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The sequence of the complex terms in the state (9,6,3)

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Abstract

The sequence of the complex terms in the state (9,6,3) is complex are proved in this work by using sketch, place polarization, Capelli identities and the injective of maps. In this work $\mathcal{R}$ is a commutative ring has 1, $\mathcal{F}$ is a free $\mathcal{R}$-module and $\mathcal{D}_i\,\mathcal{F}$ be the divided power algebra of degree **i**.

Subject Classification (2020): *13D07.*

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1. Introduction

$\mathcal{K}_{\lambda/\mu}\mathcal{F} = \mathrm{Im}(d'_{\lambda/\mu})$, for the partition λ/μ, where the Weyl map is $d'_{\lambda/\mu}: \mathcal{DF} \to \wedge\mathcal{F}$ (exterior algebra of finitely generated free $\mathcal{R}$-module $\mathcal{F}$), [1, 2]. The maps $\sum_{k>0} \mathcal{D}_{p+t_1+k} \otimes \mathcal{D}_{q-t_1-k} \otimes \mathcal{D}_r \to \mathcal{D}_p \otimes \mathcal{D}_q \otimes \mathcal{D}_r$ is k^{th} divided power of the place polarization from place 1 to place 2 ($\partial_{21}^{(k)}$), and the maps $\sum_{e>0} \mathcal{D}_p \otimes \mathcal{D}_{q+t_2+e} \otimes \mathcal{D}_{r-t_2-e} \to \mathcal{D}_p \otimes \mathcal{D}_q \otimes \mathcal{D}_r$ explicated as e^{th} divided power of the place polarization from place 2 to place 3 ($\partial_{32}^{(e)}$), and as in two-rowed case, where the resolution of the Weyl module with the three-rowed $(p, q, r; t_1, t_2)$, [2,3]. We need these Capelli identities

$$\partial_{32}^{(2)}\,\partial_{21} = \partial_{21}\,\partial_{32}^{(2)} + \partial_{32}\,\partial_{31} \qquad \ldots(1)$$

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$$\partial_{32}\,\partial_{21}^{(2)} = \partial_{21}^{(2)}\,\partial_{32} + \partial_{21}\,\partial_{31} \qquad \ldots(2)$$

$$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31} \qquad \ldots(3)$$

$$\partial_{32}\,\partial_{31} = \partial_{31}\,\partial_{32} \qquad \ldots(4)$$

The partitions (8,8)/(1,0), (6,5)/(1,0), (6,5)/(2,0), (6,6,4;0,0) and (7,7,4;0,0) studied respectively in [4-7]. We applied the same idea in [8] for the partition (9,6,3). Also we can apply that for $\mathcal{PSL}(2,\mathfrak{U})$, and $\mathcal{SL}(2,\mathfrak{U})$ where $\mathfrak{U}$ = 23, 29, 31, 37, [9-12], hyperbolic univalent function, $\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U} = 49, 81, 729$, $\mathcal{S}.\mathcal{U}.\mathcal{T}.(2,p)$, for a prime $p \leq 9$ [13-16]. Also for A(G24), A(G27), $\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U}$ = 125, 3125, 121, 169 and PG (2,8) which are study by authors in [17-20].

2. The complexness of the sequence

For the state (9,6,3)

$$\begin{array}{c} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \quad \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \to \oplus \to \oplus \to \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \quad \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{array}$$

See the sketch

$$\begin{tikzcd}
\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "d_1"] \arrow[d, "h_1"'] \arrow[dr, phantom, "Q"] & \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "d_2"] \arrow[d, "h_2"] \arrow[dr, phantom, "T"] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "h_3"] \\
\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "g_2"'] & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "g_2"'] & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
\end{tikzcd}$$

Acquaint the above maps as $d_1(v) = \partial_{32}(v)$, $h_1(v) = \partial_{21}(v)$, $h_2(v) = \partial_{21}^{(2)}(v)$.

We reign to acquaint the map g_1 and by (1) make the sketch Q commutative. Thus

$$g_1 \circ \partial_{21} = \partial_{21}^{(2)} \circ \partial_{32} = \partial_{32} \circ \partial_{21}^{(2)} - \partial_{31} \circ \partial_{21} = (\tfrac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31}) \circ \partial_{21}$$

Hence, $g_1 = \frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31}$

If we acquaint the map $g_2(v) = \partial_{32}(v)$ and $h_3(v) = \partial_{21}(v)$. We require acquainting d_2 and by (2) to get the sketch T commute. Thus $h_3 \circ d_2 = g_2 \circ h_2$.

$$\partial_{21} \circ d_2 = \partial_{32} \circ \partial_{21}^{(2)} = \partial_{21}^{(2)} \circ \partial_{32} + \partial_{21} \circ \partial_{31} = \partial_{21} \circ (\tfrac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31}).$$

Hence, $d_2 = \frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31}$.

