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Cubic intuitionistic ideals of a TM-algebra

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Abstract

This study investigated the cubic intuitionistic fuzzy set of TM-algebra as a generalization of the cubic set. First, a cubic intuitionistic ideal and a cubic intuitionistic T-ideal are defined, followed by a discussion of their properties. Furthermore, the level set of a cubic intuitionistic TM-algebra is defined, and the relationship between a cubic intuitionistic level set and the cubic intuitionistic T-ideal is established. A novel definition of a cubic intuitionistic set under homomorphism is proposed, and several significant results are demonstrated.

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Keywords: *Cubic intuitionistic set, Cubic intuitionistic ideal, Cubic intuitionistic level set.*

1. Introduction

Atanassov introduced the concept of "intuitionistic fuzzy sets" in his publication [1]. Jun and Kim [7] studied intuitionistic fuzzy ideals on BCK algebras, and Tong and Guan [17] introduced the idea of intuitionistic

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distribution numbers. Mostafa and Kareem [13] studied an intuitionistic fuzzy n-fold KU-ideal of KU algebra. Many authors introduced intuitionistic fuzzy sets and applied them in many structures (see [2, 3, 4, 5 and 10]).

Jun, Kim, and Kang [8] studied the concept of a cubic set of BCK/BCI algebras. Yaqoob et al. introduced a cubic KU algebra, which is an extension of fuzzy KU ideas in KU algebras [18]. Some research papers presented the idea of cubic sets in different ways (see [6, 11, 14, 15, and 16]).

In this paper, we will look at the concepts of a cubic intuitionistic ideal and a cubic intuitionistic T-ideal, as well as some of its features. We define the level set of a cubic intuitionistic TM-algebra and establish the relationship between a cubic intuitionistic level set and a cubic intuitionistic T-ideal. We also present a novel definition of a cubic intuitionistic ideal under a homomorphism.

2. Basic Concepts

Definition 2.1: [12] The triple order $(\mathcal{H},*,0)$ is called a TM-algebra, where "0" is a constant, " *" is a binary operation and $\mathcal{H} \neq \varphi$, if : $\forall \wp, ᴟ, \eta \in \mathcal{H}$

(tm_1) $\wp * 0 = \wp$,

(tm_2) $(\wp * ᴟ) * (\wp * \eta) = \eta * ᴟ$.

The relation $\leq$ is defined $\wp \leq ᴟ \Leftrightarrow \wp * ᴟ = 0$. Then, the following are fulfilled. $\forall \wp, ᴟ, \eta \in \mathcal{H}$

a) $\wp * \wp = 0$,

b) $\wp * (\wp * ᴟ) = ᴟ$,

c) $(\wp * \eta) * (ᴟ * \eta) \leq \wp * ᴟ$,

d) $(\wp * ᴟ) * \eta = (\wp * \eta) * ᴟ$.

Definition 2.2: [12] A set $\varphi \neq S \subseteq \mathcal{H}$ is called a TM-subalgebra if $\wp * ᴟ \in S$ whenever $\wp, ᴟ \in \mathcal{H}$.

Definition 2.3: [12] A set $\varphi \neq \psi \subseteq \mathcal{H}$ is called an ideal if, for all $\wp, ᴟ \in \mathcal{H}$.

i) $0 \in S$,

ii) If $\wp * ᴟ \in S$,$ᴟ \in S \Rightarrow \wp \in S$.

Definition 2.4: [12] A set $\varphi \neq S \subseteq \mathcal{H}$ is called a T-ideal, if

i) $0 \in S$

ii) $\forall \wp, ᴟ, \eta \in \mathcal{H}, ((\wp * ᴟ) * \eta) \in S$,$ᴟ \in \psi \Rightarrow \wp * \eta \in S$.

Definition 2.5: [12] Let $\mathcal{H}$ and $\mathcal{H}'$ be two TM-algebras. A mapping $f: \mathcal{H} \to \mathcal{H}'$ is called a homomorphism TM-algebra if $(\wp * ᴟ) = f(\wp) * f(ᴟ)$, $\forall \wp, ᴟ \in \mathcal{H}$.

Definition 2.6: [8] An interval valued fuzzy set on $\mathcal{H}$ is given by $\{(\wp, [\omega^L(\wp), \omega^U(\wp)])\}$, $\forall \wp \in \mathcal{H}$, where $\omega^L(\wp)$ *and* $\omega^U(\wp)$ are two fuzzy sets in $\mathcal{H}$ such that $\omega^L(\wp) \leq \omega^U(\wp)$. All closed sub intervals of [0,1] is denoted by D[0,1] and $\widetilde{\omega}(\wp) = [\omega^L(\wp), \omega^U(\wp)]$, $\forall \wp \in \mathcal{H}$, where $\widetilde{\omega}: \mathcal{H} \to D[0,1]$.

Definition 2.7: [8] A cubic set in a non-empty set $\mathcal{H}$ is the form

$\lambda = \langle \wp, \widetilde{\omega}_{\lambda}, \rho_{\lambda} : \wp \in \mathcal{H} \rangle$ where $\widetilde{\omega}_{\lambda} : \mathcal{H} \to D[0,1]$ is a fuzzy function of an interval valued and ρ_{λ} is a fuzzy function such that $\rho_{\lambda} : \mathcal{H} \to [0,1]$.

