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Cubic bipolar ideals of a TM-algebra

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Abstract

This paper refers to studying some types of ideals, specifically cubic bipolar ideals and cubic bipolar T-ideals of TM algebra. It also introduces a cubic bipolar sub-TM-algebra and several important properties of these concepts. The relationships between these ideals and characterizations of cubic bipolar T-ideals are investigated.

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1. Introduction

In 2010 Jun, Kim, and Kang [10] introduced the notion of a cubic set. They studied the relationship between a cubic subalgebra and a cubic ideal. This concept has been presented in several studies (see [1, 3 and 5]). The bipolar fuzzy set concept was introduced by Lee [11], which is an extension of the fuzzy set from [0, 1] to [-1, 1], and then it was studied by several authors in different structures (see [8, 9, and 15]). After that, some authors combined the two concepts as another generalization of the idea of fuzzy sets. Riaz and Tehrim [16] presented the concepts of internal and external cubic bipolar fuzzy sets. Lee, Hur, and Mostafa [12] studied a cubic bipolar BCC-ideal of BCC-algebra, by proving some properties. Some papers have studied this concept differently, (see [2, 4 and 13-15]). In our paper, we introduce a cubic bipolar sub-TM-algebra, cubic bipolar ideals, and cubic bipolar T-ideals of a TM-algebra and prove some results.

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2. Preliminaries

Definition 2.1: [14] A nonempty subset $\aleph$ and a constant "0" with a binary operation " *" is called a TM-algebra $(\aleph, *, 0)$ if :$\forall \alpha, \tau, \mu \in \aleph$

(tm$_1$) $\alpha * 0 = \alpha$,

(tm$_2$) $(\alpha * \tau) * (\alpha * \mu) = \mu * \tau$.

The operation $\leq$ is defined by $\alpha \leq \tau \Leftrightarrow \alpha * \tau = 0$. Then

a) $\alpha * \alpha = 0$,

b) $(\alpha * \tau) * \alpha = 0 * \tau$,

c) $(\alpha * \mu) * (\tau * \mu) \leq \alpha * \tau$,

d) $(\alpha * \tau) * \mu = (\alpha * \mu) * \tau, \forall \alpha, \tau, \mu \in \aleph$

Definition 2.2: [14] A set $\varphi \neq S \subseteq \aleph$ is called a TM-subalgebra if $\alpha * \tau \in S$ whenever $\alpha, \tau \in \aleph$.

Definition 2.3: [14] A set $\varphi \neq \psi \subseteq \aleph$ is called an ideal if,

i) $0 \in \psi$,

ii) $\alpha * \tau \in \psi$, $\tau \in \psi \Rightarrow \alpha \in \psi$, for any $\alpha, \tau \in \aleph$.

Definition 2.4: [14] A set $\varphi \neq E \subseteq \aleph$ is called a T-ideal, if $\forall \alpha, \tau, \mu \in \aleph$

i) $0 \in E$

ii) $(\alpha * \tau) * \mu \in E$, $\tau \in E \Rightarrow (\alpha * \mu) \in E$.

Definition 2.5: [11] A bipolar fuzzy subset in $\aleph$ is $\partial = \{(\alpha, \omega^-(\alpha), \omega^+(\alpha) | \alpha \in \aleph\}$ where $\omega^-: \aleph \to [-1,0]$ and $\omega^+: \aleph \to [0,1]$. The mapping $\omega^+(\alpha)$ represents the satisfaction degree of α with respect to the corresponding property of ∂ and $\omega^-(\alpha)$ represents the satisfaction degree of α with respect to some implicit counter-property of ∂.

Definition 2.6: [7] An interval-valued fuzzy subset A over TM-algebra $(\aleph, *, 0)$ is the form $V = \{< \alpha, \widetilde{\omega}(\alpha) >: \alpha \in \aleph\} = (\alpha, \omega^L, \omega^U)$, where $\widetilde{\omega}: \aleph \longrightarrow$ D[0,1], and D[0,1] is the set of all sub intervals of [0,1]. Then the intervals of the degree of membership and degree of non-membership of the element α to the set A is $\widetilde{\omega}(\alpha) = [\omega^L(\alpha), \omega^U(\alpha)]$, for all $\alpha \in \aleph$.

