

Union of subgraphs with respect to some ideals on near rings

Awrad A. Al-Alawani *

Ameer A. J. Al-Swidi †

Department of Mathematics

College of Education for Pure Science

University of Babylon

Babylon 51002

Iraq

Abstract

In our work, the new concept of union in certain graphs: $W_i(N)$, $AP_i(N)$, $\Gamma_i(N)$ and $PG_i(N)$ are dealt, which depends on the case of the intersection of edges in the subgraphs in which it equals an empty set, as well as a study of the case of homomorphism, where we reached many new theorems and Results that can be generalized to other graphs or algebra graph theory.

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1. Introduction

The first who consider the concept of the mathematical system: near ring is Pilz[1] and Groenwald[2] introduced completely prime ideal (simply, CPI) of a near ring N while Al-swidi [3] defined weakly completely ideal when is not equal zero in near ring and introduced [4] several ideals like almost v -prime ideals ($v=1,2,3$) in near ring.

In 1988, Beck [5] determined the idea of the zero divisor graph $G(R)$ associated with a commutative ring R . In this graph, the vertices represent the elements of the ring R , and two distinct vertices x and y are connected by only one edge if the product of them is zero in R . [5] Beck investigated various

† ORCID-ID: 0000-0002-8560-2072

* E-mail: awrad.alalawani@gmail.com (Corresponding Author)

† E-mail: ameer.alswidi@yahoo.com

properties and coloring aspects of this zero-divisor graph. Later, in 2010, Satyanarayana [6] defined a new graph structure called the prime graph $PG(R)$ of a ring R , not necessarily commutative. In the prime graph $PG(R)$, the vertices correspond to all the ring's elements, and any two different vertices (x and y) are connected by one edge iff $xRy = 0$ or $yRx = 0$, where R represents the ring operations. [6] Satyanarayana and his colleagues studied several properties and relationships associated with this prime graph $PG(R)$.

In 2023, Al-swidi [7] and colleagues introduced a novel graph concept called the WCPG, for a near ring N , denoted as $W_I(N)$. This graph is constructed using a non-zero ideal I of the near ring N as its foundation. The vertices of $W_I(N)$ correspond to elements of N , with two different vertices a and b forming an edge iff their product (in either order) yields a non-zero element in I . Additionally, they defined an almost 1-prime graph of N , represented as $AP_{\mathcal{A}}(N)$, where $\mathcal{A}$ is not an idempotent ideal of N . In this graph, vertices are again elements of N , but adjacency between distinct vertices a and b occurs when their product belongs to the set difference $\mathcal{A} \setminus \mathcal{A}^2$. These graph concepts can be further extended to other graph types and related mathematical notions [8-12] and [19-32].

2. Preliminary

Pilz [14] and Kandasamy [13] determined the idea of the radical for an ideal in N . Specifically, they defined the radical of an ideal I in N as the set of elements m in N such that m^n is contained in I for some positive integer n . [14,13]. Furthermore, Pilz [14] and others have defined an ideal I of N to be a completely prime (c-prime) ideal, if for any elements n and m in N , whenever their product nm is contained in I , then either n is an element of I or m is an element of I . [1,14] This property of completely prime ideals (CPI) is a stronger condition than the standard concept of a prime ideal in N [1].

The ideal $I \subseteq N$ is called a 3-prime ideal if for any x and y in N , whenever the product xNy is contained in I , then either x is in I or y is in I [14]. Furthermore, a non-trivial N is called a symmetric N if the product of any element in N with the additive identity 0 is equal to 0 [15].

Based on the references provided [16,17], a graph G is defined as a mathematical model consisting of two sets: the vertex set V and the edge set E . Each edge in the edge set E is represented by an unordered pair (x, y) of vertices from V .

The degree of $v \in G$ is the edges number's that are incident on that vertex v . The maximum degree among all vertices in the graph G is denoted by the symbol Δ . A subset W of the vertex set V is considered a clique if every distinct pair of vertices within W are adjacent, meaning they are connected by an edge [16-18].

