Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 28 (2025), No. 2, pp. 577-585
DOI : 10.47974/JDMSC-2260

Cutoff fuzzy dominating set of strong fuzzy graph

Hayder Jasim Yousif *
Ali Ameer †
Department of Mathematics
General Directorate of Education
Diwaniya 54001
Iraq

Hatem Kareem Sulaiman §
Department of Mathematics
General Directorate of Education
Babylon 36001
Iraq

Abstract

This paper discussed a new parameter in the fuzzy graph $F_G = (\Omega, \eta)$. This parameter is named the cutoff fuzzy domination number and is represented by (γ_{cf}) and tested in a strong fuzzy graph. Some features and limitations of this parameter are presented, such as the fuzzy path, cycle, complete, complete bipartite, wheel, and star. In addition, the relation between this parameter and other known concepts in fuzzy graphs is introduced.

Subject Classification: *45D31, 45D33, 45C42.*

Keywords: *Cutoff-fuzzy dominating set, Cutoff-fuzzy domination number, Strong fuzzy graph.*

1. Introduction

The concept of the fuzzy graph, denoted as (F_G), was first put forth by Rosenfeld back in 1975 [1], taking inspiration from L. Zadeh's earlier exploration of fuzzy sets in 1965 [2]. This innovative idea has since found its

* *ORCID-ID:* 0000-0002-0065-8393
† *ORCID-ID:* 0009-0008-5354-4294
§ *ORCID-ID:* 0000-0001-8481-2222

* *E-mail:* hayderjasim36@gmail.com (Corresponding Author)
† *E-mail:* alialquzali89@gmail.com
§ *E-mail:* hatim19861986@gmail.com

way into a plethora of graph theory disciplines, including fuzzy graphs [3-8], topological graphs [9-11], and labeled graphs [12], among others [13-14]. The fascinating exploration of the domination number shapes discussions across numerous studies and applications. Now, when we dive into the world of fuzzy graphs, domination takes a particular turn as highlighted by A. Somasundaram and S. Somasundaram [15], who pointed out that domination in (F_G) hinges on certain operative boundaries. Another intriguing angle was offered by Nagoor Gani and Chanderskeran [17], who focused on the notion of domination through strong arcs. Meanwhile, Mahioub and Soner broadened the scope by analyzing domination numbers in fuzzy graphs, investigating all potential fuzzy dominating sets and aiming to capture the minimum sum of their membership values [18]. In a twist of ideas, Xavior and colleagues [19] proposed yet another interpretation of fuzzy domination. Let $\mathcal{V}$ be a finite and non-empty set of a fuzzy graph for a pair of functions $\Omega: \mathcal{V} \rightarrow [0,1]$ and $\eta: \mathcal{V} \times \mathcal{V} \rightarrow [0,1]$ such that $\eta\,(x,y) \leq min\{\Omega(x), \Omega(y)\}\, \forall x, y \in \mathcal{V}$ and denoted by $F_G = (\Omega, \eta)$. An edge (x,y) is called effective if $\eta(x,y) = min\,[\Omega(x), \Omega(y)]$ and (F_G) is named strong if each edge belongs to it is effective. A vertex x of (F_G) is said to be a cutoff vertex if $\eta\,(x,y) < min\{\Omega(x), \Omega(y)\}$ for all $y \in \mathcal{V} - \{x\}$. A set $\mathcal{D}$ for some vertices of (F_G) is fuzzy dominating set (FDS) of (F_G) if for every vertex $x \in \mathcal{V} - \mathcal{D}$ there exists $y \in \mathcal{D}$ such that (x,y) is effective edge and we say y dominates x or x is dominated by y. If the number of vertices of $\mathcal{D}$ is minimum then it is called a minimum fuzzy dominating set $(MFDS)$ of (F_G). Fuzzy domination number of (F_G) is the sum of membership values of vertices of $(MFDS)$ and it is denoted by γ_f. In this article, we deal with the strong fuzzy graph. For more information we refer [20-29].

2. Cutoff fuzzy dominating set $(\mathbf{CFD-set})$ of strong fuzzy graph $(\mathbf{F_G})$.

Definition 2.1: If $\mathcal{D}$ has at least one fuzzy cutoff vertex, it is referred to as a cutoff fuzzy dominating set $(CFD-set)$ of F_G.

Definition 2.2: If $\mathcal{D}$ does not contain any $(CFD-sets)$ it is known as a minimal $(CFD-set)$.

