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On digclosure operators

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Abstract

The purpose of this study is to use a novel definition of the $ÐIG_m$-closure operator on out-linked digs to produce construction of topology on the power set of vertices of digs. Some topological structure's properties are investigated, and some of the instances are offered. We also define new generalization subdigraph called R-open subdigraph and look at their accuracy.

Subject Classification (2010): *05C30.*

Keywords: *Digraph, Topology, Closure operators, Accuracy.*

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1. Introduction

Two reasons make graph theory a valuable tool in many branches of mathematics and a cornerstone of certain fields of study. Firstly, grαphs are created mathematically. Chosen from an ıntellectual perspective. Grαphs are simple relational combinations, but they can also be used as collection objects, topological spaces, and many other types of mathematical groups. For a second reason, certain ideas are easier to understand when shown with grαphs. grow more sensible. In terms of how topology and graph theory are related, one of the toles used to express the graph is topological ideas, for example, converting a set of vertices or edges to topological space. The most popular and up-to-date subject is topology in a broad section of thin king mathematician. These are a few previous research works on the topic of topological grαphs. Evans and Harary [4] proposed the idea of topology on digraph in 1967. They discovered a single relationship between the set of all topologies with n vertices and the set of all topologies with n vertices. A.J.M. Khalaf and others in [1] they introduce the notion of bridge grαphs and define the hyper-Zagreb index, first multiple Zagereb index, second multiple Zagreb index, Zagreb polynomials, and M-polynomial for microstructures of bridge grαphs. And more relations between Polynomial and Topological Indices see[13-15]. In [5-20], Kh. Sh. Al Ɗzhabri and others developed a new concept in topological graph spaces related to undigrαphs and digrαphs produced by a new open set and examined various features of this idea by utilizing the closed link between topology and graph.

2. Preliminaries

Are the first stepin the process. Some fundamental definitions of topology[2] and graph theory[12,3] are ıntroduced in this study. A graph, often known as a directed graph or digraph. Ɖ$= (V, \Xi)$ consists of pairs of unordered (or ordered) elements from}, represented by the vertex set V and the edge set Ξ. To avoid misunderstanding, we consider the sets of vertices and edges to be disjoınt. If an edge of the form ύ"I exists between two vertices, ύ and ώ, of a graph (i.e., digraph), then we say that they are neighboring.(rsep., $\overrightarrow{\sigma_1\sigma_2}$or $\overrightarrow{\sigma_2\sigma_1}$) connecting them, and such an edge then occurs at the vertices ύ and ώ. A subdigraphraph H of a digraph Ɖ is a digraph, all vertices of which are part of¸ and all edges of which are part of Ξ. The number of edges that coincide with a vertex ύ in Ɖ is its degree, written deg(ύ). An isolated vertex is one whose out-degree and in-degree are both 0. In digraph, The number of edges in the form is the outdegree of a vertex ύ of $\overrightarrow{\sigma_1\sigma_2}$ and denoted by $d^{+}(\sigma)$, Likewise, the number of edges in the form$\overrightarrow{\sigma_2\sigma_1}$ is the indegree of a vertex ύ of Ɖ., and denoted by $d^{-}(\sigma)$.A digraph is said to be symmetric digraph if $\sigma_1\ \sigma_2 \in \Xi$ imples $\sigma_2\ \sigma_1 \in \Xi$ Let Ӽ represent a set. A topological set on Ӽ is a set τ⊂P(¸)of open sets, which are subsets of Ӽ such that: (1) Ӽ and ∅ are open. (2) A finite number of open sets have an open ıntersection. A union of open sets is open in any case. A set and its corresponding topology τ on constitute a topolgical space. In topological spaces, a subset is considered closed if its counterpart is open. All open sets contained corresponding topology τ on constitute a topolgical space. In topological spaces, a subset is considered closed if its counterpart is open. All open sets contained within an A are united to form the ınterior of that A., ınt Å =∪{U ⊂ Å| U is open}. Additionally, any ıntersection of closed sets contained in Å defines the closure of Å., Cl Å =∩{F ⊃ Å| F is cloosed}.

