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The complexness of the sequence in the status (10, 10, 3, 2, 2)

Zainab Subhi Mustafa *
Department of Mathematics
College of Education for Pure Science (Ibn Al-Haitham)
University of Baghdad
Baghdad 10071
Iraq

Zahraa S. M. Alhaidary †
Department of Applied Sciences
Branch of Mathematics and Computer Applications
University of Technology
Baghdad 10066
Iraq

Lemya Abd Alameer Hadi §
Department of Communication Engineering
University of Technology
Baghdad 10066
Iraq

Niran Sabah Jasim ‡
Department of Mathematics
College of Education for Pure Science (Ibn Al-Haitham)
University of Baghdad
Baghdad 10071
Iraq

‡ *ORCID-ID:* 0009-0005-0578-7409
† *ORCID-ID:* 0009-0008-2248-3644
§ *ORCID-ID:* 0000-0001-9258-9331
* *ORCID-ID:* 0000-0001-5340-3020

‡ *E-mail:* zainab.so.m@ihcoedu.uobaghdad.edu.iq
† *E-mail:* zahraa.s.mohammed@uotechnology.edu.iq
§ *E-mail:* lemya.a.hadi@uotechnology.edu.iq
* *E-mail:* niraan.s.j@ihcoedu.uobaghdad.edu.iq (Corresponding Author)

Abstract

Everywhere in this paper $\mathcal{R}$ has 1 and a commutative ring, $\mathcal{F}$ is a free $\mathcal{R}$-module and $\mathcal{D}_i\mathcal{F}$ is the divided power (di.po.) algebra of degree i.

The goal of this study is to find the complexness of the sequence of the elements of Lascoux (La.) in the status (10,10,3;2,2) by employing Capelli identities (Ca.id.), the commute of the schemes and the place polarization (p.po.).

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Keywords: *Capelli identities, Polarization, Complex of characteristic zero, Lascoux resolution.*

1. Introduction

Writers in [1, 13-16] exhibit the resolution of the Weyl module with the three rows $(p,q,r;t_1,t_2)$, the maps $\sum_{k>0}\mathfrak{D}_{p+t_1+k}\otimes\mathfrak{D}_{q-t_1-k}\otimes\mathfrak{D}_r \to \mathfrak{D}_p\otimes\mathfrak{D}_q\otimes\mathfrak{D}_r$ displayed as k^{th} di.po. of the p.po. from place 1 to place 2 ($\partial_{21}^{(k)}$), and the maps $\sum_{e>0}\mathfrak{D}_p\otimes\mathfrak{D}_{q+t_2+e}\otimes\mathfrak{D}_{r-t_2-e} \to \mathfrak{D}_p\otimes\mathfrak{D}_q\otimes\mathfrak{D}_r$ explicated as e^{th} di.po. of the p.po. from place 2 to place 3 ($\partial_{32}^{(e)}$), and as in two rowed case. Everywhere this work we poverty these Ca.id. [2].

$$\partial_{32}^{(2)}\partial_{21} = \partial_{21}\,\partial_{32}^{(2)} + \partial_{32}\,\partial_{31}\ \ldots\text{(a)}\ \partial_{32}\,\partial_{21}^{(2)} = \partial_{21}^{(2)}\partial_{32} + \partial_{21}\,\partial_{31}\ \ldots\text{(b)}$$

$$\partial_{21}^{(2)}\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31}\ \ldots\text{(c)}\ \partial_{32}\,\partial_{31} = \partial_{31}\partial_{32}\ \ldots\text{(d)}$$

For the status (10,10,3;2,2) the writers displayed in this work the same idea in [3], we can apply that also for $\mathcal{PSL}(2,\mathfrak{U})$ and $\mathcal{SL}(2,\mathfrak{U})$ where $\mathfrak{U}$ = 23,29,31,37 [4-7], $\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U}$ = 49, 81, 729, 121, 169, 125, 3125 [8-11] and $\mathcal{S.U.T.}(2,p)$, for a prime $p \le 9$. [12]

2. The scheme of Weyl module of La.

The terms of the La. in status (10,10,3;2,2) are written as the sequence

$$\begin{array}{ccccc} & & \mathfrak{D}_{10}\mathcal{F}\otimes\mathfrak{D}_7\mathcal{F}\otimes\mathfrak{D}_2\mathcal{F} & \mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_7\mathcal{F}\otimes\mathfrak{D}_3\mathcal{F} & \\ \mathfrak{D}_{10}\mathcal{F}\otimes\mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_1\mathcal{F} & \to & \oplus \to & \oplus \to & \mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_3\mathcal{F} \\ & & \mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_1\mathcal{F} & \mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_2\mathcal{F} & \end{array}$$

