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Finite discrete RGCN model for kinship verification

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Abstract

Kinship verification has been a challenging problem for generations, technology has been trying to resolve the same last two decades, without any success. It determines the blood relation between two people with help of their given pair of images. This problem has attracted significant attention in various fields, such as biometrics, forensic research, and social media, but age and gender differences make this problem more complicated, especially when we want to find the relationship between Descendants with skipped levels like grandparents and grandsons/daughters. In this paper, we proposed a novel approach by formulating kinship data as a finite discrete structure (FDS), which provides a mathematical model of kinship relations. We represent the data as a graph, where nodes denote individuals, and edges determine the relationships among them. This structured representation serves as the foundation for learning relational patterns. Using this Finite Discrete Structure framework, we employ Relational Graph Convolutional Networks (RGCN) to extract and analyze the complex relational dependencies in kinship verification. Initially EfficientNet is used to extract facial features, these feature vectors along with relations, form the graph structure.

RGCN processes this graph to derive kin relations and effectively capture complex patterns within the discrete structured space. To improve model's ability for enhancing

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Class-wise discriminability we used ArcFace and Center loss functions to enforce feature separability and robust kinship classification.

We evaluate our approach on the FIW dataset, achieving an accuracy of 89.45%, demonstrating that our method effectively addresses the kinship verification problem within the framework of finite discrete structures. This work highlights the potential of graph-based models in analyzing and classifying complex relationships in structured, discrete domains.

Subject Classification: *68R01, 68R10, 68U10, 68T10, 68T45.*

Keywords: *Finite discrete structure, Kinship verification, RGCN, Feature extraction face image, ArcFace loss, Center loss.*

1. Introduction

Finite discrete structures are finite and Discrete mathematical structures, which have countable elements. It provides a rigorous mathematical framework for representing data. In Finite discrete structures (FDS) objects are distinct and their relation form a structured representation. Graphs [1] are a natural manifestation of FDS, where nodes denote entities and edges represent their relationships.

Kinship verification is a problem that demonstrates the framework of FDS, kinship verification is a specialized problem within facial recognition [2]. the objective of this task is to discern familial relationships from facial images. Kinship data inherently conforms to an FDS, in which an individual are distinct entities, and their relation form a structured network. This perspective allows us to model kinship as a graph, where nodes representing individual person and labeled edge showing their family relation. This graph-based representation aligns perfectly with the mathematical framework of finite discrete structures, where elements are countable, and relationships are explicitly defined.

The facial features such as bone structure, lips size etc. can give vital clues to judge the kinship similarity can be interpreted as attribute within this structured framework. However, variations in age and gender introduce complexities, as increasing age gaps reduce feature similarity, and gender difference further complicate the recognition of similarity in features. Additionally, minute variations in facial similarities across different families add another layer of complexity to the task. Despite these challenges, kinship verification remains crucial across various domains, including forensic investigations, social media applications, and genetic analysis.[3] The structured nature of kinship relationships within an FDS provides a mathematically rigorous foundation for modeling and analyzing familial patterns, reinforcing the importance of graph-based and relational learning approaches in this field.

Extensive research has been conducted in kinship verification, with various approaches categorized based on their feature extraction methodologies. These approaches can be broadly classified into four categories: 1 handpicked features like the gap between eyes and lips size[4], [5] 2. Use feature extraction techniques like LBP, HOG[6], and SIFT. 3. Deep learning techniques: to detect complex and minute features, complicated and complex convolution models were designed to extract features 4. Transfer Learning: pretrained models were used to extract features. After extracting features, any classification techniques or metric learning[7], [8] approach is used for classification.

Graph Neural Network (GNN) is a powerful architecture of neural network, which is designed particularly for graph structure data [9]. In graph data, node represents feature vector of entity and edge represents relation between them. GNN is used to classify node, edge or graph classification. GCN operates by passing massaging between nodes, in massage passing operation it aggregates the information of all adjacent nodes and their relation. This iterative process enables the model to effectively capture and propagate relational dependencies

In Kinship verification problem, different relations are exists like father-son, father-daughter, grandfather-grandson etc. and every relation require different weights because in father-son mother-daughter both images have same gender while father-daughter or mother-son both image have different gender, in brother-brother or sister-sister images have almost same age person while in grandparents-grandchild relation images have a big age difference. To address this problem, we require a model which treats all relations differently and assigns different weight to different edges. But GCN does not have the capability of distinguishing between various types of relations, and it treats all relations in same way, so we used Relational Graph Convolution network (RGCN).

RGCN is an advanced version of GCN, with the property of differentiating between various relations. It treats all relations uniquely and gives different weight to different relations and learns unique characteristics of different relations. RGCN can be used for node classification and link prediction, for link prediction it works like an autoencoder, in which an encoder model generates features of entities which contain information about entities and relation of that entity with other entities and decoder use DistMult [10] factorization technique to decode the features and predict the relation.