Furthermore, if there is a path (one edge or more) between any two different vertices a and b belong to G , then G is a connected graph. Conversely, if such a path does not exist between any two vertices a and b , then the graph G is a disconnected [16-18].

3. Main Results

This part introduces the concept of union with special case and generalized it with some graphs.

Definition 3.1: A graph $G = (V, E)$ is said to be joining edge if

- $V(G_1) \cup V(G_2) = V(G)$
- $E(G_1) \cup E(G_2) = E(G)$ with $E(G_1)$ and $E(G_2)$ are disjoint

Whenever G_1 and G_2 are subgraphs of G . Then we denoted as $G = G_1 \oplus G_2$

Definition 3.2: Let the ideal I of N and S_I be a subgraph of $W_I(N)$, $AP_I(N)$, $\Gamma_I(N)$ and $PG_I(N)$ with each vertices represent the elements of I is adjacent with the subgraph $N \setminus I$ is not clique.

Example 3.3: assume that ideal $I = \{0, 4\}$ of the R -module 8, then $W_I(J)$ are illustrate in down Figure 1.1.

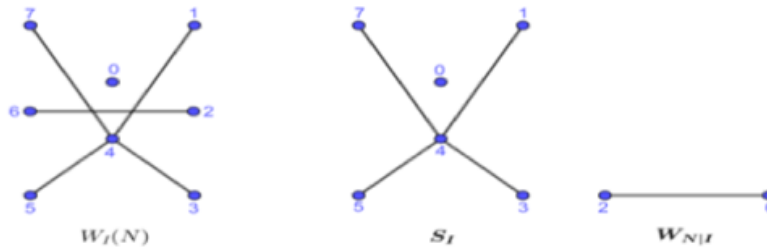


Figure 1.1
Lusion subgraphs of $W_I(N)$.

Theorem 3.4: Let ideal $I \subseteq J$, then $W_I(J) = S_I \oplus W_{N \setminus I}(J)$

Proof: As $V(S_I) \cup V(W_{N \setminus I}(J)) = N = V(W_I(J))$. Let $\overline{xy} \in E(W_I(J))$. Then, $0 \neq x, y \in I$ or $0 \neq y, x \in I$. Suppose $x, y \in I$. Then we have two cases:

Case1: If $x \in I$, or $y \in I$. Using proposition 3.2.5 in Ameer Al-swidi [8], we get x have a maximum degree or y have a maximum degree at least n , $\forall x \in I \setminus \{0\}$ or $y \in I \setminus \{0\}$, where $|N|=n$. Therefore $\overline{xy} \in E(S_I)$. Note that $\overline{xy} \notin E(W_{N \setminus I}(N))$.

Case 2: If $x \notin I$ or $y \notin I$. Then $(xy) \in E(W_{N \setminus I}(N))$. Using above definition $\overline{xy} \notin E(S_I)$. Therefore, from cases 1 and 2 implies that $E(W_I(N)) \subseteq E(S_I) \cup E(W_{N \setminus I}(N))$, With $E(S_I)$ and $E(W_{N \setminus I}(N))$ are disjoint. Now, as S_I and $W_{N \setminus I}(N)$ are subgraphs of $W_I(N)$. Then we get that, $E(S_I) \cup E(W_{N \setminus I}(N)) \subseteq E(W_I(N))$. So that, $E(W_I(N)) = E(S_I) \oplus E(W_{N \setminus I}(N))$. And similar proof if $0 \neq y, x \in I$.

Corollary 3.5: Let I be a weakly CPI of J , then $W_I(J) = S_I \oplus W_{N \setminus I}(J)$.

Proof: As I be a weakly CPI. Then directly from theorem3.4 (case (1)). We get the result.

Corollary 3.6: Assume that I be a W -semi CPI of N , Then $W_I(J) = S_I \oplus W_{N \setminus I}(J)$.

Proof: Directly from theorem 3.4 (case (1)). We get the result. $\square$

Corollary 3.7: Let I be an ideal of N_1 with $\gamma: N_1 \rightarrow N_2$ be epimorphism then $W_{\gamma(I)}(N_2) = S_{\gamma(I)} \oplus W_{N_2 \setminus \gamma(I)}(N_2)$

Proof: As $\gamma(I)$ is an ideal of N_2 then directly from theorem 3.4 (case(1)). We get the result.

