

Cubic JD-fuzzy ideals of BH-algebra

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Abstract

The concepts of cubic JD-Fuzzy Ideals are introduced, & many properties are studied. Their image and their *inverse image* in BH-algebras are defined & investigated. We'll refer to BH-algebra by BH-G, fuzzy set by F.S, interval-valued by I.V. & to cubic S. by cubic S. and we studied the P union of any collection of cubic JD-Fuzzy Ideal and we find is also cubic JD-Fuzzy Ideal, as well as we introduced R union for any given set of cubic JD-Fuzzy Ideal not always be a cubic JD-Fuzzy Ideal, for example. However, we can say A homomorph pre-image of cubic JD-Fuzzy Ideal is cubic JD-ideal.

Subject Classification: 35J88, 39B72, 58E50.

Keywords: BH-algebras, Cubic JD-Fuzzy ideals, Cubic BH-subalgebras, Homomorphism of cubic JD-Fuzzy ideals, Fuzzy set.

1. Introduction

In 1965, Zadeh [8] introduced a new concept of F.S. In Zadeh [7] presents the idea cubic AT-ideals of AT-algebra & talked about a few connected properties of it [1]. In 2018, Mohammed presented the idea of JD-Fuzzy Ideals of BH-algebra [9] & in 2010, Jun studied the homomorphi c image & inverse image of it [12].

Imai and Iseki [13] established the study of BCK-algebra and BCI algebra in 1966. Neggers and Kim introduced B-algebra [8], which is closely related to BCI/BCK-algebra in several ways. B-algebra was expanded as BG-algebra by Zadeh [7], and Jun et. al. [10] fuzzified this algebra. In 1999, Khalid and Ahmad

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[4] laid out fuzzy H-ideals in BCI algebra. Jun [13-17] originally put forward the idea of doubt fuzzy ideals in BCK/BCI-algebras in 1994. Zhan and Tan presented the idea of doubt fuzzy H ideals in BCK-algebra [6]. Xi [8] started the idea of interval-valued fuzzy sets, which are an extension of fuzzy sets. Jun [12] later developed the idea of cubic sets on top of this concept. with this method, doubt the cubic H-ideal

2. Preliminaries

Definition 2.1: [14] Suppose $\mathfrak{A}$ be a set that has a constant $\mathfrak{o}$ & the binary operation $*$. Then $(\mathfrak{A}; \neg, \mathfrak{o})$ is referred to as a BH-Gif the subsequent axioms are met: $\forall \mathfrak{o}, \mathfrak{q}, \mathfrak{z} \in \mathfrak{A}$,

$$1) \mathfrak{o} \neg \mathfrak{q} = \mathfrak{o}, \& \mathfrak{q} \neg \mathfrak{o} = \mathfrak{o} \text{ then } \mathfrak{o} = \mathfrak{q}, 2) \mathfrak{o} \neg \mathfrak{o} = \mathfrak{o}, 3) \mathfrak{o} \neg \mathfrak{o} = \mathfrak{o},$$

Definition 2.2: ([11]) suppose $(\mathfrak{A}; \neg, \mathfrak{o})$ be a BH-G, a F.S μ of $\mathfrak{A}$ is called JD-Fuzzy Ideal of BH-G if it fulfills the requirements listed below. $\forall \mathfrak{o}, \mathfrak{q}, \mathfrak{z} \in \mathfrak{A}$,

$$\mu(\mathfrak{o}) \geq \mu(\mathfrak{o}), \mu(\mathfrak{o}) \geq \min \{ \mu(\mathfrak{o} \neg \mathfrak{q}), \mu(\mathfrak{q}) \}.$$

Proposition 2.3: ([11])

- 1) Any set of JD-Fuzzy Ideals of BH-G that has an intersection is likewise a fuzzy JD-Fuzzy Ideal.
- 2) Any set of JD-Fuzzy Ideals of BH-G has a fuzzy union. JD-Fuzzy Ideal with a chain location.

Remark 2.4: [1] An interval number is $\tilde{\mathfrak{t}} = [\mathfrak{t}^-, \mathfrak{t}^+]$, $\mathfrak{o} \leq \mathfrak{t}^- \leq \mathfrak{t}^+ \leq 1$. $F = [\mathfrak{o}, 1]$. Suppose $S[\mathfrak{o}, 1]$ represents the collection of all closed subintervals. Of $F = [\mathfrak{o}, 1]$, that is, $S[\mathfrak{o}, 1] = \{ \tilde{\mathfrak{t}} = [\mathfrak{t}^-; \mathfrak{t}^+] | \mathfrak{t}^- \leq \mathfrak{t}^+, \text{ for } \mathfrak{t}^-, \mathfrak{t}^+ \in F \}$. Let's now specify what is meant by refined min (r min, briefly) of two components in $S[\mathfrak{o}, 1]$.