Definition 2.3: If all $(CFD-sets)$ have more vertices than or equal to the number of vertices in $\mathcal{D}$, then $\mathcal{D}$ of F_G is said to be minimum $(MCFD-set)$.

Definition 2.4: A cutoff fuzzy domination number of $F_G = (\Omega, \eta)$ is the minimum sum of membership values of the vertices for all $(MCFD-sets)$ and denoted by $\gamma_{cf}(F_G)$ or simply γ_{cf} (for example see **Figure 1**). $MCFD-sets$ are: $\mathcal{D}_1 = \{v_3, v_4\}$ and $\mathcal{D}_2 = \{v_2, v_5\}$. $\gamma_{cf} = min\{0.7, 0.4\} = 0.4$

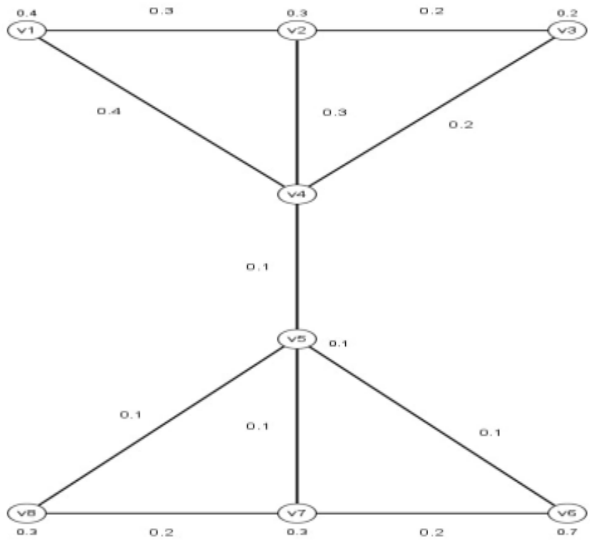

Figure 1
A cutoff fuzzy domination number of $F_G = (\Omega, \eta)$.

Remark 2.5:

i) $(MCFD - set)$ has a minimum fuzzy cardinality is said to be $\gamma_{cf} - set$.

ii) Every independent (FDS) in F_G is $(CFD - set)$.

Proposition 2.6: Let K_n^f be a complete fuzzy graph then $\gamma_{cf}(K_n^f) = min\, \Omega(v), v \in \mathcal{V}(K_n^f)$.

Proof: Since each vertex in K_n^f has $n-1$ effective neighbors, then any $(CFD - set)$ of K_n^f which $(MCF\mathcal{D} - set)$ contains only one vertex. Therefore $\gamma_{cf}(K_n^f) = min\, \Omega(v), v \epsilon \mathcal{V}(K_n^f)$

Proposition 2.7: Let W_n^f be a fuzzy wheel, then $\gamma_{cf}(W_n^f) = \Omega(v), v$ is center vertex.

Proof: Since the center vertex v of a wheel fuzzy graph W_n^f dominates all the other vertices, hence $\gamma_{IS}(W_n^f) = \Omega(v)$.

Proposition 2.8: Let S_n^f be a fuzzy star, then $\gamma_{cf}(S_n^f) = \Omega(v), v$ is center vertex of S_n^f.

Proposition 2.9: Let $K_{n,m}^f$ be a complete bipartite F_G then:

$$\gamma_{cf}\left(K_{n,m}^{f}\right) = \begin{cases} \sum \Omega(x_i), x_i \in \mathcal{V}_1 \ if\ m > n \\ \sum \Omega(y_i), y_i \in \mathcal{V}_2 \ if\ n > m \\ min\begin{cases} \sum \Omega(x_i), x_i \in \mathcal{V}_1 \\ \sum \Omega(y_i), y_i \in \mathcal{V}_2 \,| \end{cases} if\ m = n \end{cases}$$

Proof: Given $K_{n,m}^{f}$ be a complete bipartite F_G with two bipartitions (X, Y) such that $X = |\mathcal{V}|\ and\ Y = |U|$, then there are three cases:

Case 1: If $n < m$ and let $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$, $U = \{u_1, u_2, \dots, u_m\}$, then any $(MCFD - set)$ of $K_{n,m}^{f}$ contains all vertices of $\mathcal{V}_1$, therefore $\gamma_{cf}\left(K_{n,m}^{f}\right) = \sum \Omega\ (x_i), x_i \in \mathcal{V}_1$.

Case 2: if $n > m$ it is similarly to prove case 1.