The main idea of this paper is the intuitionistic fuzzy set. We will mention the basic definitions provided by [9]**.**

Definition 2.8: [9]: A nonempty subset $\beth = \{(\wp, \omega_{\beth}, \partial_{\beth}) : \wp \in \mathcal{H}\}$ is an intuitionistic fuzzy set of $\mathcal{H}$, where $\omega_{\beth} : \mathcal{H} \to [0,1]$ is refers to a degree membership, $\partial_{\beth} : \mathcal{H} \to [0,1]$ is refers to a degree non-membership of the element $\wp \in \mathcal{H}$, and $0 \le \omega_{\beth} + \partial_{\beth} \le 1$, $\forall \wp \in \mathcal{H}$.

Definition 2.9: [9] A subset $\beth = (\omega_{\beth}, \partial_{\beth})$ of $\mathcal{H}$ is named an intuitionistic fuzzy subalgebra if $\forall \wp, \text{Ƌ} \in \mathcal{H}$.

(i) $\omega_{\beth}(\wp * \text{Ƌ}) \ge \min\{\omega_{\beth}(\wp), \omega_{\beth}(\text{Ƌ})\}$,

(ii) $\partial_{\beth}(\wp * \text{Ƌ}) \le max\{\partial_{\beth}(\wp), \partial_{\beth}(\text{Ƌ})\}$.

Definition 2.10: [9] For any $t, s \in [0,1]$, the set $U(\omega_{\beth}, t) = \{\wp \in \mathcal{H} : \omega_{\beth}(\wp) \ge t\}$ is named an upper t-level of $\omega_{\beth}$ and the set $L(\partial_{\beth}, s) = \{\wp \in \mathcal{H} : \partial_{\beth}(\wp) \le s\}$ is named a lower s-level of $\partial_{\beth}$.

3. Cubic intuitionistic-ideals of a TM-algebra

The set $\beth = \{(\wp, \widetilde{\omega}_{\beth}(\wp), \tilde{\partial}_{\beth}(\wp))\}$ is refers to an interval-valued intuitionistic fuzzy set, where $\widetilde{\omega}_{\beth} : \mathcal{H} \to D[0,1]$ is a degree membership and $\tilde{\partial}_{\beth} : \mathcal{H} \to D[0,1]$ is a degree non-membership of the element $\wp \in \mathcal{H}$, where $\widetilde{\omega}_{\beth} + \tilde{\partial}_{\beth} \le D[0,1]$.

Definition 3.1: A cubic intuitionistic set (CIS) in $\mathcal{H}$ is the structure

$\psi = \{\langle \wp, \beth(\wp), \gamma(\wp) \rangle : \wp \in \mathcal{H}\}$, where $\beth$ is an interval-valued intuitionistic set such that $\beth = \{\wp \in \mathcal{H}, \widetilde{\omega}_{\beth}(\wp), \tilde{\partial}_{\beth}(\wp)\}$, $\widetilde{\omega}_{\beth} + \tilde{\partial}_{\beth} \le D[0,1]$and γ is an intuitionistic fuzzy set in $\mathcal{H}$ s. t. $\gamma = \{< \wp, \beta_{\gamma}(\wp), \rho_{\gamma}(\wp) >: \wp \in \mathcal{H}\}$, where $0 \le \beta_{\gamma}(\wp) + \rho_{\gamma}(\wp) \le 1 \ \forall \wp \in \mathcal{H}$.

Definition 3.2: A (CIS) $\psi =< \beth, \gamma >$ of a TM-algebra $\mathcal{H}$ is named a (CI)-subTM-algebra if $\forall \wp, \text{Ƌ} \in \mathcal{H}$

i) $\widetilde{\omega}_{\beth}(\wp * \text{Ƌ}) \ge rmin\{\widetilde{\omega}_{\beth}(\wp), \widetilde{\omega}_{\beth}(\text{Ƌ})\}, \tilde{\partial}_{\beth}(\wp * \text{Ƌ}) \le rmax\{\tilde{\partial}_{\beth}(\wp), \tilde{\partial}_{\beth}(\text{Ƌ})\}$.

ii) $\beta_{\gamma}(\wp * \text{Ƌ}) \ge \min\{\beta_{\gamma}(\wp), \beta_{\gamma}(\text{Ƌ})\}, \rho_{\gamma}(\wp * \text{Ƌ}) \le \max\{\rho_{\gamma}(\wp), \rho_{\gamma}(\text{Ƌ})\}$.

Proposition 3.3: If ψ is a (CI)-sub TM-algebra $\mathcal{H}$, then $\widetilde{\omega}_{\beth}(0) \ge \widetilde{\omega}_{\beth}(\wp)$, $\tilde{\partial}_{\beth}(0) \le \tilde{\partial}_{\beth}(\wp)$ and $\beta_{\gamma}(0) \ge \beta_{\gamma}(\wp)$, $\rho_{\gamma}(0) \le \rho_{\gamma}(\wp), \forall \wp \in \mathcal{H}$.

Proof: The proof is straight forward.

