

The cyclic decomposition of the special unitary groups $SU(3,9)$ and $SU(2,25)$

Semaa Hassan Aziz [†]

Department of Computer Science Governorate

University of Technology

Baghdad 10066

Iraq

Abdulahman H. Majeed *

Department of Mathematics

Al-Mamoun University College

Baghdad 10012

Iraq

Abstract

In this work we find the cyclic decomposition for the groups $SU(3,9)$ and $SU(2,25)$ after we compute the conjugacy classes and the character table of rational representations for each group.

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1. Introduction

The group of invertible $n \times n$ matrices over a field F denoted by $GL(n, F)$. The determinant of these matrices is a homomorphism from $GL(n, F)$ into $F - \{0\}$. The special unitary group of degree n , denoted by $SU(n)$, is the Lie group of $n \times n$ unitary matrices with determinant 1, [1]. Authors in [2-5] studied the cyclic decomposition for the groups $\mathcal{PSL}(2, \mathbb{U})$ and $\mathcal{SL}(2, \mathbb{U})$, where $\mathbb{U} = 23, 29, 31, 37$.

[†] ORCID-ID: 0009-0003-9663-5093

* ORCID-ID: 0000-0001-8534-0749

[†] E-mail: semaa.h.aziz@uotechnology.edu.iq

* E-mail: dulrahman.h.majeed@almamonuc.edu.iq (Corresponding Author)

We apply the same idea in [6-8] for the groups $SU(3,9)$ and $SU(3,25)$ in this work.

2. The Basic Concepts for the groups $SU(3, q^2)$

Author in [1] studied the order of the $SU(3, q^2)$ which is equal to $\prod_{i=0}^{n-1} (q^n - q^i) / (q - 1)$, the tables of the conjugacy classes and the character table of rational representations for the groups $SU(3, q^2)$ where $d = \gcd(n, q - 1) = 1$. We employ that for our groups $SU(3,9)$ and $SU(2,25)$.

2.1 The results for the group $SU(3,9)$

$|SU(3,9)| = 5616$, $d = \gcd(3,2) = 1$, $\delta = 1$, $S = q + \delta = 3+1=4$, $r = q - \delta = 3 - 1 = 2$, $t = q^2 + \delta q + 1 = 3^2 + 3 + 1 = 13$, $t' = \frac{t}{d} = 13$, $r' = 2$, $d' = \frac{(3-d)}{2} = 1$, $e^2 = 1$. Table 1 represent the conjugacy classes for the group $SU(3,9)$. While table 2 represent the character table of rational representations for the same group.

Table 1
The conjugacy classes for the group $SU(3,9)$.

Conjugacy Class	Class Canonical Representation	Parameters	No of Classes	Centralizer Order	
C_1^k	C_1^0	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$k=0$	1	1872
C_2^k	C_2^0	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$k=0$	1	18
$C_3^{(k,l)}$	$C_3^{(0,0)}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$	$k=0, l=0$	1	9
C_4^k	$C_4^{(1)}$	$\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p^2 \end{bmatrix}$	$k=1$	1	48
C_5^k	$C_5^{(1)}$	$\begin{bmatrix} p & 0 & 0 \\ 1 & p & 0 \\ 0 & 0 & p^2 \end{bmatrix}$	$k=1$	1	6
$C_6^{(k,l)}$	Not exist	Not exist	$1 \leq k, l, m \leq 2$ $k < l < m$	$\frac{1}{6} 2(2-3)$ $+\frac{1}{3} = 0$	
C_7^k	$C_7^{(1)}$ $= C_7^{(3)}$ $C_7^{(2)}$ $= C_7^{(6)}$ $C_7^{(5)}$ $= C_7^{(7)}$	$\begin{bmatrix} p & 0 & 0 \\ 0 & \sigma^{-1} & 0 \\ 0 & 0 & \sigma^{-3} \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sigma^{-2} & 0 \\ 0 & 0 & \sigma^{-6} \end{bmatrix}$ $\begin{bmatrix} p & 0 & 0 \\ 0 & \sigma^{-5} & 0 \\ 0 & 0 & \sigma^{-15} \end{bmatrix}$	$1 \leq k < 8$ $k \neq 0 \pmod{4}$ $C^{(k)} = C^{(3k)} \pmod{8}$	$\frac{1}{2} (4-1) \cdot 2$ $= 3$	8
$C_8^{(k)}$	$C_8^{(1)}$ $= C_8^{(3)}$ $= C_8^{(9)}$ $C_8^{(2)}$ $= C_8^{(6)}$ $= C_8^{(5)}$ $C_8^{(4)}$ $= C_8^{(12)}$ $= C_8^{(10)}$ $C_8^{(7)}$ $= C_8^{(8)}$ $= C_8^{(11)}$	$\begin{bmatrix} \tau & 0 & 0 \\ 0 & \tau^3 & 0 \\ 0 & 0 & \tau^9 \end{bmatrix}$ $\begin{bmatrix} \tau^2 & 0 & 0 \\ 0 & \tau^6 & 0 \\ 0 & 0 & \tau^4 \end{bmatrix}$ $\begin{bmatrix} \tau^4 & 0 & 0 \\ 0 & \tau^{12} & 0 \\ 0 & 0 & \tau^{10} \end{bmatrix}$ $\begin{bmatrix} \tau^7 & 0 & 0 \\ 0 & \tau^8 & 0 \\ 0 & 0 & \tau^{11} \end{bmatrix}$	$1 \leq k < 13$ $k \neq 0 \pmod{13}$ $C^{(k)} = C^{(3k)}$ $= C^{(9k)} \pmod{13}$ $K=1,2,\dots,12$	$\frac{1}{3} (13-1)$ $= 4$	13

