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Cryptographic approach on edge regular power fuzzy graph

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Abstract

In this paper, we newly define the notion of edge regular power fuzzy graph, based on the properties of edge regular fuzzy graph and power fuzzy graph (PFG). Totally edge regular power fuzzy graph is presented as a special case of edge regular power fuzzy graph. General formulas are derived to calculate the degree of edges for a total power fuzzy graph when it is edge regular and total degree of edges when it is totally edge regular. Further, this concept is applied in the encryption method to decipher data by authorised persons.

Subject Classification: *03B52, 05C72, 94D05, 94A60.*

Keywords: *Power fuzzy graph, Edge regular power fuzzy graph, Data security, Cryptography.*

1. Introduction

The representation of uncertainty was introduced as fuzzy set by Lotfi Zadeh in 1965 [25]. This inspired Azriel Rosenfeld to develop the concept of fuzzy graph in 1975 [23]. Over the years, fuzzy graph theory

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has developed widely, as in [1, 2, 9, 10, 15]. The notion power fuzzy graph (PFG) [5] and its regular properties on vertices [6] were introduced by Bharathi, T., et al. The edge regular fuzzy graph was introduced by Radha, K., et al. [18], and their properties on intuitionistic, bipolar, complement, and ω -complement, planar, intuitionistic m-polar, q-rung picture of fuzzy graphs are studied as in [3, 11-13, 20-22]. This is the motivation for the study of the properties of edge regular PFGs.

Cryptography is the process of safeguarding the data or information from unauthorized persons by the methods of encryption and decryption. Since cyber-crimes are on the increase day by day, new cryptosystems are emerging and improving to resolve the issues. Interesting applications on cryptography are discussed on image security by Qaid, G.R.S., et al. [17], computer network security by Misra, S., et al. [14] data privacy by Pius, A., et al. [16], transaction of digital currency by Saxena, V., et al. [24], identifying man-in-the-middle attack by Durcheva, M., et al. [8] using fuzzy graph theory. Also a cryptographic modelling using operations on beta products of fuzzy graphs are given by Borzooei, R. A., et al. [7]. The objective of this paper is to give a novel algorithm of a cryptosystem developed as public key cryptography. The required basic definitions are given in section 2. The definitions of edge regular and totally edge regular PFGs are presented in section 3. In section 4, the results on edge regular power fuzzy graph are proved and in section 5, an application on cryptosystem is discussed.

2. Preliminaries

Definition 1: [4] For any $k > 0$, the k th power G^k of an undirected connected graph $G = (V, E)$ has $V(G^k) = V(G)$, where two vertices u and v are adjacent in G^k if and only if $d_G(u, v) \leq k.G^2$ and G^3 are called square and cube of G.

Definition 2: [9] A fuzzy Graph $G_f = (V, E, \theta, \omega)$ corresponding to the crisp graph $G = (V, E)$ with a non-empty set V together with a pair of functions $\theta : V \to [0,1]$ and $\omega : V \times V \to [0,1]$ such that for all u, $v \in V$, $\omega(u, v) \leq \min\{\theta(u), \theta(v)\}$, where $\theta(u)$ and $\omega(u, v)$ represent the membership values of the vertex u and edge (u, v) in G respectively.

Definition 3: [19] Let $G_f = (V, E, \theta, \omega)$ be a fuzzy graph. The degree of an edge $e = v_1 v_2 \in E$ is defined as $deg_{G_f}(v_1, v_2) = deg_{G_f}(v_1) + deg_{G_f}(v_2) -$

$2\omega(v_1, v_2)$. The total degree of an edge $e = v_1 v_2 \in E$ is defined as $tdeg_{G_f}(v_1, v_2) = deg_{G_f}(v_1) + deg_{G_f}(v_2) - \omega(v_1, v_2)$.

Definition 4: [19] If each edge has the same degree k in G_f, then G_f is said to be an edge regular fuzzy graph of degree k. If each edge has the same total degree k in G_f, then G_f is said to be a totally edge regular fuzzy graph of degree k.