Definition 2.5: [1] The signs ($\leq, =, \geq$) rmin & rmax when there are two components in $S[\mathfrak{o}, 1]$. Consider two interval $\tilde{\mathfrak{t}} = [\mathfrak{t}^-, \mathfrak{t}^+]$ & $\tilde{\mathfrak{e}} = [\mathfrak{e}^-, \mathfrak{e}^+]$ in $S[\mathfrak{o}, 1]$: Then

$$\tilde{\mathfrak{t}} \geq \tilde{\mathfrak{e}} \leftrightarrow \text{if, } \mathfrak{e}^- \leq \mathfrak{t}^- \& \mathfrak{e}^+ \leq \mathfrak{t}^+, \tilde{\mathfrak{t}} \leq \tilde{\mathfrak{e}} \leftrightarrow \text{if, } \mathfrak{t}^- \leq \mathfrak{e}^- \& \mathfrak{t}^+ \leq \mathfrak{e}^+, \tilde{\mathfrak{t}} = \tilde{\mathfrak{e}} \leftrightarrow \text{if, } \mathfrak{t}^- = \mathfrak{e}^- \& \mathfrak{t}^+ = \mathfrak{e}^+ \text{ rmin } \{ (\tilde{\mathfrak{t}}), \tilde{\mathfrak{e}} \} = [\min \{ \mathfrak{t}^-, \mathfrak{e}^- \}, \min \{ \mathfrak{t}^+, \mathfrak{e}^+ \}], \& \text{ rmax } \{ \tilde{\mathfrak{t}}, \tilde{\mathfrak{e}} \} = [\max \{ \mathfrak{t}^-, \mathfrak{e}^- \}, \max \{ \mathfrak{t}^+, \mathfrak{e}^+ \}],$$

Remark 2.6: [1] It is clear that $\{ S[\mathfrak{o}, 1], \vee, \leq, \wedge \}$ is a full lattice that has $(\mathfrak{o})$ $\tilde{\mathfrak{a}}_i \in S[\mathfrak{o}, 1]$, $\tau \in \Lambda$. We clarify. $\text{rinf}_{\tau \in \Lambda} \tilde{\mathfrak{t}} = [\text{rinf}_{\tau \in \Lambda} \mathfrak{t}^-, \text{rinf}_{\tau \in \Lambda} \mathfrak{t}^+]$, $\text{rsup}_{\tau \in \Lambda} \tilde{\mathfrak{t}} = [\text{rsup}_{\tau \in \Lambda} \mathfrak{t}^-, \text{rsup}_{\tau \in \Lambda} \mathfrak{t}^+]$.

Definition 2.7: [14] An I.V. F.S $\tilde{\mu}_A$ on $\mathfrak{A}$ is characterized as $\tilde{\mu}_A = \{ \langle o, [\mu_A^-(o); \mu_A^+(o)] \rangle \mid o \in \mathfrak{A} \}$. Where $\mu_A^-(o) \leq \mu_A^+(o)$, $\forall o \in \mathfrak{A}$. the ordinary F.S $\mu_A^-: \mathfrak{A} \rightarrow [0, 1]$ & $\mu_A^+: \mathfrak{A} \rightarrow [0, 1]$ referred to as a lower F.S. & an upper F.S of $\tilde{\mu}_A$ respectively. suppose $\tilde{\mu}_A(o) = [\mu_A^-(o), \mu_A^+(o)]$, $\tilde{\mu}_A: \mathfrak{A} \rightarrow D[0, 1]$, we have $A = \{ \langle o, \tilde{\mu}_A(o) \rangle \mid o \in \mathfrak{A} \}$.

Definition 2.8: ([14]) Suppose $(\mathfrak{A}; *, \emptyset)$ be a set that is not empty. A cubic S. Ω in a structure $\Omega = \{ \langle o, \tilde{\mu}_\Omega(o), \nabla_\Omega(o) \rangle \mid o \in \mathfrak{A} \}$, which is indicated in brief by $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$, where $\tilde{\mu}_\Omega: \mathfrak{A} \rightarrow D[0, 1]$, $\tilde{\mu}_\Omega$ is an I.V. F.S of $\mathfrak{A}$ & $\nabla_\Omega: \mathfrak{A} \rightarrow [0, 1]$, ∇_Ω is a F.S of $\mathfrak{A}$.