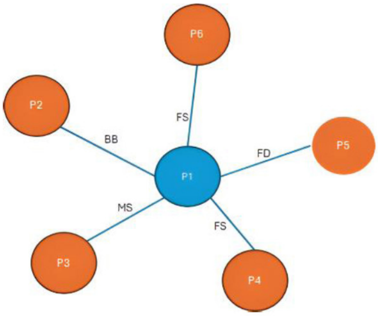

Fig a

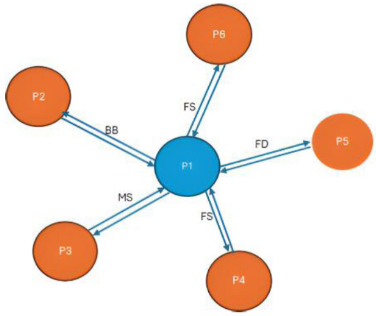

Fig b

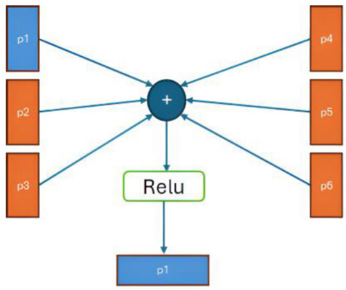

Fig c

Figure 1

a. Representing undirected graph structure of Kinship problem, b representing directed graph, c representing message passing in RGCN.

In RGCN features of every node are updated by aggregate information by neighboring node, edges and previous feature vector of same node.

$$v_i^{(l+1)} = f\left(\sum_{r \in \Re} \sum_{j \in N_i^r} \frac{1}{\alpha_{i,r}} W_r^l v_j^l + W_0^l h_i^l \right)$$

Where $v_i^{(l+1)}$ is state of node v_i at $(l+1)$ layer, N_i^r is neighboring node of v_i with relation r.

Figure 1.a representing FDS representation of kinship data. In which p1 have different relations with p2, p3…p6. But which person playing which role so we cannot give direction of relation, graph is undirected graph. To convert into directed graph a reverse edge is added and make it directed graph, figure 1.b representing directed graph. Figure 1.c representing message passing technique for RGCN.

2. Literature Review

Kinship verification, or the process of determining familial relationships using facial images, has been a topic of considerable interest in the field of computer vision. Fang et al. [4] was the first one who perform kinship verification using computer vision. They used facial appearance, like distance between two eyes, distance between nose and ear. Early methods relied on hand-crafted features like LBP, BSIF, HOG, LPQ, and SIFT to extract features and then used any classifier to identify family members, [11] [8] [12], such as texture, shape, and colour [4], [13], [14]. Alirezazadeh et al. [15] used genetic approach SPLE for selecting features. With the advent of deep learning, researchers began leveraging convolutional neural networks (CNNs) to achieve more robust and accurate kinship predictions, outperforming traditional methods significantly [11]. Patil et al [16] used a CNN model, while Dahan et al. [17] used 20-layer Sphereface CNN model. Few researcher used pre trained model for verification like Dornaika et al [18] and Chergui et al [19] used VGG-Face model, Wang et al [20] used Alexnet and Wang et al [21] used Resnet for the sameNandy et al [22] used Squeeze Net and create Siamese network. Rachmadi et al [23] combined dual CNN model and attention network to create a model for kinship verification, FaceNet CNN model and Family aware feature extraction network used to create CNN model.

Researchers also used metric learning to check similarities between features to determine kin relation. Either hand crafted methods or convolution layer [24] is used to extract features, and a metric learning method is used to check similarity. Wu Huishan et al [25] proposed Component- based Metric Learning (CML) approach for identifying kin relation. They used SIFT feature descriptor to represent the faces and Mahalanobis distance metric for similarity measures. Dong et al [26] proposed CFIL framework (Cross-generation Feature Interaction Learning) for detecting kin relation. They used ResNet-50 for feature extraction and Siamese network to ascertain similarity. Wang et al. [21] used GAN and convert old parents' image to young parents' images to counter the age factor. ResNet was used for the feature extraction. Shiwei and Haibin et al [27] used reinforcement learning and designed Deep Q-Network based model for selecting features. Bessaoudi et al [28] proposed Tensor Cross-view Quadratic Discriminant Analysis (TXQDAWCCN), it is multilinear and Multiview subspace learning method. Oruganti et al. [29] proposed a new idea in which they matched childhood images of parents and child so that age gap will not be issue. Li. et al. [30] worked on Re-Weighting technique on meta learning and proposed a framework for kinship verification called Online Re-weighting Relation Network (OR2Net).