Definition 5: [5] Let G_f be a fuzzy graph of G. Then the definition of power fuzzy graph (PFG) of G_f is $G_f^{\,k} = (V, E_k, \theta, \omega_k)$, where $E_k = E \cup E^*$, for every independent vertices $u, v \in V$ in G_f $\exists$ edges $uv \in E^*$ such that $\mathcal{L}(\mathrm{u}, \mathrm{v}) \leq \mathrm{k}$ where $\mathcal{L}$ is the length between u and v and $k > 1$ provided $\omega_k(u, v) \in [\min(\theta(u)^k, \theta(v)^k), \min(\theta(u)^{k-1}, \theta(v)^{k-1})$.

Definition 6: [5] The PFG $G_f^{\,k}$ is called total power fuzzy graph $T_f^{\,k} = (V, E_k, \theta, \omega_k)$ such that'll vertices are adjacent to one another. The notation $e(T_f^{\,k})$ indicates the number of edges incident to a vertex in $T_f^{\,k}$, also k_m indicates the maximum k value.

3. Edge Regular and Totally Edge Regular PFGs

Definition 7: A PFG $G_f^{\,k}$ is p-edge regular if for any $p > 0$, $deg_{G_f^{\,k}}(v_i, v_j) = p$, $\forall (v_i, v_j) \in E_k$.

Example 8: Consider the PFGs $G_f^{\,1}$ and $G_f^{\,2}$ as in Figure 1a and Figure 1b. $G_f^{\,1}$ is edge regular since

$$deg_{G_f^{\,1}}(v_1, v_2) = 0.7 + 0.8 - 2(0.4) = 0.7$$
$$deg_{G_f^{\,1}}(v_2, v_3) = 0.8 + 0.7 - 2(0.4) = 0.7$$
$$deg_{G_f^{\,1}}(v_3, v_4) = 0.7 + 0.6 - 2(0.3) = 0.7$$
$$deg_{G_f^{\,1}}(v_4, v_1) = 0.6 + 0.7 - 2(0.3) = 0.7$$

Similarly, $G_f^{\,2}$ is edge regular since

$$deg_{G_f^2}(v_1, v_2) = 0.95 + 1.25 - 2(0.4) = 1.4$$
$$deg_{G_f^2}(v_2, v_3) = 1.25 + 0.95 - 2(0.4) = 1.4$$
$$deg_{G_f^2}(v_3, v_4) = 0.95 + 1.05 - 2(0.3) = 1.4$$
$$deg_{G_f^2}(v_4, v_1) = 1.05 + 0.95 - 2(0.3) = 1.4$$
$$deg_{G_f^2}(v_1, v_3) = 0.95 + 0.95 - 2(0.25) = 1.4$$
$$deg_{G_f^2}\left(v_2, v_4\right) = 1.25 + 1.05 - 2(0.45) = 1.4$$

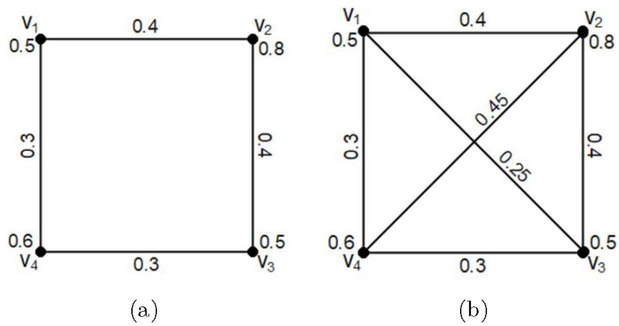

Figure 1

Edge regular power Fuzzy Graphs $G_f^{\,1}$ and $G_f^{\,2}$

Definition 9: A PFG $G_f^{\,k}$ is r-totally edge regular if for any $r > 0$, $tdeg_{G_f^{\,k}}$ $(v_i, v_j) = r$, $\forall (v_i, v_j) \in E_k$.

Example 10: Consider the PFGs $G_f^{\,1}$ and $G_f^{\,2}$ as in Figure 2a and Figure 2b. $G_f^{\,1}$ is totally edge regular since

$$tdeg_{G_f^1}(v_1, v_2) = 0.2 + 0.2 - 0.1 = 0.3$$
$$tdeg_{G_f^1}(v_2, v_3) = 0.2 + 0.2 - 0.1 = 0.3$$
$$tdeg_{G_f^1}(v_3, v_4) = 0.2 + 0.2 - 0.1 = 0.3$$
$$tdeg_{G_f^1}(v_4, v_1) = 0.2 + 0.2 - 0.1 = 0.3$$

