

The resolution of $(13,10)/(1,1)$

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Abstract

This research study the application of Weyl module and gain the elements of characteristic free for $(13,10)/(1,1)$ by using the polarization and latter place-algebra with capelli-identities and contracting homological. Through this work R is a commutative ring with identity 1 and F be a free R -module with D_b the divided power of degree b .

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1. Introduction

Author in an earlier work use of letter place algebra to the resolution for $K_{\lambda/\mu}F$ in [1], [2].

As in [3], [4] we compute the elements of characteristic free for $(13,10)/(1,1)$ and we can apply the same idea for $\mathcal{SL}(2, \mathfrak{U})$ where $\mathfrak{U} = 81, 729$,

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125,3125,121, 196, [5-7], also the ideas in [8-11], Mxn-open sets and LC-Spaces, [12-15] and the idea in [16-22].

2. The elements of the case of skew-partition (13,10)/(1,1)

The elements for the skew-partition (13,10)/(1,1) as follows

$$M_0 = D_{12} \otimes D_9$$

$$M_1 = Z_{21}^{(2)} \kappa D_{14} \otimes D_7 \oplus Z_{21}^{(3)} \kappa D_{15} \otimes D_6 \oplus Z_{21}^{(4)} \kappa D_{16} \otimes D_5 \oplus Z_{21}^{(5)} \kappa D_{17} \otimes D_4$$

$$\oplus Z_{21}^{(6)} \kappa D_{18} \otimes D_3 \oplus Z_{21}^{(7)} \kappa D_{19} \otimes D_2 \oplus Z_{21}^{(8)} \kappa D_{20} \otimes D_1 \oplus Z_{21}^{(9)} \kappa D_{21} \otimes D_0$$

$$M_2 = \bigoplus_{|b|=2}^8 Z_{21}^{(1+b_1)} x Z_{21}^{(b_2)} x D_{12+1+|b|} \otimes D_{9-1-|b|} \text{ where } |b| = b_1 + b_2 \text{ and } 2 \leq |b| \leq 8.$$

$$M_3 = \bigoplus_{|b|=3}^8 Z_{21}^{(1+b_1)} x Z_{21}^{(b_2)} x Z_{21}^{(b_3)} x D_{12+1+|b|} \otimes D_{9-1-|b|}, \text{ where}$$

$$|b| = b_1 + b_2 + b_3 \text{ and } 3 \leq |b| \leq 8.$$

$$M_4 = \bigoplus_{|b|=4}^8 Z_{21}^{(1+b_1)} x Z_{21}^{(b_2)} x Z_{21}^{(b_3)} x Z_{21}^{(b_4)} x D_{12+1+|b|} \otimes D_{9-1-|b|}, \text{ where}$$

$$|b| = b_1 + b_2 + b_3 + b_4 \text{ and } 4 \leq |b| \leq 8.$$

$$M_5 = \bigoplus_{|b|=5}^8 Z_{21}^{(1+b_1)} x Z_{21}^{(b_2)} x Z_{21}^{(b_3)} x Z_{21}^{(b_4)} x Z_{21}^{(b_5)} x D_{12+1+|b|} \otimes D_{9-1-|b|}, \text{ where } |b| =$$

$$b_1 + b_2 + b_3 + b_4 + b_5 \text{ and } 5 \leq |b| \leq 8.$$

$$M_6 = \bigoplus_{|b|=6}^8 Z_{21}^{(1+b_1)} x Z_{21}^{(b_2)} x Z_{21}^{(b_3)} x Z_{21}^{(b_4)} x Z_{21}^{(b_5)} x Z_{21}^{(b_6)} x D_{12+1+|b|} \otimes D_{9-1-|b|},$$

$$\text{where } |b| = b_1 + b_2 + b_3 + b_4 + b_5 + b_6 \text{ and } 6 \leq |b| \leq 8.$$

$$M_7 = \bigoplus_{|b|=7}^8 Z_{21}^{(1+b_1)} x Z_{21}^{(b_2)} x Z_{21}^{(b_3)} x Z_{21}^{(b_4)} x Z_{21}^{(b_5)} x Z_{21}^{(b_6)} x Z_{21}^{(b_7)} x D_{12+1+|b|} \otimes$$

$$D_{9-1-|b|}, \text{ where } |b| = b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 \text{ and } 7 \leq |b| \leq 8.$$

3. The contracting homotopy of the homological mapping

The homological mapping $S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7$ in the case of skew-partition (13,10)/(1,1) as follows:

$$S_0 \left(\left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(12)} 2^{(b)} \\ 2^{(9-b)} \end{matrix} \right) \right) = \begin{cases} Z_{21}^{(b+1)} \kappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(12+b)} \\ 2^{(9-b)} \end{matrix} \right) ; \text{ if } 1 \leq b \leq 8 \\ 0 ; \text{ if } b < 1 \end{cases}$$

$$S_1 \left(Z_{21}^{(b+1)} \kappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(12+b)} 2^{(m)} \\ 2^{(9-b-m)} \end{matrix} \right) \right) =$$

$$\begin{cases} Z_{21}^{(b+1)} \kappa Z_{21}^{(m)} \kappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(12+b+m)} \\ 2^{(9-b-m)} \end{matrix} \right) ; \text{ if } m = 1, 2, \dots, 7; \text{ where } |b| = b_1 + b_2 \\ 0 ; \text{ if } m = 0 \end{cases}$$