Definition 3.4: A (CIS) ψ is named a CI-ideal if, $\forall \wp, \text{Ƌ} \in \mathcal{H}$.

i) $\widetilde{\omega}_{\beth}(0) \geq \widetilde{\omega}_{\beth}(\wp), \tilde{\partial}_{\beth}(0) \leq \tilde{\partial}_{\beth}(\wp)\ and \beta_{\gamma}(0) \geq \beta_{\gamma}(\wp), \rho_{\gamma}(0) \leq \rho_{\gamma}(\wp)$.

ii) $\widetilde{\omega}_{\beth}(\wp) \geq rmin\{\widetilde{\omega}_{\beth}(\wp * \mathrm{ꟻ}), \widetilde{\omega}_{\beth}(\mathrm{ꟻ})\}, \tilde{\partial}_{\beth}(\wp) \leq rmax\{\tilde{\partial}_{\beth}(\wp * \mathrm{ꟻ}), \tilde{\partial}_{\beth}(\mathrm{ꟻ})\}$ and $\beta_{\gamma}(\wp) \geq \min\{\beta_{\gamma}(\wp * \mathrm{ꟻ}), \beta_{\gamma}(\mathrm{ꟻ})\}, \rho_{\gamma}(\wp) \leq \max\{\rho_{\gamma}(\wp * \mathrm{ꟻ}), \rho_{\gamma}(\mathrm{ꟻ})$.

Definition 3.5: A (CIS) $\psi =< \beth, \gamma >$ is named a CIT-ideal if, $\forall \wp, \mathrm{ꟻ}, \eta \in \mathcal{H}$.

i) $\widetilde{\omega}_{\beth}(0) \geq \widetilde{\omega}_{\beth}(\wp), \tilde{\partial}_{\beth}(0) \leq \tilde{\partial}_{\beth}(\wp)\ and \beta_{\gamma}(0) \geq \beta_{\gamma}(\wp), \rho_{\gamma}(0) \leq \rho_{\gamma}(\wp)$.

ii) $\widetilde{\omega}_{\beth}(\wp * \eta) \geq rmin\{\widetilde{\omega}_{\beth}((\wp * \mathrm{ꟻ}) * \eta), \widetilde{\omega}_{\beth}(\mathrm{ꟻ})\}$,

$$\tilde{\partial}_{\beth}(\wp * \eta) \leq rmax\{\tilde{\partial}_{\beth}((\wp * \mathrm{ꟻ}) * \eta), \tilde{\partial}_{\beth}(\mathrm{ꟻ})\} \text{ and}$$
$$\beta_{\gamma}(\wp * \eta) \geq \min\{\beta_{\gamma}((\wp * \mathrm{ꟻ}) * \eta), \beta_{\gamma}(\mathrm{ꟻ})\},$$
$$\rho_{\gamma}(\wp * \eta) \leq \max\{\rho_{\gamma}((\wp * \mathrm{ꟻ}) * \eta), \rho_{\gamma}(\mathrm{ꟻ})\}.$$

Proposition 3.6: Let $\psi =< \beth, \gamma >$ be a CIT-ideal of TM-algebra $\mathcal{H}$ and $\wp \leq \mathrm{ꟻ}$, then $\widetilde{\omega}_{\beth}(\wp) \geq \widetilde{\omega}_{\beth}(\mathrm{ꟻ})$, $\tilde{\partial}_{\beth}(\wp) \leq \tilde{\partial}_{\beth}(\mathrm{ꟻ})$ and $\beta_{\gamma}(\wp) \geq \beta_{\gamma}(\mathrm{ꟻ})$, $\rho_{\gamma}(\wp) \leq \rho_{\gamma}(\mathrm{ꟻ})$, for all $\forall \wp, \mathrm{ꟻ} \in \mathcal{H}$.

Proof: Let $\wp \leq \mathrm{ꟻ}$, then $\wp * \mathrm{ꟻ} = 0$. Now, since ψ is a CIT-ideal, then