Example 3.8: Let $N_1 = Z_6$ to $N_2 = Z_6 \times Z_2$ such that $\gamma: Z_6 \rightarrow Z_6 \times Z_2$, and $\gamma(a) = (0, a)$ be epimorphism and let $I_1 = \{0, 3\} \subseteq N_1$, then $I_2 = \{(0, 0), (0, 1)\} \subseteq N_2$, then $W_{I_2}(N_2)$ illustrates in Figure 1.2.

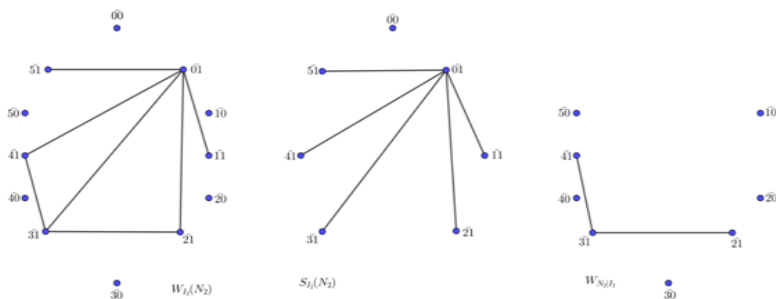


Figure 1.2
Union subgraphs of $W_i(N)$.

3.9 Theorem. Let $\mathcal{A}$ be an ideal of N . Then $AP_{\mathcal{A}}(N) = S_{\mathcal{A}} \oplus AP_{N \setminus \mathcal{A}}(N)$.

Proof: As $V(S_{\mathcal{A}}) \cup V(AP_{N \setminus \mathcal{A}}(N)) = N = V(AP_{\mathcal{A}}(N))$. Let $\overline{xy} \in E(AP_{\mathcal{A}}(N))$ then $x, y \in \mathcal{A} \setminus \mathcal{A}^2$ or $y, x \in \mathcal{A} \setminus \mathcal{A}^2$. Suppose $x, y \in \mathcal{A} \setminus \mathcal{A}^2$ so $x, y \in \mathcal{A}$. Then two cases below are valid:

Case 1: If $x \in \mathcal{A}$ or $y \in \mathcal{A}$. By proposition 3.3.5 in Ameer Al-swidi [8], we get x have a maximum degree or y its less than n ($< n$), $\forall x \in \mathcal{A} \setminus \{0\}$ or $y \in \mathcal{A} \setminus \{0\}$, where $|N| = n$. Therefore $(\overline{xy}) \in E(S_{\mathcal{A}})$. Note that $(\overline{xy}) \notin E(AP_{N \setminus \mathcal{A}}(N))$.

Case 2: If $x \notin \mathcal{A}$ and $y \notin \mathcal{A}$. Then $(\overline{xy}) \in E(AP_{N \setminus \mathcal{A}}(N))$. By above definition, $(\overline{xy}) \notin E(S_{\mathcal{A}})$.

Case-1 & case-2 implies that $E(AP_{\mathcal{A}}(N)) \subseteq E(S_{\mathcal{A}}) \cup E(AP_{N \setminus \mathcal{A}}(N))$, with $E(S_{\mathcal{A}})$ and $E(AP_{N \setminus \mathcal{A}}(N))$ are disjoint. Now, as $S_{\mathcal{A}}$ and $AP_{N \setminus \mathcal{A}}(N)$ are subgraphs of $AP_{\mathcal{A}}(N)$. Then we get that, $E(S_{\mathcal{A}}) \cup E(AP_{N \setminus \mathcal{A}}(N)) \subseteq E(AP_{\mathcal{A}}(N))$. So that, $E(AP_{\mathcal{A}}(N)) = E(S_{\mathcal{A}}) \oplus E(AP_{N \setminus \mathcal{A}}(N))$. And similar proof if $y, x \in \mathcal{A}^2$.