Definition 2.9: ([1]) Regarding a household $\Omega_\tau = \{ \langle o, \tilde{\mu}_{\Omega_\tau}(o) \rangle \mid o \in \mathfrak{A} \}$ on F.S s of $\mathfrak{A}$, where $\tau \in \Lambda$ & Λ is the index set, which we specify (V) & ($\wedge$) according to:

$$V_{\tau \in \Lambda} \Omega_\tau = (V_{\tau \in \Lambda} \tilde{\mu}_{\Omega_\tau})(o) = \sup \{ \tilde{\mu}_{\Omega_\tau}(o) \mid \tau \in \Lambda \},$$

$$\wedge_{\tau \in \Lambda} \Omega_\tau = (\wedge_{\tau \in \Lambda} \tilde{\mu}_{\Omega_\tau})(o) = \inf \{ \tilde{\mu}_{\Omega_\tau}(o) \mid \tau \in \Lambda \},$$

3. Cubic JD-Fuzzy Ideals of BH-algebra

Definition 3.1: Suppose $(\mathfrak{A}; \neg, \emptyset)$ be a BH-G. A cubic S. $\Omega = \langle \tilde{\mu}_\Omega(o), \nabla_\Omega(o) \rangle$ of $\mathfrak{A}$ is referred to as cubic JD-Fuzzy Ideal of $\mathfrak{A}$ if, $\forall o, q \in \mathfrak{A}$: 1) $\tilde{\mu}_\Omega(o) \geq \tilde{\mu}_\Omega(o)$, & $\nabla_\Omega(o) \leq \nabla_\Omega(o)$, 2) $\tilde{\mu}_\Omega(o) \geq \min \{ \tilde{\mu}_\Omega(o \neg q), \tilde{\mu}_\Omega(q) \}$, & $\nabla_\Omega(o) \leq \max \{ \nabla_\Omega(o \neg q), \nabla_\Omega(q) \}$.

Example 3.2: Suppose $\mathfrak{A} = \{ \emptyset, 1, 2, m \}$ in wherein the following table defines the operation

$\neg$	$\emptyset$	m	1	2
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
1	1	$\emptyset$	$\emptyset$	$\emptyset$
2	2	$\emptyset$	1	$\emptyset$
m	m	$\emptyset$	2	1

Then $(\mathfrak{A}; *, \emptyset)$ is an BH-G. Define a cubic S. $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ of $\mathfrak{A}$ F.S $\mu: \mathfrak{A} \rightarrow [0, 1]$ by: $\tilde{\mu}_\Omega(o) = \begin{cases} [0.3, 0.9] & \text{if } o \in \{ \emptyset, 1 \} \\ [0.1, 0.6] & \text{otherwise} \end{cases}$ & $\nabla_\Omega = \begin{cases} 0.1 & \text{if } o \in \{ \emptyset, 1 \} \\ 0.6 & \text{otherwise} \end{cases}$.

The cubic S. $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ is a cubic JD-Fuzzy Ideal of $\mathfrak{A}$.

Proposition 3.3: The R. intersection of whatever a collection of cubic JD-Fuzzy Ideal of $\mathfrak{A}$ is also cubic JD – ideal of $\mathfrak{A}$.

Proof: Suppose $\{ \Omega \mid \tau \in \nabla \}$ be a cubic family JD – ideals of $\mathfrak{A}$, then for any $o, q, z \in \mathfrak{A}$,

$$\begin{aligned}
(\cap \tilde{\mu}_{\Omega\tau})(\circ) &= \text{rinf}(\tilde{\mu}_{\Omega i}(\circ)) \geq \text{rinf}(\tilde{\mu}_{\Omega i}(\circ)) = (\cap \tilde{\mu}_{\Omega\tau})(\circ) (\cap \tilde{\mu}_{\Omega\tau})(\circ) = \text{rinf}(\tilde{\mu}_{\Omega i}(\circ)) \\
&\geq \text{rinf}(\text{rmin}\{\tilde{\mu}_{\Omega i}((\circ \neg q)), \tilde{\mu}_{\Omega i}(q)\}) = \text{rmin}\{\text{rinf}(\tilde{\mu}_{\Omega i}(\circ \neg q)), \text{rinf}(\tilde{\mu}_{\Omega i}(q))\} \\
&= \text{rmin}\{(\cap \tilde{\mu}_{\Omega\tau})(\circ \neg q), (\cap \tilde{\mu}_{\Omega\tau})(q)\} \\
(\vee v_{\Omega\tau})(\circ) &= \sup v_{\Omega\tau}(\circ) \leq \sup\{\max\{v_{\Omega\tau}((\circ \neg q)), v_{\Omega\tau}(q)\}\} = \max\{\sup \\
& (v_{\Omega\tau}((\circ \neg q))), \sup(v_{\Omega\tau}(q))\} \\
&= \max\{(\vee v_{\Omega\tau})(\circ \neg q), (\vee v_{\Omega\tau})(q)\}.
\end{aligned}$$