$$\widetilde{\omega}_{\beth}(\wp) = \widetilde{\omega}_{\beth}(\wp * 0) \geq rmin\{\widetilde{\omega}_{\beth}((\wp * \mathrm{ꟻ}) * 0), \widetilde{\omega}_{\beth}(\mathrm{ꟻ})\}$$
$$= rmin\{\widetilde{\omega}_{\beth}(\wp * \mathrm{ꟻ}), \widetilde{\omega}_{\beth}(\mathrm{ꟻ})\} = rmin\{\widetilde{\omega}_{\beth}(0), \widetilde{\omega}_{\beth}(\mathrm{ꟻ})\} = \widetilde{\omega}_{\beth}(\mathrm{ꟻ}). \text{ And}$$
$$\tilde{\partial}_{\beth}(\wp) = \tilde{\partial}_{\beth}(\wp * 0) \leq rmax\{\tilde{\partial}_{\beth}((\wp * \mathrm{ꟻ}) * 0), \tilde{\partial}_{\beth}(\mathrm{ꟻ})\}$$
$$= rmax\{\tilde{\partial}_{\beth}(\wp * \mathrm{ꟻ}), \tilde{\partial}_{\beth}(\mathrm{ꟻ})\} = rmax\{\tilde{\partial}_{\beth}(0), \tilde{\partial}_{\beth}(\mathrm{ꟻ})\} = \tilde{\partial}_{\beth}(\mathrm{ꟻ}).$$
$$\text{Also, } \beta_{\gamma}(\wp) = \beta_{\gamma}(\wp * 0) \geq min\{\beta_{\gamma}((\wp * \mathrm{ꟻ}) * 0), \beta_{\gamma}(\mathrm{ꟻ})\}$$
$$= min\{\beta_{\gamma}(\wp * \mathrm{ꟻ}), \beta_{\gamma}(\mathrm{ꟻ})\} = min\{\beta_{\gamma}(0), \beta_{\gamma}(\mathrm{ꟻ})\} = \beta_{\gamma}(\mathrm{ꟻ}). \text{ And}$$
$$\rho_{\gamma}(\wp) = \rho_{\gamma}(\wp * 0) \leq max\{\rho_{\gamma}((\wp * \mathrm{ꟻ}) * 0), \rho_{\gamma}(\mathrm{ꟻ})\}$$
$$= max\{\rho_{\gamma}(\wp * \mathrm{ꟻ}), \rho_{\gamma}(\mathrm{ꟻ})\} = max\{\rho_{\gamma}(0), \rho_{\gamma}(\mathrm{ꟻ})\} = \rho_{\gamma}(\mathrm{ꟻ}).$$

Theorem 3.7: *Each a CIT-ideal in TM-algebra $\mathcal{H}$ must be a CI-ideal.*

Proof: $\forall \wp, \mathrm{ꟻ}, \eta$ in $\mathcal{H}$, let ψ be CIT-ideal of $\mathcal{H}$, then

$\widetilde{\omega}_{\beth}(\wp * \eta) \geq rmin\{\widetilde{\omega}_{\beth}((\wp * \mathrm{ꟻ}) * \eta), \widetilde{\omega}_{\beth}(\mathrm{ꟻ})\}$,

$\tilde{\partial}_{\beth}(\wp * \eta) \leq rmax\{\tilde{\partial}_{\beth}((\wp * \mathrm{ꟻ}) * \eta), \tilde{\partial}_{\beth}(\mathrm{ꟻ})\}$ and

$\beta_{\gamma}(\wp * \eta) \geq min\{\beta_{\gamma}((\wp * \mathrm{ꟻ}) * \eta), \beta_{\gamma}(\mathrm{ꟻ})\}$,

$\rho_{\gamma}(\wp * \eta) \leq max\{\rho((\wp * \mathrm{ꟻ}) * \eta), \rho_{\gamma}(\mathrm{ꟻ})\}$. Put $\eta = 0$, we get

$\widetilde{\omega}_{\beth}(\wp) \geq rmin\{\widetilde{\omega}_{\beth}(\wp * \mathrm{ꟻ}), \widetilde{\omega}_{\beth}(\mathrm{ꟻ})\}, \tilde{\partial}_{\beth}(\wp) \leq rmax\{\tilde{\partial}_{\beth}(\wp * \mathrm{ꟻ}), \tilde{\partial}_{\beth}(\mathrm{ꟻ})\}$ and $\beta_{\gamma}(\wp) \geq min\{\beta_{\gamma}(\wp * \mathrm{ꟻ}), \beta_{\gamma}(\mathrm{ꟻ})\}$, $\rho_{\gamma}(\wp) \leq max\{\rho_{\gamma}(\wp * \mathrm{ꟻ}), \rho_{\gamma}(\mathrm{ꟻ})\}$, then ψ is CI-ideal.

The converse of Th.3.7 is not always true, the following example shows that.

Let $\mathcal{H}$ = {0, s, d, g} be a set defined by the following

$*$	0	s	d	g
0	0	0	g	d
s	s	0	g	d
d	d	d	0	g
g	g	g	d	0

Then ($\mathcal{H}$,$*$,0) is a TM-algebra. Define a (CIS) $\psi =< \beth, \gamma >$ as follows

$$\widetilde{\omega}_{\beth}(\wp) = \begin{cases} [0.2,0.8] & if \quad \wp = \{0, s, \mathrm{g}\} \\ [0.1,0.4] & if \quad \wp = d \end{cases},$$

$$\tilde{\partial}_{\beth}(\wp) = \begin{cases} [0.1,0.3] & if \quad \wp = \{0, s, \mathrm{g}\} \\ [0.1,0.6] & if \quad \wp = d \end{cases}, \text{ and}$$

$$\beta_{\gamma}(\wp) = \begin{cases} 0.9 & if \quad \wp = \{0, s, \mathrm{g}\} \\ 0.1 & if \quad \wp = d \end{cases},$$

$$\rho_{\gamma}(\wp) = \begin{cases} 0.2 & if \quad \wp = \{0, s, \mathrm{g}\} \\ 0.7 & if \quad \wp = d \end{cases}.$$

Then ψ is a CI-ideal, but not a CIT-ideal since $\widetilde{\omega}_{\beth}(d * s) \leq rmin\{\widetilde{\omega}_{\beth}((d * g) * s), \widetilde{\omega}_{\beth}(g)\}$, $\tilde{\partial}_{\beth}(d * s) \geq rmax\{\tilde{\partial}_{\beth}((d * g) * s), \tilde{\partial}_{\beth}(g)\}$ and $\beta_{\gamma}(d * s) \leq min\{\beta_{\gamma}((d * g) * s), \beta_{\gamma}(g)\}$, $\rho_{\gamma}(d * s) \geq max\{\rho_{\gamma}((d * g) * s), \rho_{\gamma}(g)\}$.

