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Result for some contracting homotopy in the status (8,8)

Zainab Subhi Mustafa ‡
Department of Mathematics
College of Education for Pure Science (Ibn Al-Haitham)
University of Baghdad
Baghdad 10071
Iraq

Marwa Riyadh Saleh †
Department of Applied Sciences
Branch of Mathematics and Computer Applications
University of Technology
Baghdad 10066
Iraq

Nibras Yasir Khudair §
Department of Mathematics and Computer Applications
College of Science
Nahrain University
Baghdad 10072
Iraq

Niran Sabah Jasim*
Department of Mathematics
College of Education for Pure Science (Ibn Al-Haitham)
University of Baghdad
Baghdad 10071
Iraq

‡ *ORCID-ID:* 0009-0005-0578-7409
† *ORCID-ID:* 0009-0001-6190-4703
§ *ORCID-ID:* 0009-0004-3815-5697
* *ORCID-ID:* 0000-0001-5340-3020

‡ *E-mail:* zainab.so.m@ihcoedu.uobaghdad.edu.iq
† *E-mail:* marwa.riad.salih@uomustansiriyah.edu.iq
§ *E-mail:* nibras.yasir@nahrainuniv.edu.iq
* *E-mail:* niraan.s.j@ihcoedu.uobaghdad.edu.iq (Corresponding Author)

Abstract

The goal of this studied is to prove that the sequence terms of the Weyl resolution (Wey. Res.) of the characteristic-free (ch.fr.) resolution is exact in the status (8,8) with overlap (2,4) thus the status of our work is (6,4).

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1. Introduction

Everywhere this work $\mathcal{R}$ has 1 and commutative ring, $\mathcal{F}$ free $\mathcal{R}$-module, $\wedge\mathcal{F}$ is the exterior algebra of $\mathcal{F}$, $\mathfrak{D}_i\mathcal{F}$ the divided power algebra (div.pow.alg.) of degree i, [1]. Writers in [1], [2] display $\mathcal{K}_{\lambda/\mu}\mathcal{F} = \mathrm{Im}(d'_{\lambda/\mu})$ where $d'_{\lambda/\mu}$ is the Weyl map $d'_{\lambda/\mu}\colon \mathfrak{D}\mathcal{F} \to \wedge\mathcal{F}$ and div.pow.alg. of the place polarization ($\partial_{21}^{(k)}$), for the partition (p, q) we have $\square = \sum_{k=t+1}^{q} \partial_{21}^{(k)}$, and

$$\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(p+k)} \\ 2^{(q-k)} \end{matrix}\right) \xrightarrow{\partial_{21}^{(k)}} \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(p)}\ 2^{(k)} \\ 2^{(q-k)} \end{matrix}\right) \to \sum_{w} \left(\begin{matrix} w_{(1)} \\ w' w_{(2)} \end{matrix}\middle| \begin{matrix} (t+1)'(t+2)' \dots (p+t)' \\ 1'2'3' \dots q' \end{matrix}\right).$$

Author in [1] defines the contracting homotopy for the maps $\{\mathcal{S}_{\mathrm{i}}\}$, we apply the same idea for the partition (6,4) in this work. Also, we can apply that for $\mathcal{PSL}(2,\mathfrak{U})$, $\mathcal{SL}(2,\mathfrak{U})$ where $\mathfrak{U}$ = 23, 29, 31, 37 [3-6], $\mathcal{S}.\mathcal{U}.\mathcal{T}.(2, p)$, for a prime $p \leq 9$ and $\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U}$ = 81, 729, 121, 169, 125, 3125 [7-10].

2. The Terms of Wey. Res.

The terms of ch.fr. Wey.Res. for the partition (6,4) are

$\mathcal{M}_0 = \mathcal{D}_6 \otimes \mathcal{D}_4$, $\mathcal{M}_1 = \mathcal{Z}_{21}^{(b)} x \mathcal{D}_{6+b} \otimes \mathcal{D}_{4-b}$; where $1 \leq b \leq 4$

$\mathcal{M}_2 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{D}_{6+|b|} \otimes \mathcal{D}_{4-|b|}$; where $2 \leq |b| = b_1 + b_2 \leq 4$

$\mathcal{M}_3 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{6+|b|} \otimes \mathcal{D}_{4-|b|}$; where $3 \leq |b| = \sum_{i=1}^{3} b_i \leq 4$

$\mathcal{M}_4 = \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x\ \mathcal{D}_{11} \otimes \mathcal{D}_0$