Case 3: If $m = n$, it is clear that $\gamma_{cf}\left(K_{n,m}^{f}\right) = min\begin{cases} \sum \Omega(x_i), x_i \in \mathcal{V}_1 \\ \sum \Omega\ (y_i), y_i \in \mathcal{V}_2 \end{cases}$

Theorem 2.10: *Let* $F_G \cong$ *Petersen fuzzy graph* F_G , *then* $\gamma_{cf}(F_G) = min \sum \Omega\ (x_i), x_i \in \mathcal{D}, \mathcal{D}\ is\ (MCFD - set)$.

Proof: Let F_G be the Petersen fuzzy graph, since any set of two vertices can dominate at most 8 vertices, then any (FDS) which is also $(CFD - set)$ of (F_G) should contain at least three vertices. Therefore, $\gamma_{cf}\left(G_f\right) = min\ [\sum \Omega\ (x_i), x_i \in \mathcal{D}_j\ , \mathcal{D}_j\ \ is\ MCF\mathcal{D} - sets.$

Theorem 2.11: *If* F_G *is a disconnected fuzzy graph with* $F_{G1}, F_{G2}, \dots, F_{Gk}$ *as its components, then:* $\gamma_{cf}(F_G)\ = mi\,n\{s_i\}, 1 \le i \le k\ \ where\ \ s_i\ =\ \gamma_{cf}(F_{Gi}) + \sum_{j=1, j\neq i}^{k} \gamma_f(F_{Gj})$.

Proof: Suppose that $d = min\{s_1, s_2, \dots, s_k\}$ and let $\mathcal{D}$ be a $\gamma_{IS} - set$ of a F_{G1} also let A_i be a $\gamma_f - set$ of $G_{fi}\ \forall i \ge 2$. Then the set $\mathcal{D} \cup (\bigcup_{i=2}^{k} A_i)$ is $(CFD - set)$ of F_G, so that $\gamma_{cf}(F_G) \le \gamma_{cf}(F_{G1}) + \sum_{j=2}^{k} \gamma_f\left(F_{Gj}\right) = d = min\{s_i\}, 1 \le i \le k$. Now, let $\mathcal{D}$ be a minimal $(CFD - set)$ of F_G. Then $\mathcal{D}$ must intersect the $\mathcal{V}(F_{Gi})$ of every component F_{Gi}, and so $\mathcal{D} \cap \mathcal{V}(F_{Gi})$ is a minimal (FDS) of F_{Gi}, $\forall i = 1,2, \dots, k$. Further, one of the sets of $\mathcal{D} \cap \mathcal{V}(F_{Gi})$, say $\mathcal{D} \cap \mathcal{V}(F_{Gj})$, must be $(CFD - set)$ of F_{Gj}. Therefore $\gamma_{cf}(F_G) \ge \gamma_{cf}\left(F_{Gj}\right) + \sum_{i=1, j\neq i}^{k} \gamma_f(F_{Gi}) = d_j \ge min\{s_i\}, 1 \le i \le k$. Hence $\gamma_{cf}(F_G) = min\{s_i\}, 1 \le i \le k$.

Corollary 2.12: *For any* F_G *contains a cutoff vertex, then* $\gamma_{cf}(F_G) = \gamma_f(F_G)$.

Theorem 2.13: *If F_G is fuzzy graph of order n, then $\gamma_{cf} + \alpha_f \leq \sum \Omega(x)\ \forall\ x \in \mathcal{V}(G_f)$, where α_f is a fuzzy covering number of F_G.*

Proof: Let S be a $\alpha_f - set$ of (F_G). If a vertex $x \in S$ is not dominated by any vertex of $\mathcal{V} - S$, then $S - \{x\}$ is a fuzzy cover of (F_G) and that is not possible. Thus, $\mathcal{V} - S$ is (FDS) of F_G, since S is a fuzzy cover set of F_G, then $\mathcal{V} - S$ is independent fuzzy set and at the same time it is independent (FDS) of F_G. Hence, $\gamma_{cf} \leq |\mathcal{V} - S| = \sum \Omega\ (x) - \alpha_f$.

Theorem 2.14: *If F_G is a triangle-free fuzzy graph without any cutoff vertices, then any $(MCFD - set)$ of $\overline{F_G}$ contains exactly two vertices.*

Proof: Given F_G is a triangle-free fuzzy graph without cutoff vertices and since F_G is strong there exists effective edge $\{x, y\}$ in F_G. As F_G is a triangle-free fuzzy graph, then no vertex in (F_G) has effective edge to both x and y, then each vertex $z \notin \{x, y\}$ has effective edge either to x or to y in $\overline{F_G}$. Hence, the set $\{x, y\}$ say $\overline{\mathcal{D}}$ is $(CFD - set)$ of $\overline{F_G}$ which it is $(MCF\mathcal{D} - set)$. If $\overline{\mathcal{D}}$ has only one vertex, then (F_G) has a cutoff vertex. This contradicts our assumption.