Take the pursue sketch and acquaint $w(v) = \partial_{32}^{(2)}(v)$

$$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{d_1} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{d_2} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

$$\begin{tikzcd}[column sep=small] {} \arrow[d, "\hbar_1"'] & {} \arrow[rrr, "\mathcal{w}"] & {} \arrow[d, "\hbar_2"] & & {} & {} \arrow[d, "\hbar_3"] \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[rr, "\mathcal{g}_2"'] \arrow[rr, phantom, "\mathcal{H}", shift left=6] & & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[rrr, "\mathcal{g}_2"'] \arrow[rrr, phantom, "\mathcal{N}", shift left=6] & & & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{tikzcd}$$

Proposition 2.1: The sketch $\mathcal{H}$ is commutative.

Proof: $d_2 \circ d_1 = (\frac{1}{2}\partial_{21} \circ \partial_{32} + \partial_{31}) \circ \partial_{32} = \partial_{32}^{(2)} \circ \partial_{21} - \partial_{32} \circ \partial_{31} + \partial_{31} \circ \partial_{32}$

$= \partial_{32}^{(2)} \circ \partial_{21} = \mathcal{w} \circ \hbar_1$

Proposition 2.2: The sketch $\mathcal{N}$ is commutative.

Proof: $\mathcal{g}_2 \circ \mathcal{g}_1 = \partial_{32} \circ (\frac{1}{2}\partial_{32} \circ \partial_{21} - \partial_{31}) = \partial_{21} \circ \partial_{32}^{(2)} + \partial_{32} \circ \partial_{31} - \partial_{32} \circ \partial_{31}$

$= \partial_{21} \circ \partial_{32}^{(2)} = \hbar_3 \circ \mathcal{w}$

Proposition 2.3: The pursue sequence is complex

$$0 \to \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\sigma_3} \begin{matrix} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \end{matrix} \xrightarrow{\sigma_2} \begin{matrix} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_5\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\sigma_1} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Where $\sigma_3(x) = \big(d_1(x), \hbar_1(x)\big)$, $\sigma_2((x_1, x_2)) = (d_2(x_1) - \mathcal{w}(x_2), \mathcal{g}_1(x_2) - \hbar_2(x_1))$ and $\sigma_1((x_1, x_2)) = \big(\hbar_3(x_1) + \mathcal{g}_2(x_2)\big)$.

Proof: By (1)-(4) and the injective of ∂_{21} and ∂_{32} [3] we gain σ_3 is also injective

$$(\sigma_2 \circ \sigma_3)(x) = \sigma_2\big(d_1(x), \hbar_1(x)\big)$$

$$= \Big(d_2\big(\partial_{32}(x)\big) - \mathcal{w}\big(\partial_{21}(x)\big), \mathcal{g}_1\big(\partial_{21}(x)\big) - \hbar_2\big(\partial_{32}(x)\big)\Big)$$

$$d_2\big(\partial_{32}(x)\big) - \mathcal{w}\big(\partial_{21}(x)\big) = (\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31} + \partial_{31}\partial_{32} - \partial_{32}^{(2)}\partial_{21})(x) = 0$$

$$\mathcal{g}_1\big(\partial_{21}(x)\big) - \hbar_2\big(\partial_{32}(x)\big) = (\partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31} - \partial_{31}\partial_{21} - \partial_{21}^{(2)}\partial_{32})(x) = 0$$

We reign $(\sigma_2 \circ \sigma_3)(x) = 0$, and $(\sigma_1 \circ \sigma_2)(x_1, x_2) = \big(\partial_{32}\partial_{21}^{(2)} - \partial_{21}\partial_{31} + \partial_{21}\partial_{31} - \partial_{32}\partial_{21}^{(2)}\big)(x_1) + \big(\partial_{32}^{(2)}\partial_{21} + \partial_{32}\partial_{31} - \partial_{32}\partial_{31} - \partial_{21}\partial_{32}^{(2)}\big)(x_2) = 0.$ So the sequence is complex.

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