

On a F-pseudo Z-b-subalgebra

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Abstract

The document outlined the definition of F-pseudo Z-b-subalgebra in addition to introducing fundamental properties. Multiple theorems have been established throughout the discussion using appropriate examples. We introduced a new concept after presenting the material.: (fuzzy set $\mu_{dz}^{(f)}$) of pseudo Z-algebra. This research examines both the homomorphic images and inverse images from this study. We denote b-sg to represent b-subalgebra in this article.

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1. Introduction

Zadeh was first introduced to the idea of fuzzy subsets in [1]. Z-algebras were introduced by S. Sowmiy and PJeyalakshmi, [2]. Adel and layth introduced the pseudo Z-algebra [3-11]. they also provided an introduction to some of its properties.

Theorem 1: A fuzzy set dz of a Z-algebra $(\mathcal{A}, \diamond, 0)$ is a fuzzy Z-ideal $\leftrightarrow$ for any $i \in [0,1]$, $U(dz; i) = \{x \in \mathcal{A} \mid \mu_{dz}(x) \geq i\}$ is an Z-ideal of $\mathcal{A}$ where $U(dz; i) \neq \emptyset$.

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Definition 2: A fuzzy subset μ of pseudo Z -algebra $\hat{A}$ is named F -pseudo Z - b -subalgebra of $\hat{A}$ if it fulfills the following axioms $\forall \hat{a}, \hat{z} \in \hat{A}, \exists \hat{b} \in \hat{A}$. Such that.

- 1) $\mu(b \blacklozenge(\hat{a} \blacksquare \hat{z})) \geq \min\{\mu(\hat{a}), \mu(\hat{z})\}$ · 2) $\mu(b \blacksquare(\hat{a} \blacklozenge \hat{z})) \geq \min\{\mu(\hat{a}), \mu(\hat{z})\}$.

Theorem 3: Let $\{\mu_{\hat{d}_j} \mid j \in \mathfrak{J}\}$ where $j \in \mathfrak{J}$ an index set, be a family of F -pseudo Z - b -sg of a pseudo Z -algebra $\hat{A}$. Then $\cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}$ is also a F -pseudo Z - b -sg of $\hat{A}$.

Proof: Let $\{\mu_{\hat{d}_j} \mid j \in \mathfrak{J}\}$ be a family of F -pseudo Z - b -sg of $\hat{A}$. Then, $\forall \hat{a}, \hat{z} \in \hat{A}, \exists \hat{b} \in \hat{A}$

- 1) $\cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}(b \blacksquare(\hat{a} \blacklozenge \hat{z})) = \inf\{\mu_{\hat{d}_j}(b \blacksquare(\hat{a} \blacklozenge \hat{z}))\} \geq \inf\{\min\{\mu_{\hat{d}_j}(\hat{a}), \mu_{\hat{d}_j}(\hat{z})\}\} = \min\{\inf\{\mu_{\hat{d}_j}(\hat{a})\}, \inf\{\mu_{\hat{d}_j}(\hat{z})\}\} = \min\{\cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}(\hat{a}), \cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}(\hat{z})\}$
- 2) $\cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}(b \blacklozenge(\hat{a} \blacksquare \hat{z})) = \inf\{\mu_{\hat{d}_j}(b \blacklozenge(\hat{a} \blacksquare \hat{z}))\} \geq \inf\{\min\{\mu_{\hat{d}_j}(\hat{a}), \mu_{\hat{d}_j}(\hat{z})\}\} = \min\{\inf\{\mu_{\hat{d}_j}(\hat{a})\}, \inf\{\mu_{\hat{d}_j}(\hat{z})\}\} = \min\{\cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}(\hat{a}), \cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}(\hat{z})\}$. Hence $\cap_{j \in \mathfrak{J}} \mu_{\hat{d}_j}$ is a F -pseudo Z - b -sg of a pseudo Z -algebra $\hat{A}$.

Theorem 4: Let $\mu_{\hat{d}_z}$ be a F -pseudo Z - b -sg of a pseudo Z -algebra $\hat{A}$ such that $\mu_{\hat{d}_z}(0) \neq 0$. Let $\mu_{\hat{b}_b}: \hat{A} \rightarrow [0, 1]$ be a fuzzy set defined by $\mu_{\hat{b}_b} = \frac{\mu_{\hat{d}_z}(\hat{a})}{\mu_{\hat{d}_z}(0)}$ for all $\hat{a} \in \hat{A}$. Then $\mu_{\hat{b}_b}$ is a F -pseudo Z - b -sg of $\hat{A}$.