Corollary 3.10: Let $\mathcal{A}$ be an almost 1- prim ideal of N . Then $AP_{\mathcal{A}}(N) = S_{\mathcal{A}} \oplus AP_{N \setminus \mathcal{A}}(N)$.

Proof: As $\mathcal{A}$ be almost 1- prime ideal of N then directly from theorem3.9 (case (1)). We get the result.

Corollary 3.11: Let $\mathcal{A}$ be an almost 1- semi prim ideal of N . Then $AP_{\mathcal{A}}(N) = S_{\mathcal{A}} \oplus AP_{N \setminus \mathcal{A}}(N)$. Where $\mathcal{A}$ is not idempotent ideal.

Proof: Directly from theorem3.9 (case (1)). We get the result.

Corollary 3.12: Let $\mathcal{A}$ be an ideal of N with $\gamma : N_1 \rightarrow N_2$ be epimorphism. Then $AP_{\gamma(\mathcal{A})}(N_2) = S_{\gamma(\mathcal{A})} \oplus AP_{N_2 \setminus \gamma(\mathcal{A})}(N_2)$.

Proof: As $\gamma(\mathcal{A})$ is an ideal of N_2 . Then from theorem3.9 (case (1)). We get the result.

Corollary 3.13: Let $\mathcal{A}$ be an almost 2- prime ideal of N . Then $AP_{\mathcal{A}}(N) = S_{\mathcal{A}} \oplus AP_{N \setminus \mathcal{A}}(N)$. Where $\mathcal{A}$ is not idempotent ideal and every $a \in \mathcal{A}$ is idempotent element.

Proof: Directly from theorem3.9 (case (1)). We get the result. $\square$

Corollary 3.14: Let $\mathcal{A}$ be an almost 2- semi prime ideal of N . Then $AP_{\mathcal{A}}(N) = S_{\mathcal{A}} \oplus AP_{N \setminus \mathcal{A}}(N)$.

Proof: Directly from theorem3.9 (case (1)).

Theorem 3.15: For a simple near ring N , then $\Gamma_{I=\{0\}}(J) = S_{I=\{0\}}(J) \oplus \Gamma_{N \setminus I=\{0\}}(J)$.

Proof: As $V(S_{I=\{0\}}(N)) \cup V(\Gamma_{N \setminus I=\{0\}}(N)) = N = V(\Gamma_{I=\{0\}}(N))$. Let $\overline{xy} \in E(\Gamma_{I=\{0\}}(N))$. Then $x.y=0$ or $y.x=0$

Let us $x.y=0$, then we have two cases:

Case 1: If $x=0$ or $y=0$, then we have already $\overline{xy} \in E(S_{I=\{0\}})$ with $\overline{xy} \notin E(\Gamma_{N \setminus I=\{0\}}(N))$.

Case 2: If $x \neq 0$ and $y \neq 0$. Then $\overline{xy} \in E(\Gamma_{N \setminus I=\{0\}}(N))$. By previous definition. $\overline{xy} \notin E(S_{N \setminus I=\{0\}})$.

Case-1 & case-2 implies that $E(\Gamma_{I=\{0\}}(N)) \subseteq E(S_{I=\{0\}}) \cup E(\Gamma_{N \setminus I=\{0\}}(N))$ with $E(S_{I=\{0\}})$ and $E(\Gamma_{N \setminus I=\{0\}}(N))$. are disjoint and similar proof if $y.x=0$.

Corollary 3.16: Assume that $I = \{0\}$ be a CPI of N . Then $\Gamma_{I=\{0\}}(N) = S_{I=\{0\}} \oplus \Gamma_{N \setminus I=\{0\}}(N)$

Proof: Directly from theorem3.15 (case (1)). We get the result. $\square$

Corollary 3.17: Let $\gamma : N_1 \rightarrow N_2$ be epimorphism of a simple near rings N_1 and N_2 . Then $\Gamma_{\gamma(I)=\{0\}}(N_2) = S_{\gamma(I)=\{0\}} \oplus \Gamma_{N_2 \setminus \gamma(\mathcal{A})}(N_2)$

Proof: As $\gamma(\mathcal{A})$ is an ideal of N_2 . Then from theorem3.15(case (1)). We get the result.