Theorem 2.15: *Let $F_G = (\Omega, \eta)$ has no cutoff vertices and $\mathcal{D}$ is minimal $(CFD - set)$ of F_G, then $\mathcal{V} - \mathcal{D}$ is (FDS) of F_G.*

Proof: Suppose $\mathcal{V} - \mathcal{D}$ is not (FDS). Then there exists $y \in \mathcal{D}$ is not dominated by any vertex of $\mathcal{V} - \mathcal{D}$, because F_G has no cutoff vertices, then y has effective edge in $\mathcal{D}$. Thus, $\mathcal{D} - \{y\}$ is $(CFD - set)$ of F_G which is incongruity with minimality of $\mathcal{D}$.

Theorem 2.16: *Let $\mathcal{D}$ be any $(CFD - set)$ of F_G. Then $\mathcal{D}$ is minimal if and only if $\mathcal{D}$ is 1-minimal.*

Proof: Let $\mathcal{D}$ is $(CFD - set)$ of F_G. Suppose $\mathcal{D}$ is 1-minimal and there exists $A \subseteq \mathcal{D}$ is $(CFD - set)$ of F_G. Then the vertices in $A - \mathcal{D}$ are only the non-cutoff of $\mathcal{D}$. Now choose a vertex $x \in A - \mathcal{D}$, then the set $\mathcal{D} - \{x\}$ is $(CFD - set)$ of F_G, this is inconsistent with our assumption. The converse is directly.

Theorem 2.17: $(\mathrm{CFD} - \mathrm{set})$ of $\mathrm{F_G}$ is minimal $(\mathrm{CFD} - \mathrm{set})$ if and only if each vertex in $\mathcal{D}$ has a private neighbour with respect to $\mathcal{D}$.

Proof: Let $\mathcal{D}$ be a minimal $(CFD - set)$ and y in $\mathcal{D}$. If y is cutoff then y is a private neighours of itself. Suppose y is not cutoff. If y has no private neigbbours with respect to $\mathcal{D}$, then $\mathcal{D} - \{y\}$ will be $(CFD - set)$ of F_G. This offers an inconsistency to the minimality of $\mathcal{D}$ and so y must have a secluded neighbour with reverence to $\mathcal{D}$. Conversely, assume that $\mathcal{D}$ is $(CFD - set)$ of F_G in which each vertex has a private neighbour with respect to $\mathcal{D}$. If $\mathcal{D}$ is not minimal, by Theorem 2.15, following $\mathcal{D}$ is not $1 -$ minimal $(CFD - set)$ of F_G

and there is a vertex y in $\mathcal{D}$ such that $\mathcal{D}-\{y\}$ is $(CFD-set)$ of F_G. Therefore, every vertex in $\mathcal{V}-(\mathcal{D}-\{y\})$ must have at least one neighbour in $\mathcal{D}-\{y\}$ and so the vertex y can have no secluded neighbor with deference $\mathcal{D}$. This denies our supposition. So, the proof is done.

Corollary 2.18: *If $\mathcal{D}$ is $(CFD-set)$ of F_G which is minimal with respect to being cutoff domination, then $\mathcal{D}$ is a minimal (FDS) of F_G.*

***Proof**:* Let $\mathcal{D}$ be a minimal $(CFD-set)$ of F_G. Then by Theorem 2.15, every vertex of $\mathcal{D}$ has a private neighbour with respect to $\mathcal{D}$ and consequently by Theorem 3.6 in [16] implies that $\mathcal{D}$ is a minimal (FDS) of F_G.

Theorem 2.19: *Every maximal independent set is a minimal $(CFD-set)$.*

Proof: Let $\mathcal{D}$ be a maximal independent set then every vertex in $\mathcal{V}-\mathcal{D}$ has effective neighbour to at least one vertex of $\mathcal{D}$. Therefore $\mathcal{D}$ is (FDS). As $\mathcal{D}$ is an independent set it is actually $(CFD-set)$ and also every vertex of $\mathcal{D}$ has a private neigbour namely itself. Hence, by Theorem 2.16, the proof is complete.