Extracting features are key points in all mentioned methods. With the help of CNN model, researchers were getting better results, but existing model did not understand relationship structure in kinship data. In all methods extracting features and based on similarity or distance between extracted features are classified the relation. Now if similarity is very less than it is possible that image pair have different relation like grandparents-grandchild. To solve such issues, we need a framework which is capable to understand this relationship structure and different relations. FDS is a perfect option to represent kinship data. Graph neural network (GNN) provides a framework for extracting features, which can be understood dependent on their relationship.

GNN used in social media [31], fraud detection [32], computer vision [33], natural language processing [34], improve traffic congestion [35], GNN also used on physics and chemistry field [36] and in recommendation system [37]. In computer vision it used for image classification [33], retrieving multiple facts of images [38], Visualizing relationship for image caption [39], also used to generate multiple images from given image [40].

3. Methodology

In this section we describe our proposed approach for kinship verification. Our approach is inspired by the recent advancements in graphical neural networks; we use Recurrent Graph Convolutional Networks and proposed a novel model for kinship verification.

In our approach, we are representing our data as an FDS. Especially in graphical format $G(V,E)$, V is set feature vectors f of individual persons extracted by EfficientNet, $f=\mathbb{E}(1)$, $f\in\mathcal{R}^{d}$, and E is set of edges representing different family relations. Each vertex $v_i \varepsilon V$ and edge (v_i, v_j, r) exits if person v_i and v_j have relation $r \varepsilon R$, where R is set of relations.

$$A_{ij}=\begin{Bmatrix} r & \text{if } (v_i, v_j, r)\varepsilon R \\ 0 & \text{Otherwise} \end{Bmatrix}$$

This graph data is feed to 4-layer RGCN model, which is used to predict the relation between images. Complete architecture shown is fig-2. To train this model we combined two loss functions, ArcFace and Center Loss, and designed a weight loss function

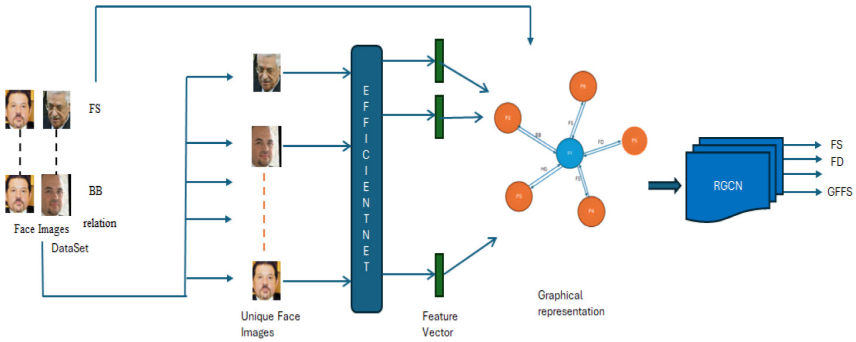

Figure 2

Architecture of Proposed Model

3.1 *EfficientNet*

EfficientNet Convolution Neural Network is the most used model for feature extraction. Initially when very few resources were available, model size was very small, but as resource increased, for better accuracy researcher increased CNN model size. Tan M. et-al [41] used compound scaling concept and scaled all dimensions of depth, width and resolution, and designed a new model called EfficientNet model. Architecture of Efficient Model is shown in Fig 3.

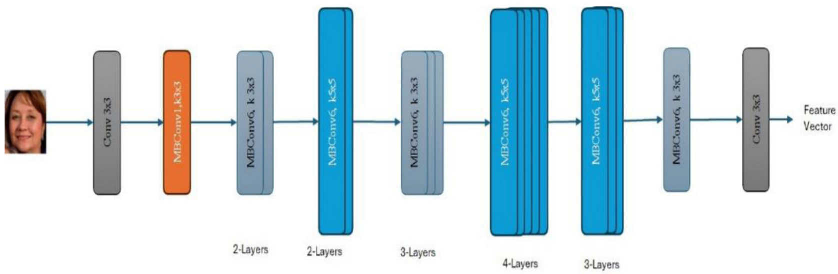

Figure 3

Architecture of EfficientNetB0

3.2 *Loss Functions*

3.2.1 ArcFace Loss

ArcFace loss is also known as Additive Angular Margin Loss. This loss function is used in face matching tasks. ArcFace loss function optimized the feature vector such inter-class feature vector goes far from each other and distance is very high with compare to intra-class feature vector. So that model is easily differentiated between inter-class and intra-class faces. ArcFace loss enhances this power by adding an angular margin to the cosine similarity between feature vector (X_i) and weights vector (W_j).

$$X_i' = \frac{X_i}{\| X_i \|} \text{ and } W_j' = \frac{W_j}{\| W_j \|}$$

X_i' and W_j' are normalized forms of feature vectors and weight vectors.

