υ' . Then $o' \in J$, we get that $o = \pi^{-1}(o') \in \pi^{-1}(J)$.

For any $\iota, \varsigma \in \Upsilon$, let $\iota * \varsigma \in \pi^{-1}(J)$and $\iota \in \pi^{-1}(J)$. It follows that $\pi(\iota) *' \pi(\varsigma) = \pi(\iota * \varsigma) \in J$ and $\pi(\iota) \in J$. Since J is an ideal of Υ' , we obtain that $\pi(\varsigma) \in J$.

Consequently $\varsigma \in \pi^{-1}(J))$, proving that $\pi^{-1}(J)$ is an ideal in Υ. Thus $\pi^{-1}(J)$is an ideal in Υ.

(6) Let J be an BDideal in Υ' , then $o' \in J$, we get that $o = \pi^{-1}(o') \in \pi^{-1}(J)$.

For any $\iota, \varsigma \in \Upsilon$, let $\iota * \varsigma \in \pi^{-1}(J)$and $\iota \in \pi^{-1}(J)$. It follows that $\pi(\iota) *' \pi(\varsigma) = \pi(\iota * \varsigma) \in J$ and $\pi(\iota) \in J$. Since J is an BDideal of Υ' , we obtain that $\pi(\varsigma) \in J$.

$\Rightarrow \varsigma \in \pi^{-1}(J))$.

$\forall \iota \in \pi^{-1}(J)$, $\varsigma \in \Upsilon$, then $\pi(\iota) = c$ and $\pi(\varsigma) = d$ for some $c, d \in J$. Thus $\pi(\iota * \varsigma) = \pi(\iota) *' \pi(\varsigma) = c *' d \in J$, as J is an BDideal. Hence $\iota *' \varsigma \in \pi^{-1}(J)$. Thus $\pi^{-1}(J)$ is an BDideal of Υ.

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