Definition 2.7: [6] A cubic TM-sub-algebra of $\aleph$ is a cubic set $\delta = \langle \widetilde{\omega}_\delta, \rho_\delta \rangle$ and satisfied: $\forall \alpha, \tau \in \aleph$

(1) $\widetilde{\omega}_\delta(\alpha * \tau) \geq rmin\{\widetilde{\omega}(\alpha), \widetilde{\omega}(\tau)\}$.

(2) $\rho_\delta(\alpha * \tau) \leq max\{\rho_\delta(\alpha), \rho_\delta(\tau)\}$.

Definition 2.8: [6] A cubic ideal of $\aleph$ is a cubic set $\delta = \langle \widetilde{\omega}_\delta, \rho_\delta \rangle$ and satisfied: for all $\alpha, \tau \in \aleph$.

(H$_1$) $\widetilde{\omega}_\delta(0) \geq \widetilde{\omega}_\delta(\alpha)$ and $\rho_\delta(0) \leq \rho_\delta(\alpha)$.

(H$_2$) $\widetilde{\omega}_\delta(\alpha) \geq rmin\{\widetilde{\omega}_\delta(\alpha * \tau), \widetilde{\omega}_\delta(\tau)\}$ and $\rho_\delta(\alpha) \leq max\{\rho_\delta(\alpha * \tau), \rho_\delta(\tau)\}$.

Definition 2.9: [6] A cubic T-ideal of $\aleph$ is a cubic set $\delta = \langle \widetilde{\omega}_\delta, \rho_\delta \rangle$ and satisfied:
$(B_1) \widetilde{\omega}_\delta(0) \geq \widetilde{\omega}_\delta(\alpha)\ and\ \rho_\delta(0) \leq \rho_\delta(\alpha),$

$$(B_2) \widetilde{\omega}_\delta(\alpha * \mu) \geq rmin\{\widetilde{\omega}_\delta((\alpha * \tau) * \mu), \widetilde{\omega}_\delta(\tau)\}\ and$$
$$\rho_\delta(\alpha * \mu) \leq max\{\rho_\delta((\alpha * \tau) * \mu), \rho_\delta(\tau)\}, \forall\ \alpha, \tau \in \aleph.$$

3. Main results

In this section, we will study two concepts together, which are bipolar set and cubic set.

Definition 3.1: The set $\lambda = \{\langle \alpha, \widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^-(\alpha), \rho_\lambda^+(\alpha), \rho_\lambda^-(\alpha) : \alpha \in \aleph \rangle\}$ is a cubic bipolar set with $M(\alpha) = \{\widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^-(\alpha)\}$ which is an interval valued fuzzy set and $F(\alpha) = \{\rho_\lambda^+(\alpha), \rho_\lambda^-(\alpha)\}$ is a bipolar fuzzy set, s. t. $\widetilde{\omega}_\lambda^+(\alpha) : \aleph \to D[0,1]$, $\widetilde{\omega}_\lambda^-(\alpha) : \aleph \to D[-1,0]$, $\rho_\lambda^+ : \aleph \to [0,1]$ and $\rho_\lambda^- : \aleph \to [-1,0]$. For simplicity, we write λ by a CBs and denoted by $\lambda = \langle M, F \rangle$.

Definition 3.2: Let $\lambda = \langle M, F \rangle$ be a CBs of $\aleph$. Then the set $U(\lambda; \tilde{t}, \tilde{r}, s, v) = \{\alpha \in \aleph :\ \widetilde{\omega}_\lambda^+(\alpha) \geq \tilde{t}, \widetilde{\omega}_\lambda^- \leq \tilde{r}\ and\ \rho_\lambda^+(\alpha) \leq s, \rho_\lambda^-(\alpha) \leq v\}$ is named a level set of λ, for any $\tilde{t}, \tilde{r} \in D[0,1]$ and $s, v \in [0,1]$.