Similarly, $G_f^{\,2}$ is totally edge regular since

$$tdeg_{G_f^2}(v_1, v_2) = 0.9 + 0.9 - 0.3 = 1.5$$
$$tdeg_{G_f^2}(v_2, v_3) = 0.9 + 0.9 - 0.3 = 1.5$$

$$tdeg_{G_f^2}(v_3,v_4) = 0.9 + 0.9 - 0.3 = 1.5$$
$$tdeg_{G_f^2}(v_4,v_1) = 0.9 + 0.9 - 0.3 = 1.5$$
$$tdeg_{G_f^2}(v_1,v_3) = 0.9 + 0.9 - 0.3 = 1.5$$
$$tdeg_{G_f^2}(v_2,v_4) = 0.9 + 0.9 - 0.3 = 1.5$$

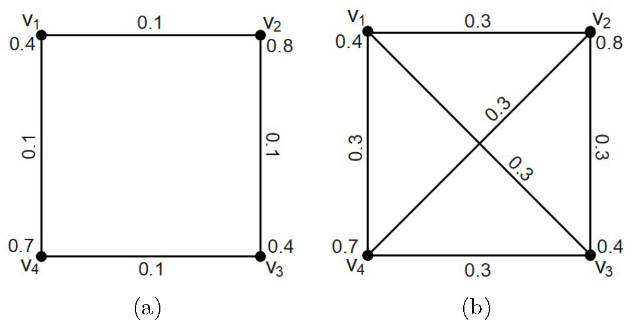

Figure 2

Totally edge regular power fuzzy graphs G_f^1 and G_f^2

Definition 11: In a PFG G_f^k, if there exist an edge regular power fuzzy graph $G_f^{k_p}$ for any particular k_p such that $1 < k_p \geq k$ then $G_f^{k_p}$ is called (k_p, p) - partly edge regular of G_f^k.

Definition 12: A PFG G_f^k is perfectly edge regular if G_f^k is both edge regular and totally edge regular.

4. Results on Edge Regularity

Theorem 13: *The PFG G_f^k is perfectly edge regular if edge membership function is constant and every vertex has same number of incident edges.*

Proof: Consider a PFG G_f^k such that edge membership values $\omega_k(u,v) = m$ such that $\min(\theta(u)^k, \theta(v)^k) \leq m \leq \min(\theta(u)^{k-1}, \theta(v)^{k-1})$ $\forall(u,v) \in E_k$. Suppose G_f^k is not edge regular (i.e.) $deg_{G_f^k}(u,v) \neq p$, a constant $\forall(u,v)$. Consider any two edges (v_1,v_2) and (v_3,v_4) such that $deg_{G_f^k}(v_1,v_2) \neq deg_{G_f^k}(v_3,v_4)$. Let e_1, e_2, e_3 and e_4 be the number of edges incident with v_1, v_2, v_3 and v_4 respectively. Then

$$deg_{G_f^k}(v_1,v_2) = deg_{G_f^k}(v_1) + deg_{G_f^k}(v_2) - 2\omega_k(v_1,v_2) \quad (1)$$

$$deg_{G_f^k}(v_1,v_2) = e_1 m + e_2 m - 2m \quad (2)$$

$$deg_{G_f^k}(v_3,v_4) = deg_{G_f^k}(v_3) + deg_{G_f^k}(v_4) - 2\omega_k(v_3,v_4) \quad (3)$$

$$deg_{G_f^k}(v_3,v_4) = e_3 m + e_4 m - 2m \quad (4)$$

From equations 2 and 4 , we get $e_1 \neq e_3$ and $e_2 \neq e_4$. This contradicts the fact that in G_f^k every vertex has same number of incident edges. Hence G_f^k is edge regular. Similarly, G_f^k is totally edge regular. Therefore, G_f^k is perfectly edge regular.