3. Closure Operators on Digs

In this section, we introduced and study the concepts of closure operators on digs that depend on the pair of edges.Various established topological characteristics on the obtained of $DIGm$-closure spaces are studied,.

Definition 3.1: Let Ɖ; = (V(Ɖ), Ξ(Ɖ)) be a dig, P(V(Ɖ)) its power set of all subdigraphs of Ɖ and $CLU_{Ɖ}$:P(Ξ(Ɖ))→P(Ξ(Ɖ)) is a mapping associating with each subdigraph H̃=(V(H̃), Ξ(H̃)) a subdigraph $CLU_{Ɖ}$ (Ξ(H̃))⊆ Ξ(Ɖ) calleed the closure subdigraph of H̃ such that: $CLU_{Ɖ}$ (Ξ(H̃))=Ξ(H̃)∪{ϭ ∈ Ξ(Ɖ)\ Ξ(H̃); for all ȩ∈ Ξ(H̃), ȩ incident with ϭ in the same direction}.

The operation $CLU_{Ɖ}$ is called dig closure operator and the pair (V(Ɖ), $CL_{Ɖ}$) is called Ɖ - closure space, where $CL_{Ɖ}$ (Ξ(Ɖ)) is the family of elements of $CLU_{Ɖ}$. The dual of the dig closure operator $CLU_{Ɖ}$ is the dig ınterior operator $INT_{Ɖ}$:P(Ξ(Ɖ))→P(Ξ(Ɖ)) defined by $INT_{Ɖ}$(Ξ(H̃))=Ξ(Ɖ)\ $CLU_{Ɖ}$ (Ξ(Ɖ)\ Ξ(H̃)) for all subdigraph H̃ ⊆ Ɖ. A family of elements of $INT_{Ɖ}$ is called ınterior subdigraph of H̃ and denoted by $IN_{Ɖ}$(Ξ(Ɖ)). Clear that (Ξ(Ɖ), $IN_{Ɖ}$) is a topological space. Then the domaiin of $CLU_{Ɖ}$is equal to the domain of $INT_{Ɖ}$ and also $CLU_{Ɖ}$ (Ξ(H̃))=Ξ(Ɖ)\ $INT_{Ɖ}$(Ξ(Ɖ))\ Ξ(H̃)). A subdigraph H̃ of Ɖ -closure space (V(Ɖ),$CL_{Ɖ}$) is called closed subdigraph if $CLU_{Ɖ}$ (Ξ(H̃))=Ξ(H̃). It is called open subdigraph if its complement is closed subdigraph, i.e., $CLU_{Ɖ}$ (Ξ(Ɖ)) \ Ξ(H̃)) = Ξ(Ɖ) \ Ξ(H̃), or equivalently $INT_{Ɖ}$(Ξ(H̃)) =Ξ(H̃).

Example 3.2: Let Ɖ (V, D) be a dig such that V(Ɖ) $= \{a, b, c, d\}$, Ξ(Ɖ) $= \{ϭ_1, ϭ_2, ϭ_3, ϭ_4\}$. (See Figure 1 and Table 1)

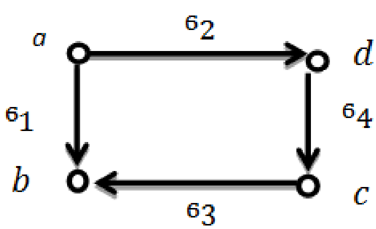

Figure 1
Digraph with 4 vertices and 4 arcs.