Definition 3.8: Let $\psi =< \beth, \gamma >$ be a (CIS) of a TM-algebra $\mathcal{H}$. Then the set $K(\psi; \tilde{t}, \tilde{r}, s, m) = \{\wp \in \mathcal{H}: \widetilde{\omega}_{\beth}(\wp) \geq \tilde{t}, \tilde{\partial}_{\beth}(\wp) \leq \tilde{r} \text{ and } \beta_{\gamma}(\wp) \geq s, \rho_{\gamma}(\wp) \leq m\}$ is called a CI-level set of ψ, for any $\tilde{t}, \tilde{r} \in D[0,1]$ and $s, m \in [0,1]$. Where $\widetilde{\omega}_{\beth}(\wp) + \tilde{\partial}_{\beth}(\wp) \leq D[0,1]$ and $0 \leq \beta_{\gamma}(\wp) + \rho_{\gamma}(\wp) \leq 1$, for all $\wp \in \mathcal{H}$.

Theorem 3.9: *The structure* $\psi =< \beth, \gamma >$ *is a CIT-ideal in* $\mathcal{H}$ *if and only if every nonempty CI-level set* $K(\psi; \tilde{t}, \tilde{r}, s, m)$ *is a T-ideal in* $\mathcal{H}$*, for any* $\tilde{t}, \tilde{r} \in D[0,1]$ *and* $s, m \in [0,1]$.

Proof: Suppose that ψ is a CIT-ideal and $\wp \in \mathcal{H}$.

Let $\wp \in K(\psi; \tilde{t}, \tilde{r}, s, m)$, then $\widetilde{\omega}_{\beth}(0) \geq \widetilde{\omega}_{\beth}(\wp) \geq \tilde{t}, \tilde{\partial}_{\beth}(0) \leq \tilde{\partial}_{\beth}(\wp) \leq \tilde{r}$ *and* $\beta_{\gamma}(0) \geq \beta_{\gamma}(\wp) \geq s, \rho_{\gamma}(0) \leq \rho_{\gamma}(\wp) \leq m$. Thus $0 \in K(\psi; \tilde{t}, \tilde{r}, s, m)$. Now, Let $\wp, \mathrm{ɑ}, \eta \in \mathcal{H}$ such that $((\wp * \mathrm{ɑ}) * \eta) \in K(\psi; \tilde{t}, \tilde{r}, s, m)$ and $\mathrm{ɑ} \in K(\psi; \tilde{t}, \tilde{r}, s, m)$ implies

$\widetilde{\omega}_{\beth}(\wp * \eta) \geq rmin\{\widetilde{\omega}_{\beth}((\wp * \mathrm{ɑ}) * \eta), \widetilde{\omega}_{\beth}(\mathrm{ɑ})\} = rmin\{\tilde{t}, \tilde{t}\} = \tilde{t}$,

$\tilde{\partial}_{\beth}(\wp * \eta) \leq rmax\{\tilde{\partial}_{\beth}((\wp * \mathrm{ɑ}) * \eta), \tilde{\partial}_{\beth}(\mathrm{ɑ})\} = rmax\{\tilde{r}, \tilde{r}\}=\tilde{r}$ and

$\beta_{\gamma}(\wp * \eta) \geq min\{\beta_{\gamma}((\wp * \mathrm{ɑ}) * \eta), \beta_{\gamma}(\mathrm{ɑ})\} = \min\{s, s\} = s$, also

$\rho_{\gamma}(\wp * \eta) \leq max\{\rho_{\gamma}((\wp * \mathrm{ɑ}) * \eta), \rho_{\gamma}(\mathrm{ɑ})\} = \max\{m, m\} = m$.

Thus, $(\wp * ᴚ) \in K(\psi; \tilde{t}, \tilde{r}, s, m)$. Hence $K(\psi; \tilde{t}, \tilde{r}, s, m)$ is T-ideal of $\mathcal{H}$.

Conversely, suppose that $K(\psi; \tilde{t}, \tilde{r}, s, m)$ is a nonempty T-ideal of $\mathcal{H}$.