Example 14: Consider the 5-cycle PFGs $G_f^1 = (V, E_1, \theta, \omega_1)$ and $G_f^2 = (V, E_2, \theta, \omega_2)$ with $\omega_1(v_i,v_j) = 0.2$, $\forall (v_i,v_j) \in E_1$ and $\omega_2(v_i,v_j) = 0.4$, $\in E_2$ as in Figure 3a and 3b respectively. For all $v_i \in V$, we have $deg_{G_f^1}(v_i) = 0.2 + 0.2 = 0.4$ and $deg_{G_f^2}(v_i) = 0.4 + 0.4 + 0.4 + 0.4 = 1.6$. Here G_f^1 is perfectly edge regular since $deg_{G_f^1}(v_i,v_j) = 0.4 + 0.4 - 2(0.2) = 0.4$ and $tdeg_{G_f^1}(v_i,v_j) = 0.4 + 0.4 - 0.2 = 0.6$, $\forall (v_i,v_j) \in E_1$. Similarly, G_f^2 is perfectly edge regular since $deg_{G_f^2}(v_i,v_j) = 1.6 + 1.6 - 2(0.4) = 2.4$ and $tdeg_{G_f^2}(v_i,v_j) = 1.6 + 1.6 - 0.4 = 2.8$, $\forall (v_i,v_j) \in E_2$.

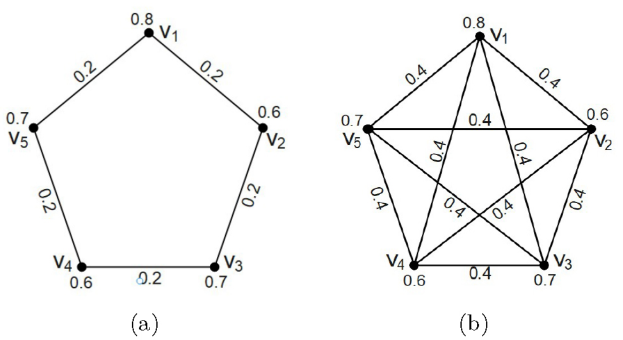

Figure 3

5-cycle perfectly edge regular power fuzzy graphs G_f^1 and G_f^2

Corollary 15: *The total PFG T_f^k is perfectly edge regular if edge membership function is constant.*

Proof: Consider a total PFG $T_f^{\,k}$ such that edge membership values $\omega_k(u,v) = m$ such that $\min(\theta(u)^k, \theta(v)^k) \le m \le \min(\theta(u)^{k-1}, \theta(v)^{k-1})$ $\forall (u,v) \in E_k$. Since $T_f^{\,k}$ has the same number of incident edges with every vertex then by Theorem $4.1 T_f^{\,k}$ is perfectly edge regular.

Suppose $T_f^{\,k}$ is not edge regular (i.e.) $deg_{T_f^k}(u,v) \ne p$, a constant $\forall (u,v)$. Consider any two edges (v_1,v_2) and (v_3,v_4) such that $deg_{T_f^k}(v_1,v_2) \ne deg_{T_f^k}(v_3,v_4)$. Let e_1, e_2, e_3 and e_4 be the number of edges incident with v_1, v_2, v_3 and v_4 respectively. Then

$$deg_{T_f^k}(v_1,v_2) = deg_{T_f^k}(v_1) + deg_{T_f^k}(v_2) - 2\omega_k(v_1,v_2) \tag{1}$$

$$deg_{T_f^k}(v_1,v_2) = e_1 m + e_2 m - 2m \tag{2}$$

$$deg_{T_f^k}(v_3,v_4) = deg_{T_f^k}(v_3) + deg_{T_f^k}(v_4) - 2\omega_k(v_3,v_4) \tag{3}$$

$$deg_{T_f^k}(v_3,v_4) = e_3 m + e_4 m - 2m \tag{4}$$

From equations 2 and 4 , we get $e_1 \ne e_3$ and $e_2 \ne e_4$. This contradicts the fact that $T_f^{\,k}$ has same number of incident edges with every vertex. Hence $T_f^{\,k}$ is edge regular. Similarly, $T_f^{\,k}$ is totally edge regular. Therefore, $T_f^{\,k}$ is perfectly edge regular.

Corollary 16: *Let $T_f^{\,k} = (V, E_k, \theta, \omega_k)$ be a total PFG. If*

(i) *$T_f^{\,k}$ is edge regular, then $deg_{T_f^k}(v_1,v_2) = 2m(e-1)$, $\forall (v_1,v_2) \in E_k$*

(ii) *$T_f^{\,k}$ is totally edge regular, then $deg_{T_f^k}(v_1,v_2) = m(2e-1)$, $\forall (v_1,v_2)\ E_k$*

Here, m represents the constant edge membership value, and e denotes the number of incident edges with a vertex in the total PFG $T_f^{\,k}$.