The homotopies $\{\mathcal{S}_i\}$; where i=0,1,2,3 below with prove that they are contracting

$$\mathcal{S}_0\left(\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6)}\ 2^{(k)} \\ 2^{(4-k)} \end{matrix}\right)\right) = \begin{cases} 0\,;\ \text{if } k \leq 0 \\ \mathcal{Z}_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)} \\ 2^{(4-k)} \end{matrix}\right)\ ;\ \text{if } k = 1,2,3,4 \end{cases}$$

$$\mathcal{S}_1\left(\mathcal{Z}_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)}\ 2^{(m)} \\ 2^{(4-k-m)} \end{matrix}\right)\right) = \begin{cases} 0;\ \text{if } m = 0 \\ \mathcal{Z}_{21}^{(k)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k+m)} \\ 2^{(4-k-m)} \end{matrix}\right)\ ;\ \text{if } m = 1,2,3 \end{cases}$$

For $|k| = k_1 + k_2$, $\mathcal{S}_2\left(\mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(6+|k|)}\ 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right)$

$$= \begin{cases} 0\ ;\ \text{if } m = 0 \\ \mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(9+|k|+m)} \\ 2^{(8-|k|-m)} \end{matrix}\right)\ ;\ \text{if } m = 1,2 \end{cases}$$

For $|k| = k_1 + k_2 + k_3$, $\mathcal{S}_3\left(\mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)}\ 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right)$

$$= \begin{cases} 0\ ;\ \text{if } m = 0 \\ \mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(k_3)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\ ;\ \text{if } m = 1 \end{cases}$$

So, we have $\mathcal{M}_4 \underset{\mathcal{S}_3}{\overset{\partial_x}{\rightleftarrows}} \mathcal{M}_3 \underset{\mathcal{S}_2}{\overset{\partial_x}{\rightleftarrows}} \mathcal{M}_2 \underset{\mathcal{S}_1}{\overset{\partial_x}{\rightleftarrows}} \mathcal{M}_1 \underset{\mathcal{S}_0}{\overset{\partial_x}{\rightleftarrows}} \mathcal{M}_0$

Now we prove that, $\mathcal{S}_{i-1}\partial_x + \partial_x \mathcal{S}_i = \mathrm{id}_{\mathcal{M}_i}$, $i = 1, 2, 3$.

$$\mathcal{S}_0 \partial_x \left(\mathcal{Z}_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)}\ 2^{(m)} \\ 2^{(4-k-m)} \end{matrix}\right)\right) = \binom{k+m}{m} \mathcal{Z}_{21}^{(k+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k+m)} \\ 2^{(4-k-m)} \end{matrix}\right),$$

and

$$\partial_x \mathcal{S}_1 \left(\mathcal{Z}_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)}\ 2^{(m)} \\ 2^{(4-k-m)} \end{matrix}\right)\right) = -\binom{k+m}{m} \mathcal{Z}_{21}^{(k+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k+m)} \\ 2^{(4-k-m)} \end{matrix}\right)$$

$+\mathcal{Z}_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)}\ 2^{(m)} \\ 2^{(4-k-m)} \end{matrix}\right)$. Hence, $\mathcal{S}_0\partial_x + \partial_x \mathcal{S}_1 = \mathrm{id}_{\mathcal{M}_1}$.

For $|k| = k_1 + k_2$, $\mathcal{S}_1 \partial_x \left(\mathcal{Z}_{21}^{(1+k_1)} x \mathcal{Z}_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)}\ 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right)$

$= -\binom{1+|k|}{k_2} \mathcal{Z}_{21}^{(1+|k|)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) +$
$\binom{k_2+m}{m} \mathcal{Z}_{21}^{(1+k_1)} x \mathcal{Z}_{21}^{(k_2+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)$, and

$\partial_x \mathcal{S}_2 \left(\mathcal{Z}_{21}^{(1+k_1)} x \mathcal{Z}_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)}\ 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right) = \binom{1+|k|}{k_2} \mathcal{Z}_{21}^{(1+|k|)} x$
$\mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) - \binom{k_2+m}{m} \mathcal{Z}_{21}^{(1+k_1)} x \mathcal{Z}_{21}^{(k_2+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) +$
$\mathcal{Z}_{21}^{(1+k_1)} x \mathcal{Z}_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)}\ 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)$. Hence $\mathcal{S}_1\partial_x + \partial_x \mathcal{S}_2 = \mathrm{id}_{\mathcal{M}_2}$

For

$|k| = k_1 + k_2 + k_3$, $\mathcal{S}_2 \partial_x \left(\mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)}\ 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right)$