For all $\tilde{t}, \tilde{r} \in D[0,1]$ *and* $s, m \in [0,1]$, assume that

$\widetilde{\omega}_ℶ(0) < \widetilde{\omega}_ℶ(\wp), \tilde{\partial}_ℶ(0) > \tilde{\partial}_ℶ(\wp)$ *and* $\beta_\gamma(0) < \beta_\gamma(\wp), \rho_\gamma(0) > \rho_\gamma(\wp)$

If we take $\tilde{t}_1 = \frac{1}{2}\{\widetilde{\omega}_ℶ(0) + \widetilde{\omega}_ℶ(\wp)\}$, $\tilde{r}_1 = \frac{1}{2}\{\tilde{\partial}_ℶ(0) + \tilde{\partial}_ℶ(\wp)\}$, and

$s_1 = \frac{1}{2}\{\beta_\gamma(0) + \beta_\gamma(\wp)\}$, $m_1 = \frac{1}{2}\{\rho_\gamma(0) + \rho_\gamma(\wp)\}$, then $\widetilde{\omega}_A(0) < \tilde{t}_1 < \widetilde{\omega}_A(\wp), \tilde{\partial}_A(0) > \tilde{r}_1 > \tilde{\partial}_A(\wp)$ *and* $\beta_\gamma(0) < s_1 < \beta_\gamma(\wp)$, $\rho_\gamma(0) > m_1 > \rho_\gamma(\wp)$

Hence $\wp \in K(\psi; \tilde{t}, \tilde{r}, s, m)$ *but* $0 \notin K(\psi; \tilde{t}, \tilde{r}, s, m)$, this is contradiction.

So, for all $\wp \in \mathcal{H}$

$\widetilde{\omega}_ℶ(0) \geq \widetilde{\omega}_ℶ(\wp), \tilde{\partial}_ℶ(0) \leq \tilde{\partial}_ℶ(\wp)$ *and* $\beta_\gamma(0) \geq \beta_\gamma(\wp)$, $\rho_\gamma(0) \leq \rho_\gamma(\wp)$.

Now, suppose that there exist $\wp, ᴚ, \eta \in \mathcal{H}$ such that

$$\widetilde{\omega}_ℶ(\wp * \eta) < rmin\{\widetilde{\omega}_ℶ((\wp * ᴚ) * \eta), \widetilde{\omega}_ℶ(ᴚ)\},$$
$$\tilde{\partial}_ℶ(\wp * \eta) > rmax\{\tilde{\partial}_ℶ((\wp * ᴚ) * \eta), \tilde{\partial}_ℶ(ᴚ)\} \text{ and}$$
$$\beta_\gamma(\wp * \eta) < min\{\beta_\gamma((\wp * ᴚ) * \eta), \beta_\gamma(ᴚ)\},$$
$$\rho_\gamma(\wp * \eta) > max\{\rho_\gamma((\wp * ᴚ) * \eta), \rho_\gamma(ᴚ)\}. \text{ Take}$$
$$\tilde{t}_1 = \frac{1}{2}\{\widetilde{\omega}_ℶ(\wp * \eta) + rmin\{\widetilde{\omega}_ℶ((\wp * ᴚ) * \eta), \widetilde{\omega}_ℶ(ᴚ)\}\},$$
$$\tilde{r}_1 = \frac{1}{2}\{\tilde{\partial}_ℶ(\wp * \eta) + rmax\{\tilde{\partial}_ℶ((\wp * ᴚ) * \eta), \tilde{\partial}_ℶ(ᴚ)\}\}.$$
$$\text{And } s_1 = \frac{1}{2}\{\beta_\gamma(\wp * \eta) + min\{\beta_\gamma((\wp * ᴚ) * \eta), \beta_\gamma(ᴚ)\}\},$$
$$m_1 = \frac{1}{2}\{\rho_\gamma(\wp * \eta) + max\{\rho_\gamma((\wp * ᴚ) * \eta), \rho_\gamma(ᴚ)\}\}.$$

Such that $(\tilde{t}, \tilde{r}) \in D[0,1]$ and $s, m \in [0,1]$, then

$$\widetilde{\omega}_ℶ(\wp * \eta) < \tilde{t}_1 < rmin\{\widetilde{\omega}_ℶ((\wp * ᴚ) * \eta), \widetilde{\omega}_ℶ(ᴚ)\},$$
$$\tilde{\partial}_ℶ(\wp * \eta) > \tilde{r}_1 > rmax\{\tilde{\partial}_ℶ((\wp * ᴚ) * \eta), \tilde{\partial}_ℶ(ᴚ)\} \text{ and}$$
$$\beta_\gamma(\wp * \eta) < s_1 < min\{\beta_\gamma((\wp * ᴚ) * \eta), \beta_\gamma(ᴚ)\},$$
$$\rho_\gamma(\wp * \eta) > m_1 > max\{\rho_\gamma((\wp * ᴚ) * \eta), \rho_\gamma(ᴚ)\} .$$

Then $\wp * \eta \notin K(\psi; \tilde{t}, \tilde{r}, s, m), ((\wp * ᴚ) * \eta) \in K(\psi; \tilde{t}, \tilde{r}, s, m)$ and $ᴚ \in K(\psi; \tilde{t}, \tilde{r}, s, m)$, this is a contradiction.

It follows that ψ is a CIT-ideal.