Table 1
The closure operator for subdigraphs
$\boldsymbol{CL_{\text{Ɖ}} = \{\Xi(\tilde{H}), \emptyset, \{\sigma_2, \sigma_4\}, \{\sigma_2, \sigma_3, \sigma_4\}, \{\sigma_3, \sigma_4\}\}},$
$\boldsymbol{mt_{\text{Ɖ}} = \{\Xi(\tilde{H}), \emptyset, \{\sigma_1\}, \{\sigma_1, \sigma_2\}\}}.$

$\Xi(\tilde{H})$	$CL_{\text{Ɖ}}$	$\Xi(\tilde{H})$	$CL_{\text{Ɖ}}$
$\Xi(\text{Ɖ})$	$\Xi(\text{Ɖ})$	$\{\sigma_1, \sigma_4\}$	$\Xi(\text{Ɖ})$
$\emptyset$	$\emptyset$	$\{\sigma_2, \sigma_3\}$	$\{\sigma_2, \sigma_3, \sigma_4\}$
$\{\sigma_1\}$	$\{\sigma_1\}$	$\{\sigma_2, \sigma_4\}$	$\{\sigma_2, \sigma_3, \sigma_4\}$
$\{\sigma_2\}$	$\{\sigma_2, \sigma_4\}$	$\{\sigma_3, \sigma_4\}$	$\{\sigma_2, \sigma_3, \sigma_4\}$
$\{\sigma_3\}$	$\{\sigma_3, \sigma_4\}$	$\{\sigma_1, \sigma_2, \sigma_3\}$	$\Xi(\text{Ɖ})$
$\{\sigma_4\}$	$\{\sigma_2, \sigma_3, \sigma_4\}$	$\{\sigma_1, \sigma_3, \sigma_4\}$	$\Xi(\text{Ɖ})$
$\{\sigma_1, \sigma_2\}$	$\{\sigma_1, \sigma_2, \sigma_4\}$	$\{\sigma_1, \sigma_2, \sigma_4\}$	$\Xi(\text{Ɖ})$
$\{\sigma_1, \sigma_3\}$	$\{\sigma_1, \sigma_3, \sigma_4\}$	$\{\sigma_2, \sigma_3, \sigma_4\}$	$\{\sigma_2, \sigma_3, \sigma_4\}$

Definition 3.3: Let $\text{Ɖ}\,(V, D)$ be a dig and $CL_{\text{Ɖ}}: P(\Xi(\text{Ɖ})) \rightarrow \mathrm{P}(\Xi(\text{Ɖ}))$ an operotor if

a) It is called Ɖ_n- closure operotor if $CLD_n\Xi(\tilde{H}) = ÇL_{\text{Ɖ}}(CL_{\text{Ɖ}}(ÇL_{\text{Ɖ}}$ $(\ldots CL_{\text{Ɖ}}\Xi(\tilde{H})$ n- timees for every subdigraph. $\tilde{H} \subseteq \text{Ɖ}$

b) It is called $\text{Ɖ} -$ topologiicalclosure operotor if $CL\ \text{Ɖ}_{n+1}\ (\Xi(\tilde{H}), CL\ \text{Ɖ}_n)$ for all subdigraph $\tilde{H} \subseteq \text{Ɖ}$ the space $(\Xi(\tilde{H}), C\ \text{Ɖ}_n)$ is called Ɖ_n- topological closure space.

Example 3.4: Let $\text{Ɖ} = (\mathrm{V}(\text{Ɖ}), \Xi(\text{Ɖ}))$ be a dig such that $V(\text{Ɖ}) = \{a, b, c, d\}$, $\Xi(\text{Ɖ}) = \{6_1, 6_2, 6_3, 6_4\}$. We note that $CL\ \text{Ɖ}_1 = \{6_1, 6_2, 6_4\}$, $CL\ \text{Ɖ}_2 = \{6_3\}$. (See Figure 2 and Table 2)