Definition 3.3: A CBs $\lambda = \langle M, F \rangle$ of $\aleph$ is called a CB sub-TM-algebra if :

$$\widetilde{\omega}_\lambda^+(\alpha * \tau) \geq rmin\{\widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^+(\tau)\}, \widetilde{\omega}_\lambda^-(\alpha * \tau) \leq rmax\{\widetilde{\omega}_\lambda^-(\alpha), \widetilde{\omega}_\lambda^-(\tau)\}$$
$$\rho_\lambda^+(\alpha * \tau) \geq min\{\rho_\lambda^+(\alpha), \rho_\lambda^+(\tau)\},$$
$$\rho_\lambda^-(\alpha * \tau) \leq max\{\rho_\lambda^-(\alpha), \rho_\lambda^-(\tau)\}, \forall \alpha, \tau \in \aleph.$$

Proposition 3.4: If $\lambda = \langle M, F \rangle$ is a CB sub-TM-algebra of $\aleph$, then $\forall \alpha \in \aleph$,

$$\widetilde{\omega}_\lambda^+(0) \geq \widetilde{\omega}_\lambda^+(\alpha), \rho_\lambda^+(0) \geq \rho_\lambda^+(\alpha) \text{ and } \widetilde{\omega}_\lambda^-(0) \leq \widetilde{\omega}_\lambda^-(\alpha), \rho_\lambda^-(0) \leq \rho_\lambda^-(\alpha).$$

Proof: $\forall \alpha \in \aleph$, we have $\alpha * \alpha = 0$, then $\widetilde{\omega}_\lambda^+(0) = \widetilde{\omega}_\lambda^+(\alpha * \alpha) \geq rmin\ \widetilde{\omega}_\lambda^+(\alpha)$, $\widetilde{\omega}_\lambda^+(\alpha) = \widetilde{\omega}_\lambda^+(\alpha)$, similarly $\rho_\lambda^+(0) \geq \rho_\lambda^+(\alpha)$ and $\widetilde{\omega}_\lambda^-(0) = \widetilde{\omega}_\lambda^-(\alpha * \alpha) \leq rmax\{\widetilde{\omega}_\lambda^-(\alpha), \widetilde{\omega}_\lambda^-(\alpha)\} = \widetilde{\omega}_\lambda^-(\alpha)$, similarly $\rho_\lambda^-(0) \leq \rho_\lambda^-(\alpha)$.

Theorem 3.5: *For a CBs $\lambda = \langle M, F \rangle$ of $\aleph$, the following are equivalent*

1) $\lambda = \langle M, F \rangle$ is a CB sub-TM-algebra $\aleph$.
2) Every nonempty level set $U(\lambda; \tilde{t}, \tilde{r}, s, v)$ is a sub-TM-algebra of $\aleph$. Where $\tilde{t}, \tilde{r} \in D[0,1]$ and $s, v \in [0,1]$.

Proof: (1) ⇒(2) Let λ be a CB sub-TM-algebra $\aleph$ and $U(\lambda; \tilde{t}, \tilde{r}, s, v) \neq \emptyset$.

Let $\alpha, \tau \in \aleph$ s. t. $\alpha, \tau \in U(\lambda; \tilde{t}, \tilde{r}, s, v)$, then $\widetilde{\omega}_\lambda^+(\alpha) \geq \tilde{t}, \widetilde{\omega}_\lambda^-(\alpha) \leq \tilde{r}$ and $\rho_\lambda^+(\alpha) \leq s, \rho_\lambda^-(\alpha) \leq v$. Also,

$$\widetilde{\omega}_\lambda^+(\tau) \geq \tilde{t}, \widetilde{\omega}_\lambda^-(\tau) \leq \tilde{r}\ and\ \rho_\lambda^+(\tau) \leq s, \rho_\lambda^-(\tau) \leq v \text{ . We have}$$
$$\widetilde{\omega}_\lambda^+(\alpha * \tau) \geq rmin\{\widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^+(\tau)\} = rmin\{\tilde{t}, \tilde{t}\} = \tilde{t}, \text{ and}$$
$$\widetilde{\omega}_\lambda^-(\alpha * \tau) \leq rmax\{\widetilde{\omega}_\lambda^-(\alpha), \widetilde{\omega}_\lambda^-(\tau)\} = rmax\{\tilde{r}, \tilde{r}\} = \tilde{r}, \text{ also}$$
$$\rho_\lambda^+(\alpha * \tau) \geq min\{\rho_\lambda^+(\alpha), \rho_\lambda^+(\tau)\} = min\{s, s\} = s, \text{ and}$$

$$\rho_\lambda^-(\alpha * \tau) \le max\{\rho_\lambda^-(\alpha), \rho_\lambda^-(\tau)\} = max\{v, v\} = v.$$

Thus, $\alpha * \tau \in U(\lambda; \tilde{t}, \tilde{r}, s, v)$. Hence $U(\lambda; \tilde{t}, \tilde{r}, s, v)$ is a sub-TM-algebra of $\aleph$.