To prove the complexness of the sequence of the elements of Weyl module of La., we use the schemes. Take the later scheme

$$\begin{tikzcd}
\mathfrak{D}_{10}\mathcal{F}\otimes\mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_1\mathcal{F} \arrow[r, "\mathfrak{g}_1"] \arrow[d, "\mathfrak{h}_1"'] \arrow[dr, phantom, "\mathcal{A}"] & \mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_1\mathcal{F} \arrow[r, "\mathcal{b}_1"] \arrow[d, "\mathfrak{h}_2"] \arrow[dr, phantom, "\mathcal{G}"] & \mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_2\mathcal{F} \arrow[d, "\mathfrak{h}_3"] \\
\mathfrak{D}_{10}\mathcal{F}\otimes\mathfrak{D}_7\mathcal{F}\otimes\mathfrak{D}_2\mathcal{F} \arrow[r, "\mathfrak{g}_2"'] \arrow[urr, "\mathcal{T}"] & \mathfrak{D}_9\mathcal{F}\otimes\mathfrak{D}_7\mathcal{F}\otimes\mathfrak{D}_3\mathcal{F} \arrow[r, "\mathcal{b}_2"'] & \mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_8\mathcal{F}\otimes\mathfrak{D}_3\mathcal{F}
\end{tikzcd}$$

Scheme (1) : Realize the above maps as $\mathfrak{g}_1(\mathfrak{r}) = \partial_{21}(\mathfrak{r})$; $\mathfrak{h}_1(\mathfrak{r}) = \partial_{32}(\mathfrak{r})$, $\mathfrak{h}_2(\mathfrak{r}) = \partial_{32}^{(2)}(\mathfrak{r})$ $\mathfrak{g}_2(\mathfrak{r}) = \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(\mathfrak{r})$, $\mathscr{b}_1(\mathfrak{r}) = \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(\mathfrak{r})$; $\mathfrak{h}_3(\mathfrak{r}) = \partial_{32}(\mathfrak{r})$, $\mathscr{b}_2(\mathfrak{r}) = \partial_{21}(\mathfrak{r})$ and $\mathcal{T}(\mathfrak{r}) = \partial_{21}^{(2)}(\mathfrak{r})$.

Proposition 2.1: $\mathfrak{h}_2 \circ \mathfrak{g}_1 = \mathfrak{g}_2 \circ \mathfrak{h}_1$ in scheme (1).

Proof: $(\mathfrak{h}_2 \circ \mathfrak{g}_1)(\mathfrak{r}) = (\partial_{32}^{(2)}\partial_{21})(\mathfrak{r})$

$$= \left(\partial_{21}\,\partial_{32}^{(2)} + \partial_{31}\partial_{32}\right)(\mathfrak{r}) \text{ by (a) and (d)}$$

$$= \left(\left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)\partial_{32}\right)(\mathfrak{r})) = (\mathfrak{g}_2 \circ \mathfrak{h}_1)(\mathfrak{r})$$

Proposition 2.2: $\mathfrak{h}_3 \circ \mathscr{b}_1 = \mathscr{b}_2 \circ \mathfrak{h}_2$ in scheme (1).

Proof: $(\mathfrak{h}_3 \circ \mathscr{b}_1)(\mathfrak{r}) = \partial_{32}\left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(\mathfrak{r})\right) = \left(\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31}\right)(\mathfrak{r})$

From (a) we gain $(\mathfrak{h}_3 \circ \mathscr{b}_1)(\mathfrak{r}) = (\mathscr{b}_2 \circ \mathfrak{h}_2)(\mathfrak{r})$.

Proposition 2.3: The commutative of scheme $\mathcal{T}$ in scheme (1).

Proof: $(\mathcal{T} \circ \mathfrak{h}_1)(\mathfrak{r}) = \left(\partial_{21}^{(2)}\partial_{32}\right)(\mathfrak{r})$. By using (c) we gain

$$(\mathcal{T} \circ \mathfrak{h}_1)(\mathfrak{r}) = \left(\partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31}\right)(\mathfrak{r}) =$$

$$\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\left(\partial_{21}(\mathfrak{r})\right) = (\mathscr{b}_1 \circ \mathfrak{g}_1)(\mathfrak{r})$$

Proposition 2.4: The commutative of schema $\mathcal{G}$ in scheme (1).