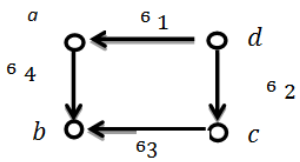

Figure 2
Digraph with 4 vertices and 4 arcs

Table 2
CL Ɖ_1 and CL Ɖ_2 for subdigraphs $\Xi(\tilde{H})$

$\Xi(\tilde{H})$	$CL\ \text{Ɖ}_1$	$CL\ \text{Ɖ}_2$	$\Xi(\tilde{H})$	$CL\ \text{Ɖ}_1$	$CL\ \text{Ɖ}_2$
$\Xi(\text{Ɖ})$	$\Xi(\text{Ɖ})$	$\Xi(\text{Ɖ})$	$\{6_1, 6_4\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\{6_2, 6_3\}$	$\Xi(\text{Ɖ})$	$\{6_1, 6_2, 6_3\}$
$\{_1\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$	$\{6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$
$\{6_2\}$	$\{6_1, 6_2\}$	$\{6_1, 6_2, 6_4\}$	$\{6_3, 6_4\}$	$\{6_1, 6_3, 6_4\}$	$\Xi(\text{Ɖ})$
$\{6_3\}$	$\{6_3\}$	$\{6_3\}$	$\{6_1, 6_2, 6_3\}$	$\Xi(\text{Ɖ})$	$\Xi(\text{Ɖ})$
$\{6_4\}$	$\{6_1, 6_4\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_3, 6_4\}$	$\Xi(\text{Ɖ})$	$\Xi(\text{Ɖ})$
$\{6_1, 6_2\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$	$\{6_1, 6_2, 6_4\}$
$\{6_1, 6_3\}$	$\Xi(\text{Ɖ})$	$\Xi(\text{Ɖ})$	$\{6_2, 6_3, 6_4\}$	$\Xi(\text{Ɖ})$	$\Xi(\text{Ɖ})$

Proposition 3.5: Let $Ð = (V(Ð), \Xi(Ð))$ be a dig and $(\Xi(\tilde{H}), C Ð_n))$ be $Ð_n$- closure space if $\tilde{H} = (V(Ð), \Xi(\tilde{H}))$, $Z = (V(Z), \Xi(Z))$ are two subdigraph of $Ð$ such that $\tilde{H} \subseteq Z \subseteq Ð$ then $CL\ Ð_n\ (\Xi(\tilde{H})) \subseteq CL\ Ð_n\ (\Xi(Z))$ and $IN_{Ð}(\Xi(Ð)) \subseteq IN_{Ð}(\Xi(Z))$.

Proof: Let $y \in CL\ Ð_n \implies y\Xi(\tilde{H}) \cup \{\sigma \in \Xi(Ð) \setminus \Xi(\tilde{H})$; for all $\hat{e} \in \Xi(\tilde{H})$, incidend with σ in the same direction$\} \implies y \in \Xi(\tilde{H})$ or $\{\sigma \in \Xi(Ð) \setminus \Xi(\tilde{H})$; for all $\hat{e} \in \Xi(\tilde{H})$, incidend with σ in the same direction$\} \implies y \in \Xi(\tilde{H})$ or $\exists h \in \Xi(\tilde{H})$; for all y $\in \Xi(\tilde{H}), y$ incidend with σ in the same direction since $\tilde{H} \subseteq Z \implies y \in \Xi(Z)$ or $\exists h \in \Xi(Z)$ or $\exists h \in \Xi(Z)$; such that y incidend with σ in the same direction $\implies y \in \Xi(Z)$ or $\{\sigma \in \Xi(Ð) \setminus \Xi(Z); \hat{e}$ incident with σ for all $\hat{e} \in \Xi(Z) \implies y \in \Xi(Z) \cup \{\sigma \in \Xi(Ð) \setminus \Xi(Z); \hat{e}$ incident with σ for all $\hat{e} \in \Xi(Z) \implies y \in CL\ Ð_n\ (\Xi(Z))$. Hence $CL\ Ð_n\ (\Xi(\tilde{H})) \subseteq CL\ Ð_n\ (\Xi(Z))$. Now, let $y \in IN\ Ð_n(\Xi(Ð) = \Xi(Ð) \setminus (CL\ Ð_n\ \Xi(\tilde{H}) \implies y \notin CL\ Ð_n\ (\Xi(Ð) \setminus \Xi(\tilde{H}))$, since $\Xi(\tilde{H}) \subseteq \Xi(Z) \implies \Xi(Ð) \setminus \Xi(Z) \subseteq \Xi(Ð) \setminus \Xi(\tilde{H}) \implies CL\ Ð_n\ (\Xi(Ð) \setminus \Xi(Z) \subseteq CL\ Ð_n\ (\Xi(Ð) \setminus \Xi(\tilde{H}))$. So $y \notin CL\ Ð_n\ (\Xi(Ð) \setminus \Xi(Z)) \implies y \in \Xi(Ð) \setminus CL\ Ð_n\ \Xi(Ð) \setminus \Xi(Z)) \implies y \in IN\ Ð_n(\Xi(Z)$. Hence $IN\ Ð_n(\Xi(\tilde{H})) \subseteq CL\ Ð_n\ (\Xi(Z))$.