Theorem 3.18: If N be a simple zero symmetric N . Then $PG_{I=\{0\}}(N) = S_{I=\{0\}} \oplus PG_{N \setminus I=\{0\}}(N)$

Proof: As $V(PG_{I=\{0\}}(N)) \cup V(PG_{N \setminus I=\{0\}}(N)) = N = V(PG_{I=\{0\}}(N))$. Let $\overline{xy} \in E(PG_{I=\{0\}}(N))$. Then $x.N.y = 0$ or $y.N.x = 0$. Suppose $x.N.y = 0$ then we have two possibilities:

Case 1: If $x = 0$ or $y = 0$, as N is zero symmetric near ring. Then $\overline{xy} \in E(PG_{I=\{0\}}(N))$, with $\overline{xy} \notin E(PG_{N \setminus I=\{0\}}(N))$.

Case 2: If $x \neq 0$ and $y \neq 0$, Then it's impossible because N is zero symmetric near ring. Therefore, from case1 and case2 implies that $\overline{xy} \notin E(S_{I=\{0\}})$. with $\overline{xy} \in E(PG_{N \setminus I=\{0\}}(N))$.

Therefore, from two cases above 1 and 2, implies that $E(PG_{I=\{0\}}(N)) \subseteq E(S_{I=\{0\}}(N)) \cup E(PG_{N \setminus I=\{0\}}(N))$ with $E(S_{I=\{0\}})$ and $E(PG_{N \setminus I=\{0\}}(N))$ are disjoint. And similar proof if $y.N.x = 0$. $\square$

Corollary 3.19: Let $I = \{0\}$ be a 3-prime ideal of N . Then $PG_{I=\{0\}}(N) = S_{I=\{0\}} \oplus PG_{N \setminus I=\{0\}}(N)$

Proof: Directly from theorem3.18 (case (1)). We get the result. $\square$

Corollary 3.20: Assume that $\gamma : N_1 \rightarrow N_2$ be epimorphism of a simple zero symmetric near rings N_1 and N_2 . Then $PG_{I=\{0\}}(N_2) = S_{\gamma(I)=\{0\}} \oplus PG_{N_2 \setminus \gamma(\mathcal{A})}(N_2)$

Proof: As $\gamma(I)$ is an ideal of N_2 . Then from theorem3.18(case (1)). We get the result.

Theorem 3.21: For an ideal $I \neq \{0\} \subseteq N$, then $W_{rad(I)}(N) = S_{rad(I)} \oplus W_{N \setminus rad(I)}(N)$.

Proof: As $V(S_{rad(I)}) \cup V(W_{N \setminus rad(I)}(N)) = N = V(W_{rad(I)}(N))$. As $I \subseteq rad(I)$, let $\overline{xy} \in E(W_{rad(I)}(N))$. Then $0 \neq x.y \in I \subseteq rad(I)$ or $0 \neq y.x \in I \subseteq rad(I)$. And in any line we take, we have two cases and can proved it same cases in theorem3.4. $\square$

Corollary 3.22: Let I be an ideal of N_1 with $\gamma : N_1 \rightarrow N_2$ be epimorphism. Then $W_{\gamma(\text{rad}(I))}(N_2) = S_{\gamma(\text{rad}(I))} \oplus \Gamma_{N_2 \setminus \gamma(\text{rad}(I))}(N_2)$

Proof: As $I \subseteq \text{rad}(I)$, we proved it directly from theorem 3.4.

Theorem 3.23: Let $\mathcal{A}$ be an ideal of N . Then $AP_{\text{rad}(\mathcal{A})}(N) = S_{\text{rad}(\mathcal{A})} \oplus AP_{N \setminus \text{rad}(\mathcal{A})}(N)$, whenever $\mathcal{A}$ is not idempotent ideal.

Proof: Same proof theorem 3.9.

Theorem 3.24: For a simple N . Then $\Gamma_{\text{rad}(I=\{0\})}(N) = S_{\text{rad}(I=\{0\})} \oplus \Gamma_{N \setminus \text{rad}(I=\{0\})}(N)$.

Proof: Same proof theorem 3.15.