Proof: Let $\hat{a}, \hat{z}, \hat{b} \in \hat{A}$ Then,

- 1) $\mu_{\hat{b}_b}(b \blacksquare(\hat{a} \blacklozenge \hat{z})) = \frac{\mu_{\hat{d}_z}(b \blacksquare(\hat{a} \blacklozenge \hat{z}))}{\mu_{\hat{d}_z}(0)} \geq \frac{1}{\mu_{\hat{d}_z}(0)} \min\{\mu_{\hat{d}_z}(\hat{a}), \mu_{\hat{d}_z}(\hat{z})\} = \min\left\{\frac{\mu_{\hat{d}_z}(\hat{a})}{\mu_{\hat{d}_z}(0)}, \frac{\mu_{\hat{d}_z}(\hat{z})}{\mu_{\hat{d}_z}(0)}\right\} = \min\{\mu_{\hat{b}_b}(\hat{a}), \mu_{\hat{b}_b}(\hat{z})\}$
- 2) the same way $\mu_{\hat{b}_b}(b \blacklozenge(\hat{a} \blacksquare \hat{z})) \geq \min\{\mu_{\hat{b}_b}(\hat{a}), \mu_{\hat{b}_b}(\hat{z})\}$ therefore $\mu_{\hat{b}_b}$ is a F -pseudo Z - b -sg of a pseudo Z -algebra $\hat{A}$.

Theorem 5: If $\mu_{\hat{d}_z}$ be a F -pseudo Z - b -sg of a pseudo Z -algebra $\hat{A}$, then the characteristic function $\mu_{\hat{d}_z} = \begin{cases} 1 & \text{if } \hat{a} \in \hat{d}_z \\ 0 & \text{if } \hat{a} \notin \hat{d}_z \end{cases}$ is a F -pseudo Z - b -sg of $\hat{A}$.

Proof: Let $\hat{a}, \hat{z}, \hat{b} \in \hat{A}$

Case (i): If $\hat{a}, \hat{z} \in \hat{d}_z, \mu_{\hat{d}_z}(\hat{a})=1, \mu_{\hat{d}_z}(\hat{z})=1$ then $\mu_{\hat{d}_z}(b \blacksquare(\hat{a} \blacklozenge \hat{z}))=1$ Thus

- 1) $\mu_{\hat{d}_z}(b \blacksquare(\hat{a} \blacklozenge \hat{z})) = \min\{\mu_{\hat{d}_z}(\hat{a}), \mu_{\hat{d}_z}(\hat{z})\} = \min\{1, 1\} = 1, \forall \hat{a}, \hat{z} \in \hat{A}$.
- 2) $\mu_{\hat{d}_z}(b \blacklozenge(\hat{a} \blacksquare \hat{z})) = 1$.

Thus $\mu_{\hat{d}_z}(b \blacklozenge(\hat{a} \blacksquare \hat{z})) = \min\{\mu_{\hat{d}_z}(\hat{a}), \mu_{\hat{d}_z}(\hat{z})\} = \min\{1, 1\} = 1, \forall \hat{a}, \hat{z} \in \hat{A}$.

Case (ii): If $\acute{a}, z \notin \mathcal{D}_Z$ then $\mu_{\mathcal{D}_Z}(\acute{a})=0, \mu_{\mathcal{D}_Z}(z)=0$ and

- 1) $\mu_{\mathcal{D}_Z}(\mathbf{b}(\acute{a} \blacklozenge z)) = \min \{\mu_{\mathcal{D}_Z}(\acute{a}), \mu_{\mathcal{D}_Z}(z)\} = \min \{0, 0\} = 0$
- 2) $\mu_{\mathcal{D}_Z}(\mathbf{b}(\acute{a} \blacklozenge z)) = \min \{\mu_{\mathcal{D}_Z}(\acute{a}), \mu_{\mathcal{D}_Z}(z)\} = \min \{0, 0\} = 0, \forall \acute{a}, z \in \mathcal{A}.$

Case (iii): If $\acute{a} \in \mathcal{D}_Z, z \notin \mathcal{D}_Z$ then $\mu_{\mathcal{D}_Z}(\acute{a})=1, \mu_{\mathcal{D}_Z}(z)=0$ and

- 1) $\mu_{\mathcal{D}_Z}(\mathbf{b}(\acute{a} \blacklozenge z)) = \min \{\mu_{\mathcal{D}_Z}(\acute{a}), \mu_{\mathcal{D}_Z}(z)\} = \min \{1, 0\} = 0$
- 2) $\mu_{\mathcal{D}_Z}(\mathbf{b}(\acute{a} \blacklozenge z)) = \min \{\mu_{\mathcal{D}_Z}(\acute{a}), \mu_{\mathcal{D}_Z}(z)\} = \min \{1, 0\} = 0, \forall \acute{a}, z \in \mathcal{A}.$

Case (iv): If $\acute{a} \notin \mathcal{D}_Z, z \in \mathcal{D}_Z$ then $\mu_{\mathcal{D}_Z}(\acute{a})=0, \mu_{\mathcal{D}_Z}(z)=1$ and

- 1) $\mu_{\mathcal{D}_Z}(\mathbf{b}(\acute{a} \blacklozenge z)) = \min \{\mu_{\mathcal{D}_Z}(\acute{a}), \mu_{\mathcal{D}_Z}(z)\} = \min \{0, 1\} = 0$
- 2) $\mu_{\mathcal{D}_Z}(\mathbf{b}(\acute{a} \blacklozenge z)) = \min \{\mu_{\mathcal{D}_Z}(\acute{a}), \mu_{\mathcal{D}_Z}(z)\} = \min \{0, 1\} = 0, \forall \acute{a}, z \in \mathcal{A}.$

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