Propostion 3.6: Let $Ð = (V(Ð), \Xi(Ð))$ be a dig $(\Xi(Ð), C\ Ð_n)$ be $Ð_n-$ closure space and $\tilde{H} = \left(V(\tilde{H}), \Xi(\tilde{H})\right), Z = \left(V(Z), \Xi(Z)\right)$ be two subdigraphs of $Ð$ then

a) $CL\ Ð_n\ (\Xi(\tilde{H}))\cap\Xi(Z) \subseteq CL\ Ð_n\ (\Xi(\tilde{H}))\cap CL\ Ð_n\ (\Xi(Z))$ and

b) $IN\ Ð_n\ (\Xi(\tilde{H}))\cup\Xi(Z) \subseteq IN\ Ð_n\ (\Xi(\tilde{H}))\cup IN\ Ð_n\ (\Xi(Z))$

Proof: Since $\Xi(\tilde{H}))\cap\Xi(Z) \subseteq (\Xi(\tilde{H})$ and $\Xi(\tilde{H}))\cap\Xi(Z) \subseteq (\Xi(Z) \implies CL\ Ð_n\ (\Xi(\tilde{H}))\cap\Xi(Z) \subseteq CL\ Ð_n\ (\Xi(\tilde{H}))$ and $CL\ Ð_n\ (\Xi(\tilde{H}))\cap\Xi(Z) \subseteq CL\ Ð_n\ (\Xi(Z)) \implies CL\ Ð_n\ (\Xi(\tilde{H}))\cap\Xi(Z) \subseteq CL\ Ð_n\ (\Xi(\tilde{H}))\cap CL\ Ð_n\ (\Xi(Z))$

b) Since $\Xi(\tilde{H})) \subseteq (\Xi(\tilde{H}) \cup (\Xi(Z))$ and $\Xi(Z)) \subseteq (\Xi(\tilde{H}) \cup (\Xi(Z) \implies IN\ Ð_n\ (\Xi(\tilde{H})) \subseteq IN\ Ð_n\ (\Xi(\tilde{H}) \cup (\Xi(Z))$ and $IN\ Ð_n\ (\Xi(Z)) \subseteq IN\ Ð_n\ (\Xi(\tilde{H}) \cup (\Xi(Z)) \implies IN\ Ð_n\ (\Xi(\tilde{H})) \cup IN\ Ð_n)\Xi(Z)) \subseteq IN\ Ð_n\ (\Xi(\tilde{H})) \cup (\Xi(Z))$

Definition 3.7: Let $Ð\ (V, D)$ be a dig and $(\Xi(Ð), C_{Ðn} -$ closure space

a) An open subdigraph $\tilde{H} = (V(\tilde{H}), \Xi(\tilde{H}))$ in $(\Xi(Ð), C_{Ðn})$ is called regular open subdigraph (barfly $R-$ open subdigraph) if $\Xi(\tilde{H}) = IN\ Ð_n\ (CL\ Ð_n\ (\Xi(\tilde{H}))$, the complement of $R-$ open subdigraph is called $R-$ closed subdigraph of $\tilde{H}$ is $Cl^R_{Ðn}(\Xi(\tilde{H})) = \cap\{\Xi(f); \Xi(f)$ is $R-$ closed subdigraph $\Xi(\tilde{H}) \subseteq \Xi(f)$, and $IN\ Ð^R_{Ðn}\ (\Xi(\tilde{H})) = \{\Xi(Ð) \setminus Cl^R_{Ðn}(\Xi(Ð) \setminus \Xi(\tilde{H})$ the family of all $R-$open subdigraph is denoted by $O^R_{Ðn}(V(Ð))$.