Theorem 3.25: Let N be a simple zero symmetric near ring. Then $PG_{\text{rad}(I=\{0\})}(N) = S_{\text{rad}(I=\{0\})} \oplus PG_{N \setminus \text{rad}(I=\{0\})}(N)$.

Proof: Same proof theorem 3.18.

References

- [1] G. Pilz, Near-rings: The Theory and Its Applications, North Holland (1983). doi: 10.1016/C2013-0-03739-6.
- [2] N. Deo, Graph Theory with Applications to Engineering and Computer Science, Courier Dover Publications (2017).
- [3] A. A. AL-Swidi and A. Omran, "Weakly completely prime graph," *Wasit Journal for Pure Sciences*, vol. 2, no. 1, pp. 156-163 (2023).
- [4] A. A. AL-alawani, A. A. AL-Swidi, "New types of ideals on near rings," in AIP Conference Proceedings, accepted for publication (2024).
- [5] I. Beck, "Coloring of commutative rings," *Journal of Algebra*, vol. 116, no. 1, pp. 208-226 (1988). doi: 10.1016/0021-8693(88)90202-5.
- [6] S. Bhavanari, S. Kuncham, and N. Dasari, "Prime graph of a ring," *Journal of Combinatorics, Information and System Sciences*, vol. 35, no. 1-2, pp. 1-2 (2010).

- [7] S. M. Kadham and A. N. Alkiffai, "Generalization of Fuzzy SHA-Transform with Medical Application," *Mathematical Modelling of Engineering Problems*, vol. 9, no. 5 (2022).
- [8] A. A. AL-Swidi and A. Omran, "Completely equiprime graph of a near ring," in *AIP Conference Proceedings* (2021).
- [9] A. A. AL-Swidi and A. Omran, "Uniquely colorable completely equiprime graph of a near ring," in *AIP Conference Proceedings* (2021).
- [10] A. A. AL-Swidi, E. H. AL-Saadi, and L. H. AL-Saadi, "Soft public key cipher," *Periodicals of Engineering and Natural Sciences*, vol. 7, no. 3, pp. 1433-1438 (2019).
- [11] M. A. Mustafa, S. M. Kadham, N. K. Abbass, S. Karupusamy, H. Y. Jasim, B. A. Alreda, et al., "A novel fuzzy M-transform technique for sustainable ground water level prediction," *Applied Geomatics*, vol. 16, no. 1, pp. 9-15 (2024).
- [12] A. A. AL-Swidi and A. A. Omran, "Study of some graph types via soft graph," *ARPN Journal of Engineering*, vol. 14, no. 8, pp. 10375-10379 (2019).
- [13] W. V. Kandasamy, *Smarandache Near-Rings*, Infinite Study (2002).
- [14] A. Sundud Najah Jabir, S. Maki Kadham, M. A. Ahmed Mustafa, and N. K. Abbass, "Encryption and enhance medical image using hybrid transform ($\hat{\alpha}$ -module and partial fuzzy $\hat{H}$ -transform)," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 26, no. 7, pp. 1903–1910 (2023). doi: 10.47974/JDMSC-1683.
- [15] H. E. Bell, "Near-rings in which each element is a power of itself," *Bulletin of the Australian Mathematical Society*, vol. 2, no. 3, pp. 363-368 (1970). doi: 10.1017/S0004972700041563.
- [16] J. L. Gross, J. Yellen, and M. Anderson, *Graph Theory and Its Applications*, 3rd ed., Chapman and Hall/CRC (2018). doi: 10.1201/9780429425134.
- [17] J. L. Gross, J. Yellen, and M. Anderson, *Graph Theory and Its Applications*, ACM Digital Library (2018). doi: 10.1145/3274895.