$= \binom{k_1+k_2}{k_2} \mathcal{Z}_{21}^{(k_1+k_2)} x \mathcal{Z}_{21}^{(k_3)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) -$
$\binom{k_2+k_3}{k_3} \mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2+k_3)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) +$
$\binom{k_3+m}{m} \mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(k_3+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)$, and

$$\partial_x \mathcal{S}_3\left(\mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)}\, 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right)$$

$$= -\binom{k_1+k_2}{k_2} \mathcal{Z}_{21}^{(k_1+k_2)} x \mathcal{Z}_{21}^{(k_3)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) +$$

$$\binom{k_2+k_3}{k_3} \mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2+k_3)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) -$$

$$\binom{k_3+m}{m} \mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(k_3+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) +$$

$\mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \mathcal{Z}_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)}\, 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)$. Hence, $\mathcal{S}_2 \partial_x + \partial_x \mathcal{S}_3 = \mathrm{id}_{\mathcal{M}_3}$.

Thus for $i = 0,1,2,3$ we gain that $\{\mathcal{S}_i\}$; is a contracting homotopy [2], which implies the exactness of the later complex $0 \to \mathcal{M}_4 \to \mathcal{M}_3 \to \mathcal{M}_2 \to \mathcal{M}_1 \to \mathcal{M}_0$.

References

[1] L. R. Vermani, An Elementary Approach to Homological Algebra, Chapman and Hall, Monographs and Surveys in Pure and Applied Mathematics. CRC Press (2003). [Online]. Available: https://doi.org/10.1201/9780203484081.

[2] N. Y. Khudair and H. R. Hassan, "Application of the two-rowed Weyl module in the case of partitions (7,7) and (7,7)/(1,0)," *Iraqi Journal of Science,* vol. 61, no. 5, pp. 1123-1135 (2020). [Online]. Available: https://doi.org/10.24996/ijs.2020.61.5.21.

[3] N. A. S. Salem and N. S. Jasim, "Circularity segmentation for the groups PSL(2,23) and PSL(2,29)," in 1st Samarra International Conference for Pure and Applied Sciences (SICPS2021), AIP Conf. Proc., vol. 2394, 070032-1-070032-5 (2022). [Online]. Available: https://doi.org/10.1063/5.0121782.

[4] S. A. Khalaf and N. S. Jasim, "Consequences for the groups SL(2, U), U = 23 and 29," in 1st Samarra International Conference for Pure and Applied Sciences (SICPS2021), AIP Conf. Proc., vol. 2394, 070031-1-070031-7 (2022). [Online]. Available: https://doi.org/10.1063/5.0121781.

[5] A. K. Sherouk and S. J. Niran, "Calculation for the groups SL(2, U), U = 31 and 37," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 25, no. 2, pp. 609-613 (2022). [Online]. Available: https://doi.org/10.1080/09720529.2021.1972614.

[6] A. S. S. Noor and S. J. Niran, "Periodical split for the groups PSL(2,31) and PSL(2,37)," *Journal of Discrete Mathematical Sciences and Cryptogra-*

phy, vol. 25, no. 2, pp. 605-608 (2022). [Online]. Available: https://doi.org/10.1080/09720529.2021.1982490.

[7] M. I. Lfta and N. S. Jasim, "Computations for the groups SL(2,81) and SL(2,729)," *Journal of Interdisciplinary Mathematics,* vol. 26, no. 7, pp. 1373-1378 (2023). [Online]. Available: https://doi.org/10.47974/JIM-1553.

[8] G. A. Khalaf and N. S. Jasim, "Score for the groups SL(2,121) and SL(2,169)," *Journal of Interdisciplinary Mathematics*, vol. 26, no. 7, pp. 1385-1390 (2023). [Online]. Available: https://doi.org/10.47974/JIM-1555.

[9] Z. M. S. Aldeen and N. S. Jasim, "Enforcement for the groups SL(2,125) and SL(2,3125)," *Journal of Interdisciplinary Mathematics*, vol. 26, no. 7, pp. 1379-1384 (2023). [Online]. Available: https://doi.org/10.47974/JIM-1554.

[10] L. A. Alameer, M. S. Fiadh, N. S. Jasim and J. A. Eleiwy, "Applications for the groups S.U.T.(2, p), where p prime upper than 9," *Iraqi Journal for Computer Science and Mathematics,* vol. 5, no. 1, pp. 78-84 (2024). [Online]. Available: https://doi.org/10.52866/ijcsm.2024.05.01.005.

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