$$S_2 \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(12+|b|)} 2^{(m)} \\ 2^{(9-|b|-m)} \end{matrix} \right) \right) =$$

$$\begin{cases} Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) ; \text{if } m = 1, 2, \dots, 6 ; \\ 0 ; \text{if } m = 0 \end{cases}$$

where $|b| = b_1 + b_2$

$$\begin{aligned} S_3 \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) = \\ \begin{cases} Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) ; \text{if } m = 1, 2, \dots, 5 ; \\ 0 ; \text{if } m = 0 \end{cases} \end{aligned}$$

where $|b| = b_1 + b_2 + b_3$

$$\begin{aligned} S_4 \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) = \\ \begin{cases} Z_{21}^{(b_1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) ; \text{if } m = 1, 2, 3, 4 ; \\ 0 ; \text{if } m = 0 \end{cases} \end{aligned}$$

where $|b| = b_1 + b_2 + b_3 + b_4$

$$\begin{aligned} S_5 \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa Z_{21}^{(b_5)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) = \\ \begin{cases} Z_{21}^{(b_1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa Z_{21}^{(b_5)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) ; \text{if } m = 1, 2, 3 ; \\ 0 ; \text{if } m = 0 \end{cases} \end{aligned}$$

where $|b| = b_1 + b_2 + b_3 + b_4 + b_5$

$$\begin{aligned} S_6 \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa Z_{21}^{(b_5)} \kappa Z_{21}^{(b_6)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) = \\ \begin{cases} Z_{21}^{(b_1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa Z_{21}^{(b_5)} \kappa Z_{21}^{(b_6)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) ; \text{if } m = 1, 2 ; \\ 0 ; \text{if } m = 0 \end{cases} \end{aligned}$$

where $|b| = b_1 + b_2 + b_3 + b_4 + b_5 + b_6$

$$\begin{aligned} S_7 \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa Z_{21}^{(b_5)} \kappa Z_{21}^{(b_6)} \kappa Z_{21}^{(b_7)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) = \\ \begin{cases} Z_{21}^{(b_1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(b_3)} \kappa Z_{21}^{(b_4)} \kappa Z_{21}^{(b_5)} \kappa Z_{21}^{(b_6)} \kappa Z_{21}^{(b_7)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) ; \text{if } m = 1 \\ 0 ; \text{if } m = 0 \end{cases} \end{aligned}$$

Where $|b| = b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7$.

Now,

$$\begin{aligned} S_0 \partial_\kappa \left(Z_{21}^{(b+1)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+b)} 2^{(m)}}{2^{(9-b-m)}} \right) \right) &= S_0 \binom{b+1+m}{m} \left(\frac{w}{w'} \middle| \frac{1^{(12)} 2^{(b+1+m)}}{2^{(9-b-m)}} \right) \\ &= \binom{b+1+m}{m} Z_{21}^{(b+1+m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+b+m)}}{2^{(9-b-m)}} \right), \text{ and} \end{aligned}$$

$$\begin{aligned} \partial_{\kappa} S_1 \left(Z_{21}^{(b+1)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+b)} 2^{(m)}}{2^{(9-b-m)}} \right) \right) &= \partial_{\kappa} \left(Z_{21}^{(b+1)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+b+m)}}{2^{(9-b-m)}} \right) \right) \\ &= - \binom{b+1+m}{m} Z_{21}^{(b+1+m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+b+m)}}{2^{(9-b-m)}} \right) + Z_{21}^{(b+1)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+b)} 2^{(m)}}{2^{(9-b-m)}} \right), \end{aligned}$$

Then $S_0 \partial_{\kappa} + \partial_{\kappa} S_1 = id_{M_1}$.

$$\begin{aligned} & S_1 \partial_{\kappa} \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) \\ = & S_1 \left(- \binom{|b|+1}{b_2} Z_{21}^{|b|+1} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) + Z_{21}^{(b_1+1)} \kappa \partial_{21} \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) \\ &= - \binom{|b|+1}{b_2} Z_{21}^{|b|+1} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) + \\ & \quad \binom{b_2+m}{m} Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2+m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right), \text{ and} \\ & \quad \partial_{\kappa} S_2 \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right) \right) = \\ & \quad \partial_{\kappa} \left(Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+b+m)}}{2^{(9-b-m)}} \right) \right) \\ &= \binom{|b|+1}{b_2} Z_{21}^{|b|+1} \kappa Z_{21}^{(m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) - \\ & \quad \binom{b_2+m}{m} Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2+m)} \kappa \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|+m)}}{2^{(9-|b|-m)}} \right) + Z_{21}^{(b_1+1)} \kappa Z_{21}^{(b_2)} \left(\frac{w}{w'} \middle| \frac{1^{(12+|b|)} 2^{(m)}}{2^{(9-|b|-m)}} \right), \end{aligned}$$

So that $S_1 \partial_{\kappa} + \partial_{\kappa} S_2 = id_{M_2}$.

By that same way, we can get that, $S_{i-1} \partial_x + \partial_x S_i = id_{M_i}$, $i = 3, 4, \dots, 7$.

Thus $\{S_i\}$; $i = 0, 1, 2, \dots, 7$ is a contracting homotopy [5], which implies the exactness of the following complex.

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