4. Homomorphism of cubic intuitionistic-ideals

Definition 4.1: Let $g: \mathcal{H} \to \mathcal{H}'$ be homomorphism of two TM-algebras $\mathcal{H}$ *and* $\mathcal{H}'$. For any a (CIS) $\psi = < ℶ, \gamma >$ of $\mathcal{H}$. Define a new (CIS)

$\psi^g = (\wp, \widetilde{\omega}_ℶ^g, \tilde{\partial}_ℶ^g, \beta_\gamma^g, \rho_\gamma^g)$ of $\mathcal{H}$ as follows $\widetilde{\omega}_ℶ^g(\wp) = \widetilde{\omega}_ℶ(g(\wp))$, $\tilde{\partial}_ℶ^g(\wp) = \tilde{\partial}_ℶ(g(\wp))$ and $\beta_\gamma^g(\wp) = \beta_\gamma(g(\wp)), \rho_\gamma^g(\wp) = \rho_\gamma(g(\wp)), \forall \wp \in \mathcal{H}$.

Theorem 4.2: *Let $\mathcal{H}$ and $\mathcal{H}'$ be two TM-algebras and $g: \mathcal{H} \to \mathcal{H}'$ be an epimorphism from $\mathcal{H}$ into $\mathcal{H}'$. Then $\psi = (\wp', \widetilde{\omega}_A, \tilde{\partial}_A, \beta_\gamma, \rho_\gamma)$ is a CIT-ideal of $\mathcal{H}'$ if and only if $\psi^g = (\wp, \widetilde{\omega}_ℶ^g, \tilde{\partial}_ℶ^g, \beta_\gamma^g, \rho_\gamma^g)$ is a CIT-ideal of $\mathcal{H}$.*

Proof: Since g is an onto mapping, then $\exists \wp \in \mathcal{H}$, such that $g(\wp) = \wp'$, for any $\wp' \in \mathcal{H}'$. Now, we have

$$\tilde{\omega}_{\gimel}^{g}(0) = \tilde{\omega}_{\gimel}\big(g(0)\big) = \tilde{\omega}_{\gimel}(0') \geq \tilde{\omega}_{\gimel}(\wp') = \tilde{\omega}_{\gimel}\big(g(\wp)\big) = \tilde{\omega}_{\gimel}^{g}(\wp)\,,$$

$$\tilde{\partial}_{\gimel}^{g}(0) = \tilde{\partial}_{\gimel}\big(g(0)\big) = \tilde{\partial}_{\gimel}(0') \leq \tilde{\partial}_{\gimel}(\wp') = \tilde{\partial}_{\gimel}\big(g(\wp)\big) = \tilde{\partial}_{\gimel}^{g}(\wp) \text{ and}$$

$$\beta_{\gamma}^{g}(0) = \beta_{\gamma}\big(g(0)\big) = \beta_{\gamma}(0') \geq \beta_{\gamma}(\wp') = \beta_{\gamma}\big(g(\wp)\big) = \beta_{\gamma}^{g}(\wp),$$

$$\rho_{\gimel}^{g}(0) = \rho_{\gimel}\big(g(0)\big) = \rho_{\gimel}(0') \leq \rho_{\gimel}(\wp') = \rho_{\gimel}\big(g(\wp)\big) = \rho_{\gimel}^{g}(\wp).$$

Now, Let $\wp, \eta \in \mathcal{H}$ and $, \text{ꓷ}' \in \mathcal{H}'$, then there exists $\text{ꓷ} \in \mathcal{H}$, s.t. $g(\text{ꓷ}) = \text{ꓷ}'$. Then $\tilde{\omega}_{\gimel}^{g}(\wp * \eta) = \tilde{\omega}_{\gimel}\big(g(\wp * \eta)\big) = \tilde{\omega}_{\gimel}(g(\wp) * g(\eta)) \geq rmin\{\tilde{\omega}_{\gimel}(g(\wp) * \text{ꓷ}') * g(\eta)), \tilde{\omega}_{\gimel}(\text{ꓷ}')\} = rmin\{\tilde{\omega}_{\gimel}((g(\wp) * g(\text{ꓷ})) * g(\eta)), \tilde{\omega}_{\gimel}(g(\text{ꓷ}))\} = rmin\{\tilde{\omega}_{\gimel}^{g}((\wp * \text{ꓷ}) * \eta)), \tilde{\omega}_{\gimel}^{g}(\text{ꓷ})\}$, $\tilde{\partial}_{\gimel}^{g}(\wp * \eta) = \tilde{\partial}_{\gimel}(g(\wp * \eta)) = \tilde{\partial}_{\gimel}(g(\wp) * g(\eta)) \leq rmax\{\tilde{\partial}_{\gimel}((g(\wp) * \text{ꓷ}') * g(\eta)), \tilde{\partial}_{\gimel}(\text{ꓷ}')\} = rmax\{\tilde{\partial}_{\gimel}((g(\wp) * g(\text{ꓷ})) * g(\eta)), \tilde{\partial}_{\gimel}(g(\text{ꓷ}))\} = rmax\left\{\tilde{\partial}_{\gimel}^{g}\big((\wp * \text{ꓷ}) * \eta\big)\right), \tilde{\partial}_{\gimel}^{g}(\text{ꓷ})\}$.