Hence, R-intersection of Ω_τ is a cubic JD-Fuzzy Ideal of $\mathfrak{A}$. $\triangle$

Remark 3.4: The P. intresection of any combination of cubic JD-Fuzzy Ideal not required to be a cubic BH-sub-algebra, for example:

Example 3.5: suppose $\mathfrak{A} = \{\circ, \iota, c, e, d\}$ be a set with the following table 1:

Table 1
A combination of cubic JD-Fuzzy Ideal

$\neg$	$\circ$	c	ι	e	d
$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
ι	ι	ι	$\circ$	ι	ι
d	d	d	c	d	$\circ$
c	c	$\circ$	c	c	$\circ$
e	e	e	e	$\circ$	e

Then $(\mathfrak{A}; \neg, \circ)$ is a BH-G, $I = \{\circ, c\}$ & $J = \{\circ, e\}$ are JD-Fuzzy Ideals of $\mathfrak{A}$.

Two cubics S . $\Omega_1 = \langle \tilde{\mu}_{\Omega_1}, v_{\Omega_1} \rangle$ & $\Omega_2 = \langle \tilde{\mu}_{\Omega_2}, v_{\Omega_2} \rangle$ of $\mathfrak{A}$:-

$$\begin{aligned}
\tilde{\mu}_{\Omega_1}(\circ) &= \begin{cases} [0.6; 0.7], & \text{if } \circ \in \{\circ, c\}, \\ [1, 0.2], & \text{if } \circ \in \{d\}, \\ [0.3, 0.4], & \text{otherwise} \end{cases} \quad v_{\Omega_1}(\circ) = \begin{cases} 0.1, & \text{if } \circ \in \{\circ, c\}, \\ 0.6, & \text{if } \circ \in \{d\}, \\ 0.2, & \text{otherwise} \end{cases} \\
\tilde{\mu}_{\Omega_2}(\circ) &= \begin{cases} [0; 8, 0.9], & \text{if } \circ \in \{\iota, d\}, \\ [0.3; 0.4], & \text{otherwise,} \end{cases} \quad v_{\Omega_2}(\circ) = \begin{cases} 0.1, & \text{if } \circ \in \{\circ, e\}, \\ 0.4, & \text{otherwise.} \end{cases}
\end{aligned}$$

Following that Ω_1 & Ω_2 are cubic JD-Fuzzy Ideal of $\mathfrak{A}$, P.intersection of Ω_1 & Ω_2 are not cubic JD-Fuzzy Ideals of $\mathfrak{A}$. Given that

$$\begin{aligned}
(\cap \tilde{\mu}_{\Omega\tau})(d) &= [0.1, 0.2] \not\geq [0.3, 0.4] = \text{rmin}\{(\cap \tilde{\mu}_{\Omega\tau})(d * \iota), (\cap \tilde{\mu}_{\Omega\tau})(\iota)\} \\
&\& (\wedge v_{\Omega\tau})(d) = 0.4 \not\leq 0.1 = \min\{(\wedge v_{\Omega\tau})(d * \iota), (\wedge v_{\Omega\tau})(\iota)\}.
\end{aligned}$$

Proposition 3.6: suppose $\Omega_\tau = \langle \tilde{\mu}_{\Omega\tau}, v_{\Omega\tau} \rangle$ be a cubic JD-Fuzzy Ideal of BH-G $(\mathfrak{A}; \neg, \circ)$, where $\tau \in \Lambda$, $\sup\{\text{rmin}\{\tilde{\mu}_{\Omega\tau}(\circ), \tilde{\mu}_{\Omega\tau}(q)\}\} = \text{rmin}\{\text{rsup}$

$\tilde{\mu}_{\Omega_\tau}(o), \text{rsup } \tilde{\mu}_{\Omega_\tau}(q)\}, \forall o \in \mathfrak{L}$, following that, the P. intersection of Ω_τ is even a cubic JD-Fuzzy Ideal of $\mathfrak{L}$.