Proof: Let m represents the edge membership value of every edge, and e denotes the number of incident edges with a vertex in $T_f^{\,k}$, then degree of vertices v_1 and v_2 is $deg_{T_f^k}(v_1) = \sum_{v_1 \ne v_2} \omega_k(v_1,v_2) = em$, and $deg_{T_f^k}(v_2) = \sum_{v_2 \ne v_1} \omega_k(v_1,v_2) =$ em respectively.

(i) If $T_f^{\,k}$ is edge regular, then

$$deg_{T_f^k}(v_1,v_2) = deg_{T_f^k}(v_1) + deg_{T_f^k}(v_2) - 2\omega_k(v_1,v_2) = em + em - 2m = 2m(e-1).$$

(ii) If $T_f^{\,k}$ is totally edge regular, then

$$deg_{T_f^k}(v_1, v_2) = deg_{T_f^k}(v_1) + deg_{T_f^k}(v_2) - \omega_k(v_1, v_2) = em + em - m = m(2e-1).$$

Remark 17: For a p-edge regular PFG, the size equals $\frac{e_k \times p}{4}$ where e_k is the number of edges.

Corollary 18: *Any PFG with at least one end vertex can neither be edge regular nor totally edge regular.*

Proof: Let $G_f{}^1$ be a PFG with more than two vertices and let v_1 be the end (pendant) vertex. Then $deg_{G_f{}^1}(v_1) = \omega_1(v_1, v_2)$. Suppose $G_f{}^1 = (V, E_1, \theta, \omega_1)$ is edge regular, then $deg_{G_f{}^1}(v_i, v_j) = p, \forall (v_i, v_j) \in E_1$.

Let (v_1, v_2) and (v_2, v_3) be two edges incident at non-pendant vertex v_2 such that $deg_{G_f{}^1}(v_1, v_2) = deg_{G_f{}^1}(v_2, v_3)$,

$$\Rightarrow deg_{G_f{}^1}(v_1) + deg_{G_f{}^1}(v_2) - 2\omega_1(v_1, v_2) = deg_{G_f{}^1}(v_2) + deg_{G_f{}^1}(v_3) - 2\omega_1(v_2, v_3)$$

$$\Rightarrow deg_{G_f{}^1}(v_1) - 2\omega_1(v_1, v_2) = deg_{G_f{}^1}(v_3) - 2\omega_1(v_2, v_3)$$

$$\Rightarrow -deg_{G_f{}^1}(v_1) = deg_{G_f{}^1}(v_3) - 2\omega_1(v_2, v_3) (\because deg_{G_f{}^1}(v_1) = \omega_1(v_1, v_2)).$$

Since $deg_{G_f{}^1}(v_3) > \omega_1(v_2, v_3)$, the right-side value is positive in the above equation. Hence $deg_{G_f{}^1}(v_1, v_2) \neq deg_{G_f{}^1}(v_2, v_3)$. Therefore $G_f{}^1$ is not edge regular. Similarly, $G_f{}^1$ is not totally edge regular.

Proposition 19: A path PFG, $G_f{}^1$ is neither edge regular nor totally edge regular.

Proof: Any path PFG $G_f{}^1$ consists of pendant vertices at the end. Hence by Corollary 18, $G_f{}^1$ is neither edge regular nor totally edge regular.

Theorem 20: *Any PFG $G_f{}^1$ of a star graph S_n with n vertices such that internal vertex is u and v_i where $i = 1, 2, \ldots, n-1$ are the other vertices, is perfectly edge regular if and only if the edge membership values $\omega_1(u, v_i) = m$ and $deg_{G_f{}^1}(v_i) = m$ where $i = 1, 2, \ldots, n-1$ and m is a constant.*

Proof: Consider PFG $G_f^{\,1}$ of a star graph S_n with u as internal vertex and v_i as the other vertices where $i = 1, 2, \ldots, n-1$. Assume $G_f^{\,1}$ is edge regular, then $deg_{G_f^{\,1}}(u, v_i) = p = deg_{G_f^{\,1}}(u) + deg_{G_f^{\,1}}(v_i) - 2\omega_1(u, v_i), \forall v_i$ where p is a constant. Since u is the only vertex incident with other vertices v_i, $\omega_1(u, v_i) = deg_{G_f^{\,1}}(v_i)$. Hence, we need to prove $\omega_1(u, v_i)$ equals same value for $i = 1, 2, \ldots, n-1$. Suppose $\omega_1(u, v_1) \neq \omega_1(u, v_2)$, then $deg_{G_f^{\,1}}(u, v_1) \neq deg_{G_f^{\,1}}(u, v_2)$. This contradicts the assumption.