(2) $\Rightarrow$(1) Let $U(\lambda; \tilde{t}, \tilde{r}, s, v)$ be a sub-TM-algebra of $\aleph$ and $\alpha, \tau \in \aleph$ such that $\widetilde{\omega}_\lambda^+(\alpha * \tau) < rmin\{\widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^+(\tau)\}$ and $\widetilde{\omega}_\lambda^-(\alpha * \tau) > rmax\ \{\widetilde{\omega}_\lambda^-(\alpha), \widetilde{\omega}_\lambda^-(\tau)\}$, also $\rho_\lambda^+(\alpha * \tau) < min\{\rho_\lambda^+(\alpha), \rho_\lambda^+(\tau)\}$ and $\rho_\lambda^-(\alpha * \tau) > max\ \{\rho_\lambda^-(\alpha), \rho_\lambda^-(\tau)\}$.

Taking $\tilde{t}_1 = \frac{1}{2}\{\widetilde{\omega}_\lambda^+(\alpha * \tau) + rmin\{\widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^+(\tau)\}\}$,

$\tilde{r}_1 = \frac{1}{2}\{\widetilde{\omega}_\lambda^-(\alpha * \tau) + rmax\{\widetilde{\omega}_\lambda^-(\alpha), \widetilde{\omega}_\lambda^-(\tau)\}\}$. And

$s_1 = \frac{1}{2}\{\rho_\lambda^+(\alpha * \tau) + min\{\rho_\lambda^+(\alpha), \rho_\lambda^+(\tau)\}\}$,

$v_1 = \frac{1}{2}\{\rho_\lambda^-(\alpha * \tau) + max\{\rho_\lambda^-(\alpha), \rho_\lambda^-(\tau)\}\}$,

where $\tilde{t}_1, \tilde{r}_1 \in D[0,1]$ and $s_1, v_1 \in [0,1]$, then $\widetilde{\omega}_\lambda^+(\alpha * \tau) < \tilde{t}_1 < rmin\ \{\widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^+(\tau)\}$, $\widetilde{\omega}_\lambda^-(\alpha * \tau) > \tilde{r}_1 > rmax\{\widetilde{\omega}_\lambda^-(\alpha), \widetilde{\omega}_\lambda^-(\tau)\}$, also $\rho_\lambda^+(\alpha * \tau) < s_1 < min\{\rho_\lambda^+(\alpha), \rho_\lambda^+(\tau)\}$ and $\rho_\lambda^-(\alpha * \tau) > v_1 > max\{\rho_\lambda^-(\alpha), \rho_\lambda^-(\tau)\}$. It follows that $\alpha * \tau \notin U(\lambda; \tilde{t}, \tilde{r}, s, v)$, this is a contradiction and it follows that λ is a CB sub-TM-algebra of $\aleph$.

Definition 3.6: A CBs $\lambda = \langle M, F\rangle$ of $\aleph$ is called a CB-ideal if, for all $\alpha, \tau \in \aleph$

(CB_1) $\widetilde{\omega}_\lambda^+(0) \ge \widetilde{\omega}_\lambda^+(\alpha), \widetilde{\omega}_\lambda^-(0) \le \widetilde{\omega}_\lambda^-(\alpha)$ and $\rho_\lambda^+(0) \ge \rho_\lambda^+(\alpha), \rho_\lambda^-(0) \le \rho_\lambda^-(\alpha)$

(CB_2) $\widetilde{\omega}_\lambda^+(\alpha) \ge rmin\{\widetilde{\omega}_\lambda^+(\alpha * \tau), \widetilde{\omega}_\lambda^+(\tau)\}, \widetilde{\omega}_\lambda^-(\alpha) \le rmax\{\widetilde{\omega}_\lambda^-(\alpha * \tau), \widetilde{\omega}_\lambda^-(\tau)\}$ and $\rho_\lambda^+(\alpha) \ge min\{\rho_\lambda^+(\alpha * \tau), \rho_\lambda^+(\tau)\}$, $\rho_\lambda^-(\alpha) \le max\{\rho_\lambda^-(\alpha * \tau), \rho_\lambda^-(\tau)\}$.