Proof: Suppose $\Omega_\tau = \{\langle o, \tilde{\mu}_{\Omega_\tau}(o), v_{\Omega_\tau}(o) \rangle \mid o \in \mathfrak{L}\}$ where $\tau \in \Lambda$, be a collection of cubic JD-Fuzzy Ideal of $\mathfrak{L}$, $\forall o \in \mathfrak{L}$, then for $o, q, z \in \mathfrak{L}$,

$$\begin{aligned} (\cap \tilde{\mu}_{\Omega_\tau})(o) &= \text{rinf}(\tilde{\mu}_{\Omega_\tau}(o)) \geq \text{rinf}(\tilde{\mu}_{\Omega_\tau}(o)) = (\cap \tilde{\mu}_{\Omega_\tau})(o) \\ (\cap \tilde{\mu}_{\Omega_\tau})(o) &= \text{rinf } \tilde{\mu}_{\Omega_\tau}(o) \geq \text{rsup}\{\text{rmin}\{\tilde{\mu}_{\Omega_\tau}(o \neg q), \tilde{\mu}_{\Omega_\tau}(q)\}\} = \\ &= \text{rmin}\{(\cap \tilde{\mu}_{\Omega_\tau})(o * q), (\cap \tilde{\mu}_{\Omega_\tau})(q)\} \& (\wedge v_{\Omega_\tau})(o) = \inf v_{\Omega_\tau}(o) \leq \\ &= \inf\{\max\{v_{\Omega_\tau}(o \neg q), v_{\Omega_\tau}(q)\}\} = \max\{(\wedge v_{\Omega_\tau})(o \neg q), (\wedge v_{\Omega_\tau})(q)\} \leq \\ &= \min\{(\wedge v_{\Omega_\tau})(o \neg q), (\wedge v_{\Omega_\tau})(q)\}. \end{aligned}$$

Hence, P.intersection of Ω_τ is a JD-Fuzzy Ideal of $\mathfrak{L}$. $\square$

Proposition 3.7: The P.union of any collection of cubic JD -Fuzzy Ideal of $\mathfrak{L}$ is also cubic JD-Fuzzy Ideal of $\mathfrak{L}$.

Proof: Suppose $\Omega_\tau = \{\langle o, \tilde{\mu}_{\Omega_\tau}(o), v_{\Omega_\tau}(o) \rangle \mid o \in \mathfrak{L}\}$ where $\tau \in \Lambda$, be a collection of cubic JD-Fuzzy Ideal of $\mathfrak{L}$ & $o, q, z \in \mathfrak{L}$, then

$$\begin{aligned} (\cup \tilde{\mu}_{\Omega_\tau})(o) &= \text{rinf}(\tilde{\mu}_{\Omega_\tau}(o)) \geq \text{rinf}(\tilde{\mu}_{\Omega_\tau}(o)) = (\cup \tilde{\mu}_{\Omega_\tau})(o) \\ (\cup \tilde{\mu}_{\Omega_\tau})(o) &= \text{rsup } \tilde{\mu}_{\Omega_\tau}(o) \geq \text{rsup}\{\text{rmax}\{\tilde{\mu}_{\Omega_\tau}(o \neg q), \tilde{\mu}_{\Omega_\tau}(q)\}\} \geq \\ &= \text{rmax}\{\text{rsup } \tilde{\mu}_{\Omega_\tau}(o \neg q), \text{rsup } \tilde{\mu}_{\Omega_\tau}(q)\} \\ &= \text{rmax}\{(\cup \tilde{\mu}_{\Omega_\tau})(o \neg q), (\cup \tilde{\mu}_{\Omega_\tau})(q)\}. \\ (\vee v_{\Omega_\tau})(o) &= \sup v_{\Omega_\tau}(o) \leq \sup\{\max\{v_{\Omega_\tau}(o \neg q), v_{\Omega_\tau}(q)\}\} \\ &= \max\{(\vee v_{\Omega_\tau})(o \neg q), (\vee v_{\Omega_\tau})(q)\}, \text{ Therefore, P.union of } \Omega_\tau \text{ is a cubic} \\ &\text{JD-Fuzzy Ideal of } \mathfrak{L}. \end{aligned}$$

Remark 3.9: R.union for any given set of cubic JD-Fuzzy Ideal not always be a cubic JD-Fuzzy Ideal, for example:

Proposition 3.1:0 Suppose $\Omega_\tau = \langle \tilde{\mu}_{\Omega_\tau}, v_{\Omega_\tau} \rangle$ become a cubic JD-Fuzzy Ideal of BH-G $(\mathfrak{L}; \neg, \circ)$, where $\tau \in \Lambda$ $\text{rsup}\{\text{rmin}\{\tilde{\mu}_{\Omega_\tau}(o), \tilde{\mu}_{\Omega_\tau}(q)\}\} = \text{rmin}\{\text{rsup } \tilde{\mu}_{\Omega_\tau}(o), \text{rsup } \tilde{\mu}_{\Omega_\tau}(q)\}, \forall o \in \mathfrak{L}$, following that, the R.union of Ω_τ is also a cubic JD-Fuzzy Ideal of $\mathfrak{L}$.