Example 3.8: Consider the dig in example (3.2) then we have $O^R_{Ðn}(\Xi(Ð)) = (\Xi(Ð), \emptyset, \{\sigma_2\}, \{\sigma_2, \sigma_3, \sigma_4\}, \{\sigma_3\}\}$

$$C^R_{Ꝓn}(\Xi(Ꝓ)) = \{\Xi(Ꝓ), \emptyset, \{\sigma_1, \sigma_3, \sigma_4\}, \{\sigma_1\}, \{\sigma_1, \sigma_2, \sigma_4\}\}$$

Example 3.9: Consider the dig in example (3.4) then we have $O^R_{Ꝓn}(\Xi(Ꝓ)) = \{\Xi(Ꝓ), \emptyset, \{\sigma_1, \sigma_2, \sigma_4\}\}$ $C^R_{Ꝓn}(\Xi(Ꝓ)) = \Xi(Ꝓ), \emptyset\{\sigma_3\}\}$

Definition 3.10: Let $Ꝓ = (V(Ꝓ), \Xi(Ꝓ))$be a dig $(V(Ꝓ), C_{cn})$ be $,Ꝓ_n$ −closure space and $\tilde{H} = (V(\tilde{H}), \Xi(\tilde{H}))$ be a subdigraph $Ꝓ = (V(Ꝓ), \Xi(\tilde{H}))$ then

The accuracy of $\tilde{H}$ is denoted by $M_{Ꝓn}\Xi(\tilde{H}) = \frac{|IN_{Ꝓn}(\Xi(\tilde{H}))|}{|CL_{Ꝓn}(\Xi(\tilde{H}))|}$ the R −accuracy of $\tilde{H}$ is denoted by $M^R_{Ꝓn}(\Xi(\tilde{H}) = \frac{|IN\, Ꝓ^R_{Ꝓn}(\Xi(\tilde{H}))|}{|Cl^R_{Ꝓn}(\Xi(\tilde{H}))|}$

Example 3.11: Consider the dig in example (3.2) and example (3.8) the accuracy of any subdigraph is (See Table 3)

Table 3
Accuracy $\mathbf{M_{Ꝓn}\Xi(\tilde{H})}$ and $\mathbf{M^R_{Ꝓn}(\Xi(\tilde{H})}$.

$\Xi(\tilde{H})$	$M_{Ꝓn}\Xi(\tilde{H})$	$M^R_{Ꝓn}(\Xi(\tilde{H})$	$\Xi(\tilde{H})$	$M_{Ꝓn}\Xi(\tilde{H})$	$M^R_{Ꝓn}(\Xi(\tilde{H})$
$\Xi(Ꝓ)$	1	1	$\{\sigma_1, \sigma_4\}$	0	1/4
$\emptyset$	0	0	$\{\sigma_2, \sigma_3\}$	1/3	0
$\{\sigma_1\}$	1/4	0	$\{\sigma_2, \sigma_4\}$	0	1/4
$\{\sigma_2\}$	0	0	$\{\sigma_3, \sigma_4\}$	0	0
$\{\sigma_3\}$	0	0	$\{\sigma_1, \sigma_2, \sigma_3\}$	1/4	0
$\{\sigma_4\}$	0	0	$\{\sigma_1, \sigma_3, \sigma_4\}$	0	0
$\{\sigma_1, \sigma_2\}$	1/2	1/4	$\{\sigma_1, \sigma_2, \sigma_4\}$	0	3/4
$\{\sigma_1, \sigma_3\}$	1/2	0	$\{\sigma_2, \sigma_3, \sigma_4\}$	0	0

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