- [18] J. Huo, S. Wen, Y. Chen, and M. Li, "Neighbor distinguishing colorings of graphs with the restriction for maximum average degree," *Axioms*, vol. 12, no. 12, p. 1132 (2023). doi: 10.3390/axioms12121132.
- [19] W. Gao and M. R. Farahani, "The hyper-Zagreb index for an infinite family of nanostar dendrimer," *J. Discrete Math. Sci. Cryptography*, vol. 20, no. 2, pp. 515-523 (2017). doi: 10.1080/09720529.2016.1220088.
- [20] W. Gao, L. Shi, and M. R. Farahani, "Szeged related indices of $TUAC6[p, q]$," *J. Discrete Math. Sci. Cryptography* vol. 20, no. 2, pp. 553-563 (2017). doi: 10.1080/09720529.2016.1228312.
- [21] H. Yang and X. Zhang, "The independent domination numbers of strong product of two cycles," *J. Discrete Math. Sci. Cryptography*, vol. 21, no. 7-8, pp. 1495-1507 (2018). doi: 10.1080/09720529.2017.1316988.
- [22] S. Ediz, İ. Çiftçi, M. Cancan, and M. R. Farahani, "On k-total distance degrees and k-total Wiener polarity index," *J. Inform. Optim. Sci.*, vol. 42, no. 7, pp. 1469-1477 (2021). doi: 10.1080/02522667.2021.1896652.
- [23] F. Afzal, Shah Rukh, D. Afzal, M. R. Farahani, M. Cancan, and S. Ediz, "A study of non-commutative Von-Neumann regular rings," *J. Inform. Optim. Sci.*, vol. 42, no. 7, pp. 1425-1436 (2021). doi: 10.1080/02522667.2021.1896650.
- [24] M. S. Sardar, M. Alaeiyan, M. R. Farahani, M. Cancan, and S. Ediz, "Resistance distance in some classes of rooted product graphs obtained by Laplacian generalized inverse method," *J. Inform. Optim. Sci.*, vol. 42, no. 7, pp. 1447-1467 (2021). doi: 10.1080/02522667.2021.1899210.
- [25] M. Nadeem, S. Ahmad, M. K. Siddiqui, M. A. Ali, M. R. Farahani, and A. J. M. Khalaf, "On some applications related with algebraic structures through different well known graphs," *J. Discrete Math. Sci. Cryptography*, vol. 42, no. 2, pp. 451-471 (2021). doi: 10.1080/09720529.2021.1885806.
- [26] M. R. Farahani, S. Jafari, S. A. Mohiuddine, and M. Cancan, "Intuitionistic fuzzy stability of generalized additive set-valued functional equation via fixed point method," *Mathematical Statistician and Engineering Applications*, vol. 71, no. 3s3, pp. 142-154 (2022). [Online].

Available: <https://www.philstat.org.ph/index.php/MSEA/article/view/355>.

- [27] R. F. Kadam and K. M. Mohammed Al-Abrahemee, "Neuro-fuzzy system for solving fuzzy singular perturbation problems," *J. Interdiscip. Math.*, vol. 25, no. 5, pp. 1509-1524 (2022). doi: 10.1080/09720502.2022.2079229.
- [28] D. E. Abdulrasool and A. N. Alkiffai, "Population growth equation by fuzzy common integral transforms," *J. Interdiscip. Math.*, vol. 25, no. 6, pp. 1909-1918 (2022). doi: 10.1080/09720502.2022.2095960.
- [29] A. H. Alwan, "Small intersection graph of subsemimodules of a semimodule," *Commun. Combin. Cryptogr. Comput. Sci.*, vol. 2022, no. 1, pp. 15-22 (2022).
- [30] S. Ediz, M. Cancan, and M. R. Farahani, "Eccentric connectivity and connective eccentricity indices of generalized Petersen graphs," *Commun. Combin. Cryptogr. Comput. Sci.*, vol. 2021, no. 1, pp. 1-6 (2021).
- [31] R. Ponraj, S. Prabhu, and M. Sivakumar, "Pair mean cordial labeling of total graph of some graphs and prism related graphs," *Commun. Combin. Cryptogr. Comput. Sci.*, vol. 2024, no. 2, pp. 181-192 (2024).
- [32] F. O. Oduol and I. O. Okoth, "Counting vertices among all noncrossing trees by levels and degrees," *Commun. Combin. Cryptogr. Comput. Sci.*, vol. 2024, no. 2, pp. 193-212 (2024).

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