Conversely, assume that $\omega_1(u, v_i) = deg_{G_f^{\,1}}(v_i) = m$ where $i = 1, 2, \ldots, n-1$ and m is a constant. To prove $deg_{G_f^{\,1}}(u, v_i) = p, \forall v_i$ where p is any constant. Suppose $deg_{G_f^{\,1}}(u, v_1) \neq deg_{G_f^{\,1}}(u, v_2)$, then $deg_{G_f^{\,1}}(u) + deg_{G_f^{\,1}}(v_1) - 2\omega_1(u, v_1) \neq deg_{G_f^{\,1}}(u) + deg_{G_f^{\,1}}(v_2) - 2\omega_1(u, v_2)$. This implies, $\omega_1(u, v_1) = deg_{G_f^{\,1}}(v_1)$ and $\omega_1(u, v_2) = deg_{G_f^{\,1}}(v_2)$. This contradicts the assumption. Similarly, this is true for totally edge regular. Hence the theorem.

Example 21: Consider a PFG $G_f^{\,1}$ of a star graph S_6 in Figure 4 with $\theta(v_i) = 0.8$ for $i = 1$ to 6, $\omega_1(v_i, v_j) = 0.8$ for $i = 1$ to 5 and $j = 6$. Then we have $deg_{G_f^{\,1}}(v_i) = 0.8$ for $i = 1$ to 5 and $deg_{G_f^{\,1}}(v_6) = 4$. Here $G_f^{\,1}$ is perfectly edge regular since $deg_{G_f^{\,1}}(v_i, v_6) = 0.8 + 4 - 2(0.8) = 3.2$ and $tdeg_{G_f^{\,1}}(v_i, v_6) = 0.8 + 4 - 0.8 = 4$, $\forall (v_i, v_j) \in E_1$.

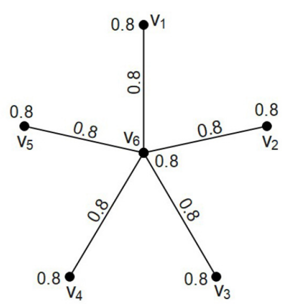

Figure 4

Perfectly edge regular PFG $G_f^{\,1}$ of a star graph S_6

5. Application

Many applications such as messaging apps, banking apps, including password protected documents, cloud storage services, online purchases, and virtual private networks use the combination of symmetric and asymmetric encryption. The security of the cryptosystem would depend on various factors, including the complexity of the graph structure, the distribution of fuzzy values, the condition to be edge regular and the effectiveness of encryption/decryption algorithms. An asymmetric encryption algorithm is newly derived such that the data is more securely decrypted.

5.1 *Asymmetric algorithm with edge regular power fuzzy logic:*

Step 1: Let power fuzzy graph $G_f^{\,k} = (V, E_k, \theta, \omega_k)$ with n vertices and corresponding membership values $\theta(v_i)$ and $\omega_k(v_i, v_j)$ such that $v_i \neq v_j$ and $k < k_m$.

Step 2: Generate a public key k_p such that $k < k_p \geq k_m$, where $G_f^{\,k_p} = (V, E_{k_p}, \theta, \omega_{k_p})$ is (k_p, p) - partly edge regular.

Step 3: Let the server private/secret key be the $deg_{G_f^{\,k_p}}(v_i) \forall v_i \in V$.

Step 4: Encrypting the plain text, $deg_{G_f^{\,k_p}}(v_i, v_j) = p$ of $G_f^{\,k_p}$. The encrypted message generated is $size = \frac{e \times p}{4}$.

Step 5: Using encrypted message $size = \sum_{(v_1, v_2) \in V \times V} \omega_{k_p}(v_i, v_j)$, public key k_p, and secret key $deg_{G_f^{\,k_p}}(v_i)$, the plain text p is decrypted.

The diagram in Figure 5 illustrates the process of this asymmetric encryption.