Calculate the angle between the feature vector and each weight vector.

$$Cos(\theta_j) = X_i'.W_j'$$

Add an angular margin 'm' to the angle to true class

$$Cos(\theta_{y_i} + m)$$

y_i is the corresponding label.

Compute logits

$$logits_j = \begin{cases} s.\cos(\theta_{y_i} + m) & \text{if } j = y_i \\ s.\cos(\theta_j) & \text{if } j \neq y_i \end{cases}$$

ArcFace Loss

$$L_{ArcFace} = -\frac{1}{N}\log\left(\frac{e^{logits_{y_i}}}{\sum_{j=1}^{N} e^{logits_j}}\right)$$

3.2.2 Center Loss

Center loss is used to minimize intra-class variance. It calculates the class center and pulls same class feature toward center point and reduce the distance between intra-class features.

$$L_{center} = \frac{1}{2}\sum_{i=1}^{m} \| X_i - C_{y_i} \|^2$$

Where $X_{i \text{ is}}$ the feature vector and C_{y_i} is the mean of the feature vectors of all samples belonging to class y_i.

3.2.3 Combined Loss Function

To train the model, we combine both loss functions with different weights and designed a weighted loss function to train model for FDS data. This weighted loss function enhances the discriminative power of the model by ensuring that feature vectors corresponding to different classes are well-separated in the feature space.

$$L_{total} = \alpha . L_{ArcFace} + \beta . L_{Center}$$

4. Result

We demonstrate our approach on FIW data set with different loss functions, results are shown in table 1. Result with combined loss function of Arcface and center loss gives better result. ArcFace increases distance between inter-class features and center loss function decreases the distance between intra-class feature vector. Resultant same class features came close to each other and far from other class.

Table 1

Comparison with different loss functions

Loss Function	Accuracy
Cross Entropy Loss	76.74%
Cross Entropy Loss +ArcFace	81.09%
ArcFace + Center Loss	89.45%

We also compared our results with existing approaches and found that our approach provides better results as shown in Table 2. Our approach gave 95.34% accuracy on brother-brother relation, which is maximum, in this relation persons have very less age gape and also have same gender, therefore high degree of feature matching is achieved. And the lowest is 84.67% of grandfather-granddaughter relation. In this case the age difference is high, and gender is also different. Overall accuracy is 89.45% which is better than other approaches.

Our model achieved an accuracy of 89.45% on the Families in the Wild (FIW) dataset. This result demonstrates the effectiveness of our RGCN-based approach in accurately classifying kinship relationships within the finite discrete structure represented by the FIW dataset's relational graph. By leveraging the RGCN's ability to learn complex

Table 2
Comparative Analysis of different Approaches of FIW data set

Approach	SS	BB	Sibs	Fd	FS	MD	MS	GFGD	GFGS	GMGD	GMGS	Avg
CNN + pyramid Attention network[23]	76.2	70.3	71.5	70.1	69.0	73.0	71.5	63.4	62.7	63.5	64.4	68.73
SPHEREFACE CNN [17]	77.3	69.1	68.6	68.9	68.9	73.8	70.9	61.9	63.7	63.9	63.3	68.2
DEEP-TENSOR +ELM [42]	-	-	-	-	-	-	-	68.36	68.18	70.14	67.77	68.61
RESNET +SDM-Loss [21]	-	-	-	69.02	68.60	72.28	69.59	65.89	65.12	66.41	64.90	69.47
TXQDA +MSLPQ +MSBSIF [43]								66.43	66.79	65.24	65.67	66.03
Dual VGG-Face classifier + NAG [44]	73.23	66.32	67.20	65.34	64.11	67.39	66.36	60.53	59.12	61.66	60.60	64.71
DML [45]			75.27	68.08	71.03	70.36	70.76	64.90	64.81	67.37	66.50	68.79
Proposed Approach	94.8	95.34	94.6	90.4	93.5	94.02	91.4	84.67	89.87	88.43	85.89	89.45

patterns within this finite discrete relational network, we were able to significantly outperform state-of-the-art methods.

5. Conclusion

In this paper, we addressed kinship verification problem by explicitly modeling kinship relationships as a finite discrete structure. We used EfficientNet for extracting facial features and RGCN for verifying kin relations. RGCN treats all relations in different ways and gives different weights to different types of relation and learn complex relation pattern in graph format. To improve the discriminative power of the model, we used ArcFace and Center loss functions. ArcFace increase distance between inter-class relation data and center loss take close intra-class data, combination of both create a significant difference between inter-class and intra-class.

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