Definition 3.7: A CBs $\lambda = \langle M, F\rangle$ is called a CB-T-ideal if, $\forall \alpha, \tau, \mu \in \aleph$.

(a) $\widetilde{\omega}_\lambda^+(0) \ge \widetilde{\omega}_\lambda^+(\alpha), \rho_\lambda^+(0) \ge \rho_\lambda^+(\alpha)$ and $\widetilde{\omega}_\lambda^-(0) \le \widetilde{\omega}_\lambda^-(\alpha), \rho_\lambda^-(0) \le \rho_\lambda^-(\alpha)$.

(b) $\widetilde{\omega}_\lambda^+(\alpha * \mu) \ge rmin\{\widetilde{\omega}_\lambda^+((\alpha * \tau) * \mu), \widetilde{\omega}_\lambda^+(\tau)\}$,

$\widetilde{\omega}_\lambda^-(\alpha * \mu) \le rmax\{\widetilde{\omega}_\lambda^-((\alpha * \tau) * \mu), \widetilde{\omega}_\lambda^-(\tau)\}$ and

$\rho_\lambda^+(\alpha * \mu) \ge min\{\rho_\lambda^+((\alpha * \tau) * \mu), \rho_\lambda^+(\tau)\}$,

$\rho_\lambda^-(\alpha * \mu) \le max\{\rho_\lambda^-((\alpha * \tau) * \mu), \rho_\lambda^-(\tau)\}$.

Proposition 3.8: Let $\lambda = \langle M, F\rangle$ be a CB-T-ideal of $\aleph$. For all $\alpha, \tau \in \aleph$, if $\alpha \le \tau$ implies

(a) $\widetilde{\omega}_\lambda^+(\alpha) \ge \widetilde{\omega}_\lambda^+(\tau)$ *and* $\widetilde{\omega}_\lambda^-(\alpha) \le \widetilde{\omega}_\lambda^-(\tau)$,

(b) $\rho_\lambda^+(\alpha) \ge \rho_\lambda^+(\tau)$ and $\rho_\lambda^-(\alpha) \le \rho_\lambda^-(\tau)$.

Proof: Since $\alpha \le \tau$, then $\alpha * \tau = 0$ for any $\alpha, \tau \in \aleph$, thus

a) $\widetilde{\omega}_\lambda^+(\alpha) = \widetilde{\omega}_\lambda^+(\alpha * 0) \ge rmin\{\widetilde{\omega}_\lambda^+((\alpha * \tau) * 0), \widetilde{\omega}_\lambda^+(\tau)\}$

$= rmin\{\widetilde{\omega}_\lambda^+(\alpha * \tau), \widetilde{\omega}_\lambda^+(\tau)\} = rmin\{\widetilde{\omega}_\lambda^+(0), \widetilde{\omega}_\lambda^+(\tau)\} = \widetilde{\omega}_\lambda^+(\tau)$. And

$\widetilde{\omega}_\lambda^-(\alpha) = \widetilde{\omega}_\lambda^-(\alpha * 0) \le rmax\{\widetilde{\omega}_\lambda^-((\alpha * \tau) * 0), \widetilde{\omega}_\lambda^-(\tau)\}$

$= rmax\{\widetilde{\omega}_\lambda^-(\alpha * \tau), \widetilde{\omega}_\lambda^-(\tau)\} = rmax\{\widetilde{\omega}_\lambda^-(0), \widetilde{\omega}_\lambda^-(\tau)\} = \widetilde{\omega}_\lambda^-(\tau)$.

b) $\rho_\lambda^+(\alpha) = \rho_\lambda^+(\alpha * 0) \geq min\{\rho_\lambda^+((\alpha * \tau) * 0), \rho_\lambda^+(\tau)\}$
$= min\{\rho_\lambda^+(\alpha * \tau), \rho_\lambda^+(\tau)\} = min\{\rho_\lambda^+(0), \rho_\lambda^+(\tau)\} = \rho_\lambda^+(\tau)$. And
$\rho_\lambda^-(\alpha) = \rho_\lambda^-(\alpha * 0) \leq max\{\rho_\lambda^-((\alpha * \tau) * 0), \rho_\lambda^-(\tau)\}$
$= max\{\rho_\lambda^-(\alpha * \tau), \rho_\lambda^-(\tau)\} = max\{\rho_\lambda^-(0), \rho_\lambda^-(\tau)\} = \rho_\lambda^-(\tau)$.