Proof: Suppose $\Omega_\tau = \{\langle o, \tilde{\mu}_{\Omega_\tau}(o), v_{\Omega_\tau}(o) \rangle \mid o \in \mathfrak{L}\}$ where $\tau \in \Lambda$, be a collection of cubic BH-subalgebra of $\mathfrak{L}$, then for $o, q, z \in \mathfrak{L}$, $v(\cup \tilde{\mu}_{\Omega_\tau})(o) = \text{rinf}(\tilde{\mu}_{\Omega_\tau}(o)) \geq \text{rinf}(\tilde{\mu}_{\Omega_\tau}(o)) = (\cap \tilde{\mu}_{\Omega_\tau})(o)$

$$\begin{aligned} (\cup \tilde{\mu}_{\Omega_\tau})(o) &= \text{rinf } \tilde{\mu}_{\Omega_\tau}(o) \geq \text{rsup}\{\text{rmin}\{\tilde{\mu}_{\Omega_\tau}(o * q), \tilde{\mu}_{\Omega_\tau}(q)\}\} = \\ &= \text{rmin}\{\text{rsup } \mu_{\Omega_\tau}(o * q), \text{rsup } \mu_{\Omega_\tau}(q)\} = \text{rmin}\{(\cup \tilde{\mu}_{\Omega_\tau})(o \neg q), (\cup \tilde{\mu}_{\Omega_\tau})(q)\} \\ &\& \\ (\wedge v_{\Omega_\tau})(o) &= \inf v_{\Omega_\tau}(o) \leq \inf\{\max\{v_{\Omega_\tau}(o \neg q), v_{\Omega_\tau}(q)\}\} = \\ &= \max\{\inf v_{\Omega_\tau}(o \neg q), \inf v_{\Omega_\tau}(q)\} \end{aligned}$$

$= \max\{(\wedge v_{\Omega_T})(o \neg q), (\wedge v_{\Omega_T})(q)\}$ Hence, R.union of Ω_T is a cubic JD-Fuzzy Ideal of $\mathfrak{L}$.

Proposition 3.11: Suppose $B \neq \emptyset$ and subset of $\mathfrak{L}$ & $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ be a cubic S. of $\mathfrak{L}$ as specified by

$$\tilde{\mu}_\Omega(o) = \begin{cases} [\alpha_1, \alpha_2], & \text{if } o \in B \\ [\beta_1, \beta_2], & \text{otherwise} \end{cases} \text{ \& } v_\Omega(o) = \begin{cases} \gamma, & \text{if } o \in B \\ \delta, & \text{otherwise} \end{cases}$$

$$\forall [\alpha_1, \alpha_2], [\beta_1, \beta_2] \in D[0, 1] \text{ \& } \gamma, \delta \in [0, 1] \text{ with } [\alpha_1, \alpha_2] \geq [\beta_1, \beta_2] \text{ \& } \gamma \leq \delta.$$

Then Ω is a cubic JD-Fuzzy Ideal of $\mathfrak{L} \leftrightarrow$ if, B is a BH – ideal of $\mathfrak{L}$.
Moreover, $I_{\tilde{\mu}_\Omega} = B = I_{v_\Omega}$.

Proof: Suppose Ω be a cubic JD – Fuzzy Ideal of $\mathfrak{L}$ & $o, q, z \in B$, then $\tilde{\mu}_\Omega(o) \geq \min\{\tilde{\mu}_\Omega(o \neg q), \tilde{\mu}_\Omega(q)\} = \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2]$ & $v_\Omega(o) \leq \max\{v_\Omega(o * q), v_\Omega(q)\} = \{\gamma, \gamma\} = \gamma$.

So $o * z \in B$. Hence B is an JD-Fuzzy Ideal of $\mathfrak{L}$.

In contrast, let's say B is JD-F.I of $\mathfrak{L}$ & let $o, q, z \in \mathfrak{L}$. Examine two instances.

Case 1: If $(o * q), q \in B$ then $o \in B$, thus $\tilde{\mu}_\Omega(o) = [\alpha_1, \alpha_2] = \min\{\tilde{\mu}_\Omega(o \neg q), \tilde{\mu}_\Omega(q)\}$ & $v_\Omega(o) = \gamma = \max\{v_\Omega(o \neg q), v_\Omega(q)\} = \max\{\gamma, \gamma\}$.

Case 2: If $(o \neg q) \in B$ & $q \notin B$, thus $\tilde{\mu}_\Omega(o) \geq [\beta_1, \beta_2] = \min\{\tilde{\mu}_\Omega(o \neg q), \tilde{\mu}_\Omega(q)\}$ & $v_\Omega(o) \leq \delta = \max\{v_\Omega(o \neg q), v_\Omega(q)\}$.