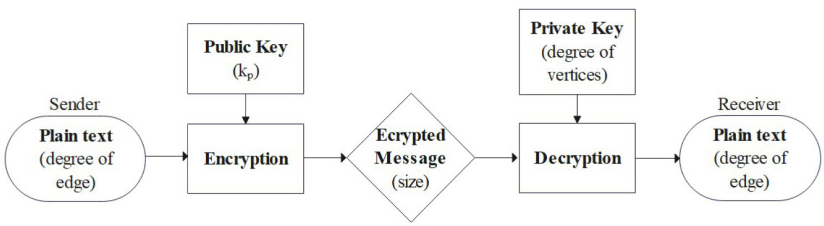

Figure 5

Block diagram of asymmetric encryption

5.2 *Illustration for the asymmetric algorithm*

Step 1: Let $G_f^{\,1} = (V, E_1, \theta, \omega_1)$ be the PFG with membership values in the following Tables 1 and 2.

Table 1

Vertex membership values of $G_f^{\,1}$

Vertices	Membership values
v_1	$\theta(v_1) = 0.5$
v_2	$\theta(v_2) = 0.6$
v_3	$\theta(v_3) = 0.7$
v_4	$\theta(v_4) = 0.5$

Step 2: The public key is $k_p = 3,$ such that PFG $G_f^{\,3} = (V,\ E_3,\ \theta,\ \omega_3)$ is partly edge regular with the vertex membership values in Table 1 and edge membership values in the following Table .

Table 2

Edge membership values of $G_f^{\,1}$

Edge	Interval	Edge membership value
(v_1, v_2)	$[0, 0.5]$	$\omega_1(v_1, v_2) = 0.5$
(v_2, v_3)	$[0, 0.6]$	$\omega_1(v_2, v_3) = 0.4$
(v_3, v_4)	$[0, 0.5]$	$\omega_1(v_3, v_4) = 0.3$

Table 3

Edge membership values of $G_f^{\,3}$

Edge	Interval	Edge membership value
(v_1, v_2)	$[0.125, 0.25]$	$\omega_3(v_1, v_2) = 0.24$
(v_2, v_3)	$[0.216, 0.36]$	$\omega_3(v_2, v_3) = 0.24$
(v_3, v_4)	$[0.125, 0.25]$	$\omega_3(v_3, v_4) = 0.23$
(v_1, v_3)	$[0.125, 0.25]$	$\omega_3(v_1, v_3) = 0.22$
(v_2, v_4)	$[0.125, 0.25]$	$\omega_3(v_2, v_4) = 0.25$
(v_1, v_4)	$[0.125, 0.25]$	$\omega_3(v_1, v_4) = 0.23$

Step 3: The private key $deg_{G_f^{k_p}}(v_i),\ \forall v_i \in V$ is generated using the edge membership values from Table 3.

$$deg_{G_f}(v_1) = 0.69$$
$$deg_{G_f}(v_2) = 0.73$$
$$deg_{G_f}(v_3) = 0.69$$
$$deg_{G_f}(v_4) = 0.71$$

Step 4: Using public key $k_p = 3$ we get number of edges $e_k = 6,$ and the plain text $p = 0.94$ the message $size = \frac{6\times 0.94}{4} = 1.41$ is encrypted.

Step 5: Using private key $deg_{G_f^{k_p}}(v_i),$ public key $k_p = 3,$ and the encrypted message $size$ = 1.41 = 0.24 + 0.24 + 0.23 + 0.22 + 0.25 + 0.23, we find $deg_{G_f^3}(v_i, v_j) = deg_{G_f^3}(v_i) + deg_{G_f^3}(v_j) - 2\omega_3(v_i, v_j),\ \forall (v_i, v_j) \in E_3$. Therefore, we get $deg_{G_f^3}(v_1, v_2) = deg_{G_f^3}(v_2, v_3) = deg_{G_f^3}(v_3, v_4) = deg_{G_f^3}(v_1, v_3) = deg_{G_f^3}(v_2, v_4) = deg_{G_f^3}(v_1, v_4) = 0.94$. Hence the plain text is $p = 0.94$.

6. Conclusion

An edge regular PFG's size is given using a standard formula. It is proved that the total power fuzzy graph is totally regular if the edge membership function is constant. Similarly, the PFG $G_f^{\,1}$ of a star graph is edge regular if and only if the edge membership function is constant. In this study a better key exchange system based on PFG is presented. This system uses the edge regular property of PFG to securely share the data or information. This study can be further extended to analyse irregular properties and adjacency sequences of PFGs.

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