Proof: $(\mathfrak{h}_3 \circ \mathcal{G})(\mathfrak{r}) = \left(\partial_{32}\,\partial_{21}^{(2)}\right)(\mathfrak{r})$

By using (c) we gain

$$(\mathfrak{h}_3 \circ \mathcal{G})(\mathfrak{r}) = \left(\partial_{21}\left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)\right)(\mathfrak{r}) = (\mathscr{b}_2 \circ \mathfrak{g}_2)(\mathfrak{r}).$$

Take the sequence of elements of La.

$$\begin{array}{ccccc} & & \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_1\mathcal{F} & & \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_2\mathcal{F} & & \\ \mathfrak{D}_{10}\mathcal{F} \otimes \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_1\mathcal{F} & \to & \oplus & \to & \oplus & \to & \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_3\mathcal{F} \\ & & \mathfrak{D}_{10}\mathcal{F} \otimes \mathfrak{D}_7\mathcal{F} \otimes \mathfrak{D}_2\mathcal{F} & & \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_7\mathcal{F} \otimes \mathfrak{D}_3\mathcal{F} & & \end{array}$$

Realize the maps

$\mathfrak{S}_3: \mathfrak{D}_{10}\mathcal{F} \otimes \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_1\mathcal{F} \to \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_1\mathcal{F} \oplus \mathfrak{D}_{10}\mathcal{F} \otimes \mathfrak{D}_7\mathcal{F} \otimes \mathfrak{D}_2\mathcal{F}$

$\mathfrak{S}_2$: $\mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_1\mathcal{F} \oplus \mathfrak{D}_{10}\mathcal{F} \otimes \mathfrak{D}_7\mathcal{F} \otimes \mathfrak{D}_2\mathcal{F} \to \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_2\mathcal{F} \oplus \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_7\mathcal{F} \otimes \mathfrak{D}_3\mathcal{F}$, and

$\mathfrak{S}_1: \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_2\mathcal{F} \oplus \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_7\mathcal{F} \otimes \mathfrak{D}_3\mathcal{F} \to \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_3\mathcal{F}$

By $\mathfrak{S}_3(\mathfrak{r}) = (\partial_{21}(\mathfrak{r}), \partial_{32}(\mathfrak{r}))$

$\mathfrak{S}_2(\mathfrak{r},\mathfrak{s}) = \left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(\mathfrak{r}) - \partial_{21}(\mathfrak{s}), \left(\frac{1}{2}\partial_{21}\,\partial_{32} + \partial_{31}\right)(\mathfrak{s}) - \partial_{32}^{(2)}(x)\right)$,
and

$$\mathfrak{S}_1(\mathfrak{r}) = \partial_{21}(\mathfrak{s}) + \partial_{32}(\mathfrak{r})$$

Proposition 2.5: $\ker(\mathfrak{S}_2) \supseteq \mathrm{Im}(\mathfrak{S}_3)$

Proof: $(\mathfrak{S}_2 \circ \mathfrak{S}_3)\,(\mathfrak{r}) = \wp_2(\partial_{21}(\mathfrak{r}), \partial_{32}(\mathfrak{r}))$
$= \left(\left(-\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\left(\partial_{21}(\mathfrak{r})\right) - \partial_{21}^{(2)}\left(\partial_{32}(\mathfrak{r})\right), \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)\left(\partial_{32}(\mathfrak{r})\right) - \partial_{32}^{(2)}\left(\partial_{21}(\mathfrak{r})\right)\right)$
$= \left(\partial_{32}\,\partial_{21}^{(2)}(\mathfrak{r}) - \partial_{31}\,\partial_{21}(\mathfrak{r}) - \partial_{21}^{(2)}\,\partial_{31}(\mathfrak{r}), \partial_{21}\,\partial_{32}^{(2)}(\mathfrak{r}) + \partial_{31}\,\partial_{32} - \partial_{32}^{(2)}\partial_{21}(\mathfrak{r})\right)$