Table 2
The character table of rational representations for the group $\mathcal{SU}(2, 9)$.

K parameters	$u = 1$			$1 \leq u \leq 8$ $u \neq 0 \pmod{4}$ $(u) = (3u) \pmod{8}$	$1 \leq u \leq 13$ $u \neq 0 \pmod{13}$ $(u) = (3u) = (27u) \pmod{13}$
Number of characters	1 1 1	1	1	$\frac{1}{2}(s-1)r$ $= \frac{1}{2}(4-1)2 = 3$	$\frac{1}{3}(t-d) = \frac{1}{3}(13-1) = 4$
	$\chi_1 \chi_{12} \chi_{27}$	$\chi_{13^{(1)}}$	$\chi_{39^{(1)}}$	$\chi_{26}^{(1)} \chi_{26}^{(2)} \chi_{26}^{(5)}$	$\chi_{16}^{(1)} \chi_{16}^{(2)} \chi_{16}^{(4)} \chi_{16}^{(7)}$
$C_1^{(0)}$	1 12 27	13	39	26 26 26	16 16 16 16
$C_2^{(0)}$	1 3 0	4	3	-1 -1 -1	-2 -2 -2 -2
$C_3^{(0,0)}$	1 0 0	1	0	-1 -1 -1	1 1 1 1
$C_4^{(1)}$	1 4 3	5	4	2 2 2	0 0 0 0
$C_5^{(1)}$	1 1 0	2	1	-1 -1 -1	0 0 0 0
$C_7^{(1)} = C_7^{(3)}$	1 0 -1	1	-1	-2 2 2	0 0 0 0
$C_7^{(2)} = C_7^{(6)}$	1 0 -1	1	-1	1 -1 -1	0 0 0 0
$C_7^{(5)} = C_7^{(7)}$	1 0 -1	1	-1	2 2 2	0 0 0 0
$C_8^{(1)} = C_8^{(3)} = C_8^{(9)}$	1 -1 1	0	0	0 0 0	0 2 -2 -2
$C_8^{(2)} = C_8^{(5)} = C_8^{(6)}$	1 -1 1	0	0	0 0 0	2 -1 -1 -1
$C_8^{(4)} = C_8^{(10)} = C_8^{(12)}$	1 -1 1	0	0	0 0 0	-2 -1 -1 -1
$C_8^{(7)} = C_8^{(8)} = C_8^{(11)}$	1 -1 1	0	0	0 0 0	-4 -2 -2 -2

By applying the same idea in [9], and $\mathcal{K}(G) = \bigoplus_{i=1}^n \mathbb{Z}_{p^i}$ for the p -group G , [10-16]. We get the diagonalization matrix of the above table is

5616	0	0	0	0	0	0	0	0	0	0	0
0	1872	0	0	0	0	0	0	0	0	0	0
0	0	13	0	0	0	0	0	0	0	0	0
0	0	0	3	0	0	0	0	0	0	0	0
0	0	0	0	2	0	0	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0	0	0
0	0	0	0	0	0	2	0	0	0	0	0
0	0	0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	2	0	0	0
0	0	0	0	0	0	0	0	0	2	0	0
0	0	0	0	0	0	0	0	0	0	2	0
0	0	0	0	0	0	0	0	0	0	0	1

And the cyclic decomposition equal to

$$\mathbb{Z}_{5616} \oplus \mathbb{Z}_{1872} \oplus \mathbb{Z}_{13} \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_1.$$

2.2 The results for the group $\mathcal{SU}(2, 25)$

$\mathcal{SU}(2, 25) \mid = 372000$, $d = \gcd(3, 2) = 1$, $\delta = 1$, $S = q + \delta = 5 + 1 = 6$, $r = q - \delta = 5 - 1 = 4$, $t = q^2 + \delta q + 1 = 5^2 + 5 + 1 = 31$, $t' = \frac{t}{d} = 31$, $r' = 4$, $d' = \frac{(3-d)}{2} = 1$, $\epsilon^4 = 1$. As in section 2.1 the results for this group is

[illegible]

- [1] W. A. Simpson and J. S. Frame, "The Character Tables for $Sl(3,q)$, $SU(3,q^2)$, $PSL(3,q)$, $PSU(3,q^2)$," *Can. J. Math.*, vol. 25, no. 3, pp. 486-494 (1973). [Online]. Available: <https://doi.org/10.4153/cjm-1973-049-7>.
- [2] N.A.S. Salem and N.S. Jasim, "Circularity Segmentation for The Groups $PSL(2,23)$ and $PSL(2,29)$," 1st Samarra International Conference for Pure and Applied Sciences (SICPS2021) AIP Conf. Proc., 2394: 070032-1-070032-5 (2022). [Online] Available: <https://doi.org/10.1063/5.0121782>.