Case 3: If $(o * q) \notin B$ & $q \in B$, thus $\tilde{\mu}_\Omega(o) \geq [\beta_1, \beta_2] = \min\{\tilde{\mu}_\Omega(o \neg q), \tilde{\mu}_\Omega(q)\}$ & $v_\Omega(o) \leq \delta = \max\{v_\Omega(o \neg q), v_\Omega(q)\}$.

Case 4: If $o \notin B$ or $q \notin B$, then $\tilde{\mu}_\Omega(o * q) \geq [\beta_1, \beta_2] = \min\{\tilde{\mu}_\Omega(o \neg q), \tilde{\mu}_\Omega(q)\}$ & $v_\Omega(o \neg q) \leq \delta = \max\{v_\Omega(o), v_\Omega(q)\}$.

Thus, Ω is cubic JD – Fuzzy Ideal of $\mathfrak{L}$. Now, $I_{\tilde{\mu}_\Omega} = \{o \in \mathfrak{L} \mid \tilde{\mu}_\Omega(o) = \tilde{\mu}_\Omega(o)\} = \{o \in \mathfrak{L} \mid \tilde{\mu}_\Omega(o) = [\alpha_1, \alpha_2]\} = B$ & $I_{v_\Omega} = \{o \in \mathfrak{L} \mid v_\Omega(o) = v_\Omega(o)\} = \{o \in \mathfrak{L} \mid v_\Omega(o) = \gamma\} = B$.

Theorem 3.12: A homomorph pre image of cubic JD-Fuzzy Ideal is cubic JD-ideal.

Proof: Suppose $p : (\mathfrak{L}; \neg, \odot) \rightarrow (Y; \neg', \odot')$ as a homomorphism between a BH-G $\mathfrak{L}$ into a BH – algebra Y .

If $\beta = \langle \tilde{\mu}_\beta, \nabla_\beta \rangle$ is a cubic JD – Fuzzy Ideal of Y & $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ the pre – image of β under p , then $\tilde{\mu}_\Omega(o) = \tilde{\mu}_\beta(p(o))$, $\nabla_\Omega(o) = \nabla_\beta(p(o))$, $\forall o \in \mathfrak{L}$. suppose $o \in \mathfrak{L}$, then $(\tilde{\mu}_\Omega)(o) = \tilde{\mu}_\beta(p(o)) \geq \tilde{\mu}_\beta(p(o)) = \tilde{\mu}_\Omega(o)$, & $(\nabla_\Omega)(o) = \nabla_\beta(p(o)) \leq \nabla_\beta(p(o)) = \nabla_\Omega(o)$.

Now, let $o, q, z \in \mathfrak{L}$, then $\tilde{\mu}_\Omega(o) = \tilde{\mu}_\beta(p(o)) \geq \text{rmin} \{ \tilde{\mu}_\beta(p((o \neg q)), \tilde{\mu}_\beta(p(q)) \} = \text{rmin} \{ \tilde{\mu}_\Omega(o \neg (q)), \tilde{\mu}_\Omega(q) \}$, & $\nabla_\Omega(o) = \nabla_\beta(p(o)) \leq \max \{ \nabla_\beta(p(o \neg q)), \nabla_\beta(p(q)) \} = \max \{ \nabla_\Omega(o \neg q), \nabla_\Omega(q) \}$.

Theorem 3.13: Suppose $p : (\mathfrak{L}; \neg, \circ) \rightarrow (Y; \neg', \circ')$ be an epimorphism from a BH-G $\mathfrak{L}$ into a BH-algebra Y . For every cubic JD-Fuzzy Ideal $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ of $\mathfrak{L}$ with properties of sup and inf, then $p(\Omega)$ is a cubic JD-Fuzzy Ideal of Y .