From (b) and (c) we gain
$\partial_{32}\,\partial_{21}^{(2)}(\mathfrak{r}) - \partial_{31}\,\partial_{21}(\mathfrak{r}) - \partial_{21}^{(2)}\,\partial_{31}(\mathfrak{r}) = \partial_{21}^{(2)}\partial_{32}(\mathfrak{r}) + \partial_{21}\,\partial_{31}(\mathfrak{r}) - \partial_{21}\,\partial_{31}(\mathfrak{r}) - \partial_{21}^{(2)}\,\partial_{32}(\mathfrak{r}) = 0$, and
$\partial_{21}\,\partial_{32}^{(2)}(\mathfrak{r}) + \partial_{31}\,\partial_{32} - \partial_{32}^{(2)}\partial_{21}(\mathfrak{r}) =$
$\partial_{32}^{(2)}\,\partial_{21}(\mathfrak{r}) - \partial_{32}\,\partial_{31}(\mathfrak{r}) + \partial_{32}\,\partial_{31}(\mathfrak{r}) - \partial_{32}^{(2)}\,\partial_{21}(\mathfrak{r}) = 0$

So, $(\mathfrak{S}_2 \circ \mathfrak{S}_3)\,(\mathfrak{r}) = 0$ which mean that $\ker(\mathfrak{S}_2) \supseteq \mathrm{Im}(\mathfrak{S}_3)$.

Proposition 2.6: $\ker(\mathfrak{S}_1) \supseteq \mathrm{Im}(\mathfrak{S}_2)$

Proof: $(\mathfrak{S}_1 \circ \mathfrak{S}_2)\,(\mathfrak{r},\mathfrak{s}) = \wp_1\left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(\mathfrak{r}) - \partial_{21}^{(2)}(\mathfrak{s}), \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(\mathfrak{s}) - \partial_{32}^{(2)}(\mathfrak{r})\right)$
$= \partial_{21}\left(\left(\frac{1}{2}\partial_{21}\,\partial_{32} + \partial_{31}\right)(\mathfrak{s}) - \partial_{32}^{(2)}(\mathfrak{r})\right) + \partial_{32}\left(\left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(\mathfrak{r}) - \partial_{21}^{(2)}(\mathfrak{s})\right)$
$=\partial_{21}^{(2)}\,\partial_{32}(\mathfrak{s}) + \partial_{21}\,\partial_{31}(\mathfrak{s}) - \partial_{21}\,\partial_{32}^{(2)}(\mathfrak{r}) + \partial_{32}^{(2)}\,\partial_{21}(\mathfrak{r}) - \partial_{32}\,\partial_{31}(\mathfrak{r}) - \partial_{32}\,\partial_{21}^{(2)}(\mathfrak{s})$

By (a), (b) and (c) we gain

$$(\mathfrak{S}_1 \circ \mathfrak{S}_2)\,(\mathfrak{r},\mathfrak{s}) = \partial_{21}^{(2)}\,\partial_{32}(\mathfrak{s}) + \partial_{21}\,\partial_{31}(\mathfrak{s}) - \partial_{21}\,\partial_{32}^{(2)}(\mathfrak{r}) + \partial_{21}\,\partial_{32}^{(2)}(\mathfrak{r}) + \partial_{32}\,\partial_{31}(\mathfrak{r}) - \partial_{32}\,\partial_{31}(\mathfrak{r}) - \partial_{21}^{(2)}\,\partial_{32}(\mathfrak{s}) - \partial_{21}\,\partial_{31}(\mathfrak{s}) = 0.$$

Thus, $\ker(\mathfrak{S}_1) \supseteq \mathrm{Im}(\mathfrak{S}_2)$.

Theorem 2.7: *The following sequence of elements of La. is complex*

$$\begin{array}{c} \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_9\mathcal{F} \otimes \mathfrak{D}_2\mathcal{F} \quad \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ 0 \to \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\mathfrak{S}_3} \oplus \xrightarrow{\mathfrak{S}_2} \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \quad \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_4\mathcal{F} \end{array}$$

$$\xrightarrow{\mathfrak{S}_1} \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_8\mathcal{F} \otimes \mathfrak{D}_4\mathcal{F} \xrightarrow{\acute{d}_{(10,10,3;2,2)}(\mathcal{F})} \mathcal{K}_{(10,10,3;2,2)}(\mathcal{F}) \to 0$$

Proof: The injectiveness of $\partial_{i\,j}$ by [1] gain the injectiveness of $\wp_3$. $\ker(\mathfrak{S}_2) \supseteq \mathrm{Im}(\mathfrak{S}_3)$ and $\ker(\mathfrak{S}_1) \supseteq \mathrm{Im}(\mathfrak{S}_2)$ by propositions (2.5) and (2.6), so we gain the complexness of the sequence.

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