- [3] S. S. Noor Alhuda and S. J. Niran, Periodical split for the groups $\text{PSL}(2,31)$ and $\text{PSL}(2,37)$,," *J. Discrete Math. Sci. Cryptography*, vol. 25, no. 2, pp. 605-608 (2022). [Online] Available: <https://doi.org/10.1080/09720529.2021.1982490>.
- [4] S. A. Khalaf and S. J. Jasim, Calculation for the groups $\text{SL}(2,U)$, $U = 31$ and 37 ,," *J. Discrete Math. Sci. Cryptography*, vol. 25, no. 2, pp. 609-613 (2022). [Online] Available: <https://doi.org/10.1080/09720529.2021.1972614>.
- [5] S. A. Khalaf and S. J. Jasim, Consequences for The Groups $\text{SL}(2,U)$, $U = 23$ and 29 , 1st Samarra International Conference for Pure and Applied Sciences (SICPS2021) AIP Conf. Proc., 2394: 070031-1-070031-7. (2022). [Online] Available: <https://doi.org/10.1063/5.0121781>.
- [6] M. I. Lfta and N. S. Jasim, "Computations for the groups $\text{SL}(2,81)$ and $\text{SL}(2,729)$," *J. Interdiscip. Math.*, vol. 26, no. 7, pp. 1373-1378 (2023). [Online] Available: <https://doi.org/10.47974/jim-1553>.
- [7] M. Salah and N. S. Jasim, "Enforcement for the groups $\text{SL}(2,125)$ and $\text{SL}(2,3125)$," *J. Interdiscip. Math.*, vol. 26, no. 7, pp. 1379-1384 (2023). [Online] Available: <https://doi.org/10.47974/jim-1554>.
- [8] G. A. Khalaf and N. S. Jasim, "Score for the groups $\text{SL}(2,121)$ and $\text{SL}(2,169)$," *J. Interdiscip. Math.*, 26, no. 7, pp. 1385-1390 (2023). [Online] Available: <https://doi.org/10.47974/jim-1555>.
- [9] S. J. Niran, H. L. Hadeel and N. M. Rana, "Computations for the special linear group $(2, 49)$," *J. Interdiscip. Math.*, vol. 24, no. 6, pp. 1677-1683 (2021). [Online] Available: <https://doi.org/10.1080/09720502.2021.1892273>.
- [10] D. M. Hameed, A. K. Mohsin and I.Z. Mushtt, "Score for some groups $(\text{SUT}(2, p))$," *International journal of nonlinear analysis and applications*, vol. 12, no. 2, pp. 2539-2543 (2021). [Online] Available: <https://doi.org/10.22075/ijnaa.2021.5408>.
- [11] S. Hussain, F. Afzal, D. Afzal, M. Cancan, S. Ediz and M.R. Farahani, "Analyzing the boron triangular nanotube through topological indices via M-polynomial," *J. Discrete Math. Sci. Cryptography*, vol. 24, no. 2, pp. 415-426 (2021). [Online] Available: <https://doi.org/10.1080/09720529.2021.1882158>.
- [12] F. Afzal, S. Rukh, D. Afzal, M. R. Farahani, M. Cancan and S. Ediz, "A study of non-commutative Von-Neumann regular rings," *J. Inf.*

- Optim. Sci.*, vol. 42, no. 7, pp. 1425-1436 (2021). [Online] Available: <https://doi.org/10.1080/02522667.2021.1896650>.
- [13] M. Nadeem, S. Ahmad, M. K. Siddiqui, M. A. Ali, M. R. Farahani and A. J. M. Khalaf, "On some applications related with algebraic structures through different well known graphs," *J. Discrete Math. Sci. Cryptography*, vol. 24, no. 2, pp. 451-471 (2021). [Online] Available: <https://doi.org/10.1080/0972052>.
- [14] S. Ediz, M. Cancan and M. R. Farahani, "Eccentric connectivity and connective eccentricity indices of generalized Petersen graphs," *Commun. Combinatorics Cryptogr. Comput. Sci.*, vol. 2021, no. 1, pp. 1-6 (2021).
- [15] R. Ponraj, S. Prabhu and M. Sivakumar, "Pair mean cordial label-ing of total graph of some graphs and prism-related graphs," *Commun. Combinatorics Cryptogr. Comput. Sci.*, vol. 2024, no. 2, pp. 181-192 (2024).
- [16] F. O. Oduol and I. O. Okoth, "Counting vertices among all noncross-ing trees by levels and degrees," *Commun. Combinatorics Cryptogr. Comput. Sci.*, vol. 2024, no. 2, pp. 193-212 (2024).

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