Proof: By definition $\tilde{\mu}_\beta(q') = p(\tilde{\mu}_\Omega)(q') = \text{rsup}_{o \in p^{-1}(q')} \tilde{\mu}_\Omega(o)$ & $\nabla_\beta(q') = f(\nabla_\Omega)(q') = \inf_{o \in f^{-1}(q')} \nabla_\Omega(o) \forall q' \in Y$ & $\inf(\emptyset) = \circ$, $\text{rsup}(\emptyset) = [\circ, \circ]$. We have prove that $\tilde{\mu}_\beta(o') \geq \text{rmin} \{ \tilde{\mu}_\beta((o' \neg q')), \tilde{\mu}_\beta(q') \}$, & $\nabla_\beta(o') \leq \max \{ \nabla_\beta(o' \neg q'), \nabla_\beta(q') \}$, $\forall o', q', z' \in Y$. suppose $p : (\mathfrak{L}; \neg, \circ) \rightarrow (Y; \neg', \circ')$ be epimorphism of BH-Gs, $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ is a cubic JD-Fuzzy Ideal of $\mathfrak{L}$ has properties of sup and inf & $\beta = \langle \tilde{\mu}_\beta, \nabla_\beta \rangle$ the image of $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ under f . Given that $\Omega = \langle \tilde{\mu}_\Omega, \nabla_\Omega \rangle$ is a cubic JD-Fuzzy Ideal of $\mathfrak{L}$, we have $(\tilde{\mu}_\Omega)(\circ) \geq \tilde{\mu}_\Omega(o)$, & $(\nabla_\Omega)(\circ) \leq \nabla_\Omega(o)$, $\forall o \in \mathfrak{L}$.

Note that, $\circ \in p^{-1}(\circ')$ where $\circ, \circ'$ are the zero of $\mathfrak{L}$ & Y , respectively. Thus $\tilde{\mu}_\beta(0') > \tilde{\mu}_\beta(x')$, $\forall \circ \in \mathfrak{L}$, which implies that $\tilde{\mu}_\beta(0') \geq \tilde{\mu}_\beta(o') & \nabla_\Omega(o_\circ) \geq \nabla_\beta\{f(o_\circ)\} \forall o' \in Y$. $\forall o', q', z' \in Y$, let $x_0 \in p^{-1}(o')$, $y_0 \in p^{-1}(q')$, & $z_0 \in p^{-1}(z')$ $\tilde{\mu}_\Omega(o_\circ \neg q_\circ) = \text{rsup}_{t \in p^{-1}((o' * q'))} \tilde{\mu}_\Omega(t)$, $\tilde{\mu}_\Omega(y_0) = \text{rsup}_{t \in p^{-1}(y')} \tilde{\mu}_\Omega(t)$ & $\tilde{\mu}_\Omega(o_\circ) = \tilde{\mu}_\beta\{f(o_\circ)\} = \tilde{\mu}_\beta(o') = \text{rsup}_{(o_\circ) \in p^{-1}(o')} \tilde{\mu}_\Omega(o_\circ) = \text{rsup}_{t \in p^{-1}(o')} \tilde{\mu}_\Omega(t)$. Also, $\nabla_\Omega(o_\circ \neg q_\circ) = \inf_{t \in p^{-1}((o' * q'))} \nabla_\Omega(t)$, $\nabla_\Omega(q_\circ) = \inf_{t \in p^{-1}(q')} \nabla_\Omega(t)$ & $\nabla_\Omega(o_\circ) = \nabla_\beta\{p(o_\circ)\} = \nabla_\beta\{p(o')\} = \inf_{(o_\circ) \in p^{-1}(o')} \nabla_\Omega(o_\circ) = \inf_{t \in p^{-1}(o')} \nabla_\Omega(t)$. Then $\tilde{\mu}_\beta(o') = \text{rsup}_{t \in p^{-1}(o')} \tilde{\mu}_\Omega(t) = \tilde{\mu}_\Omega(o_\circ) \geq \text{rmin} \{ \tilde{\mu}_\Omega(x_0 \neg y_0), \tilde{\mu}_\Omega(y_0) \} = \text{rmin} \{ \tilde{\mu}_\beta(o' \neg q'), \tilde{\mu}_\beta(q') \}$ & $\tilde{\nabla}_\beta((o') \inf_{t \in p^{-1}(o')} \nabla_\Omega(t) = \nabla_\Omega(o') \leq \max \{ \nabla_\Omega((o_\circ \neg q_\circ), \nabla_\Omega(q_\circ) \} = \max \{ \inf_{t \in p^{-1}((o' * q'))} \nabla_\Omega(t), \inf_{t \in p^{-1}(q')} \nabla_\Omega(t) \}$ Hence, β is a cubic JD-Fuzzy Ideal of Y .

Conclusions

Many properties of cubic JD-Fuzzy Ideals are studied and their concepts are introduced. We define and study their image and their inverse image in BH-algebras. The terms BH-algebra, fuzzy set, interval valued, and cubic S. are used interchangeably. For example, we studied the P.union of any collection of cubic JD-Fuzzy Ideals and discovered that is also a cubic JD-Fuzzy Ideal. We also introduced the idea that R union for any given set of cubic JD-Fuzzy Ideals need not always be a cubic JD-Fuzzy Ideal. Furthermore, we can state that cubic JD-ideal is a homomorph pre-image of cubic JD-Fuzzy Ideal.

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Received July, 2024