$\beta_{\gimel}^{g}(\wp * \eta) = \beta_{\gimel}\big(g(\wp * \eta)\big) = \beta_{\gimel}\big(g(\wp) * g(\eta)\big) \geq min\{\beta_{\gimel}\big((g(\wp) * \text{ꓷ}') * g(\eta)\big), \beta_{\gimel}(\text{ꓷ}')\} = min\{\beta_{\gimel}((g(\wp) * g(\eta)\big) * g(\eta)), \beta_{\gimel}\big(g(\mathrm{k})\big)\} = min\left\{\beta_{\gimel}^{g}\big((\wp * \text{ꓷ}) * \eta\big)\right), \beta_{\gimel}^{g}(\text{ꓷ})\}$, and

$\rho_{\gimel}^{g}(\wp * \eta) = \rho_{\gimel}\big(g(\wp * \eta)\big) = \rho_{\gimel}\big(g(\wp) * g(\eta)\big) \leq max\{\rho_{\gimel}\big((g(\wp) * \text{ꓷ}') * g(\eta)\big), \rho_{\gimel}(\text{ꓷ}')\} = max\{\rho_{\gimel}((g(\wp) * g(\text{ꓷ})\big) * g(\eta)), \rho_{\gimel}\big(g(\text{ꓷ})\big)\} = max\left\{\rho_{\gimel}^{g}\big((\wp * \text{ꓷ}) * \eta\big)\right), \rho_{\gimel}^{g}(\text{ꓷ})\}$. Hence $\psi^{g} = \big(\wp, \tilde{\omega}_{\gimel}^{g}, \tilde{\partial}_{\gimel}^{g}, \beta_{\gamma}^{g}, \rho_{\gamma}^{g}\big)$ is a CIT-ideal of $\mathcal{H}$.

Conversely, since $g\colon \mathcal{H} \to \mathcal{H}'$ is an onto, then $\forall \wp', \text{ꓷ}', \eta' \in \mathcal{H}'$, $\exists \wp, \text{ꓷ}, \eta \in \mathcal{H}$ s.t. $g(\wp) = \wp'$, $g(\text{ꓷ}) = \text{ꓷ}'$ and $g(\eta) = \eta'$. It follows that $\tilde{\omega}_{\gimel}\big(g(\wp) * g(\eta)\big) = \tilde{\omega}_{\gimel}\big(g(\wp * \eta)\big) = \tilde{\omega}_{\gimel}^{g}(\wp * \eta) \geq rmin\{\tilde{\omega}_{\gimel}^{g}\big((\wp * \text{ꓷ}) * \eta\big), \tilde{\omega}_{\gimel}^{g}(\text{ꓷ})\} = rmin\left\{\tilde{\omega}_{\gimel}\left(\big(g(\wp) * g(\text{ꓷ})\big) * g(\eta)\right), \tilde{\omega}_{\gimel}\big(g(\text{ꓷ})\big)\right\}$. And

$\tilde{\partial}_{\gimel}(g(\wp) * g(\eta)) = \tilde{\partial}_{\gimel}\big(g(\wp * \eta)\big) = \tilde{\partial}_{\gimel}^{g}(\wp * \eta) \leq rmax\{\tilde{\partial}_{\gimel}^{g}\big((\wp * \text{ꓷ}) * \eta\big), \tilde{\partial}_{\gimel}^{g}(\text{ꓷ})\} = rmax\left\{\tilde{\partial}_{\gimel}\left(\big(g(v) * g(\text{ꓷ})\big) * g(\eta)\right), \tilde{\partial}_{\gimel}\big(g(\text{ꓷ})\big)\right\}$. Also,

$\beta_{\gimel}\big(g(\wp) * g(\eta)\big) = \beta_{\gimel}\big(g(\wp * \eta)\big) = \beta_{\gimel}^{g}(\wp * \eta) \geq min\{\beta_{\gimel}^{g}\big((\wp * \text{ꓷ}) * \eta\big), \beta_{\gimel}^{g}(\text{ꓷ})\} = min\left\{\beta_{\gimel}\left(\big(g(\wp) * g(\text{ꓷ})\big) * g(\eta)\right), \beta_{\gimel}\big(g(\text{ꓷ})\big)\right\}$, and

$\rho_{\gimel}\big(g(\wp) * g(\eta)\big) = \rho_{\gimel}\big(g(\wp * \eta)\big) = \rho_{\gimel}^{g}(\wp * \eta) \leq max\{\rho_{\gimel}^{g}\big((\wp * \text{ꓷ}) * \eta\big), \rho_{\gimel}^{g}(\text{ꓷ})\} = max\left\{\rho_{\gimel}\left(\big(g(\wp) * g(\text{ꓷ})\big) * g(\eta)\right), \rho_{\gimel}\big(g(\text{ꓷ})\big)\right\}$.

Hence $\psi = (\wp', \tilde{\omega}_{A}, \tilde{\partial}_{A}, \beta_{\gamma}, \rho_{\gamma})$ is a CIT-ideal of $\mathcal{H}'$.

5. Discussion and Conclusion

We studied cubic intuitionistic structures on TM-algebra as a generalization of the cubic set of TM-algebra. We discussed a few properties of this concept and investigated some related results. Cubic intuitionistic T-ideals and the homomorphism of new cubic intuitionistic fuzzy sets in a TM-algebra are defined. The future work is to study cubic intuitionistic fuzzy ideals for other algebraic structures. Also, we can examine the notion of a cubic intuitionistic fuzzy graph for a TM-algebra.

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