Theorem 3.9: *In TM-algebra ℵ. Every CB-T-ideal is a CB-ideal.*

Proof: For all α, τ, μ in ℵ, let $\lambda = \langle M, F\rangle$ be a CB-T-ideal of ℵ, then

$$\widetilde{\omega}_\lambda^+(\alpha * \mu) \geq rmin\{\widetilde{\omega}_\lambda^+((\alpha * \tau) * \mu), \widetilde{\omega}_\lambda^+(\tau)\},$$

$$\widetilde{\omega}_\lambda^-(\alpha * \mu) \leq rmax\{\widetilde{\omega}_\lambda^-((\alpha * \tau) * \mu), \widetilde{\omega}_\lambda^-(\tau)\} \text{ and}$$

$$\rho_\lambda^+(\alpha * \mu) \geq min\{\rho_\lambda^+((\alpha * \tau) * \mu), \rho_\lambda^+(\tau)\},$$

$\{\rho_\lambda^-(\alpha * \mu) \leq max\{\rho_\lambda^-((\alpha * \tau) * \mu), \rho_\lambda^-(\tau)\}$, put $\mu = 0$, we get
$\widetilde{\omega}_\lambda^+(\alpha) \geq rmin\{\widetilde{\omega}_\lambda^+(\alpha * \tau), \widetilde{\omega}_\lambda^+(\tau)\}, \widetilde{\omega}_\lambda^-(\alpha) \leq rmax\{\widetilde{\omega}_\lambda^-(\alpha * \tau), \widetilde{\omega}_\lambda^-(\tau)\}$ and
$\rho_\lambda^+(\alpha) \geq min\{\rho_\lambda^+(\alpha * \tau), \rho_\lambda^+(\tau)\}, \quad \rho_\lambda^-(\alpha) \leq max\{\rho_\lambda^-(\alpha * \tau), \rho_\lambda^-(\tau)\}$, it follows that $\lambda = \langle M, F\rangle$ is a CB-ideal of ℵ.

To prove the opposite direction of the theory, we take the following TM-algebra

*	0	a	b	c
0	0	0	c	b
a	a	0	c	b
b	b	b	0	c
c	c	c	b	0

Define $\lambda = \langle M, F\rangle$ as follows

$$M(\alpha) = \begin{cases} \{[-0.4, -0.1], [0.2, 0.9]\} & if \quad \alpha = 0, a, c \\ \{[-0.6, -0.2], [0.1, 0.4]\} & if \quad \alpha = b \end{cases},$$

$$\rho_\lambda^+(\alpha) = \begin{cases} 0.5 & if \quad \alpha = 0, a, c \\ 0.3 & if \quad \alpha = b \end{cases},$$

$$\rho_\lambda^-(\alpha) = \begin{cases} -0.6 & if \quad \alpha = 0, a, c \\ -0.1 & if \quad \alpha = b \end{cases}.$$

Then $\lambda = \langle M, F\rangle$ is a CB-ideal, but not a CB-T-ideal since
$\widetilde{\omega}_\lambda^+(b * a) < rmin\{\widetilde{\omega}_\lambda^+((b * c) * a), \widetilde{\omega}_\lambda^+(c)\}, \widetilde{\omega}_\lambda^-(b * a) > rmax\{\widetilde{\omega}_\lambda^-((b * c) * a), \widetilde{\omega}_\lambda^-(c)\}$ and $\rho_\lambda^+(b * a) < min\{\rho_\lambda^+((b * c) * a), \rho_\lambda^+(c)\}, \quad \rho_\lambda^-(b * a) > max\{\rho_\lambda^-((b * c) * a), \rho(c)\}$.

5. Discussion and Conclusion

We studied cubic bipolar sets of TM-algebra as a generalization of the cubic set and discussed a few properties of this concept. Some important results are investigated such that (Every CB-T-ideal is a CB-ideal. But the converse is not true). Also, the relationships between a CB sub-TM-algebra and the level set $U(\lambda; \tilde{t}, \tilde{r}, s, v)$ are discussed. The future work is to study cubic bipolar sets for other algebraic structures and a cubic bipolar graph for TM- algebra.

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