3. Conclusion

This article provided,some features on cutoff fuzzy dominating set of strong fuzzy graphs. Furthermore, for the particular graphs referred above, its cutoff fuzzy domination number is deliberated.

References

[1] L.A. Zadeh, " Fuzzy Sets", *Information and Control*, vol. 8, pp. 338-353 (1965). [Online]. Available: https://doi.org/10.2307/2272014.

[2] A. Rosenfeld, "Fuzzy graphs, The support of the Office of Computing Activities, National Science Foundation, under Grant GJ-32258X, is gratefully acknowledged, as is the help of Shelly Rowe in preparing this paper.," pp. 77-95 (1975). [Online]. Available: https://doi.org/10.1016/b978-0-12-775260-0.50008-6.

[3] A. Kaufmann and A.P. Bonaert, "Introduction to the Theory of Fuzzy Subsets-vol. 1: Fundamental Theoretical Elements," *IEEE Transactions on Systems Man and Cybernetics*, vol. 7, no. 6, pp. 495-496 (1977). [Online]. Available: https://doi.org/10.1109/tsmc.1977.4309751.

[4] S.S. Kahat, A.A. Omran and M. Al-Harere, " Fuzzy equality co-neighborhood domination of graphs, "*Int. J. Nonlinear Anal. Appl.*, vol. 12, no. 2, pp. 537-545 (2021). [Online]. Available: https://doi.org/10.22075/ijnaa.2021.5101.

[5] H.H. Aljanaby and A.A. Omran, “, “Fuzzy hn-domination of strong fuzzy graphs,” *AIP Conference Proceedings,* vol. 2386, no. 1, p. 060001 (2022). [Online]. Available: https://doi.org/10.1063/5.0066879.

[6] H.J. Yousif and A.A. Omran, “The Split Anti Fuzzy Domination in Anti Fuzzy Graphs,” *J. Phys. Conf. Ser.* vol. 1591, no. 1, pp. 012054-012054 (2020). [Online]. Available: https://doi.org/10.1088/1742-6596/1591/1/012054.

[7] H.J. Yousif and A.A. Omran, “, “Closed Fuzzy Dominating Set in Fuzzy Graphs,” *J. Phys. Conf. Ser.* vol. 1879, no. 3, p. 032022 (2021). [Online]. Available: https://doi.org/10.1088/1742-6596/1879/3/032022.

[8] H.J. Yousif and A.A. Omran, “Some Results On The N —Fuzzy Domination in Fuzzy Graphs,” *J. Phys. Conf. Ser.* vol. 1879, no. 3, pp. 032009-032009 (2021). [Online]. Available: https://doi.org/10.1088/1742-6596/1879/3/032009.

[9] H.J. Yousif and A.A. Omran, “2-anti fuzzy domination in anti fuzzy graphs,” *IOP Conference Series Materials Science and Engineering*, vol. 928, no. 4, pp. 042027-042027 (2020). [Online]. Available: https://doi.org/10.1088/1757-899x/928/4/042027.

[10] A.A. Jabor and A.A. Omran, “Domination in discrete topology graph,” *Third International Conference of Mathematical Sciences* (ICMS), vol. 2183, p. 030006 (2019). [Online]. Available: https://doi.org/10.1063/1.5136110.

[11] A.A. Jabor and A.A. Omran, “Topological domination in graph theory,” *AIP Conference Proceedings* (2021). [Online]. Available: https://doi.org/10.1063/5.0042840.

[12] A.A. Jabor and A. Abd-Ali Omran, “ Hausdorff of a cycle in topological graph, “*Bol. Soc. Parana. Mat.* vol. 41, pp. 1-10, Dec. (2022). [Online]. Available: https://doi.org/10.5269/bspm.52013.

[13] M.N. Al-Harere and A.A. Omra, “Binary operation graphs,” *AIP conference proceedings,* vol. 2091, pp. 030008-030008 (2019). [Online]. Available: https://doi.org/10.1063/1.5095093.

[14] A.A. Omran and H.F. Ghazi, “The radius, diameter and chromatic number of some zero divisor graph”, *Int. J. Nonlinear Anal. Appl.,* vol. 13, no. 1, pp. 3891-3896 (2022). [Online]. Available: https://doi.org/10.22075/ijnaa.2022.6189.

[15] S. Al-Asadi, A. Omran, and F. Al-Maamori, “ Some Properties of Mobius Function Graph, “*Adv. Math. Models Appl.,* vol. 7, no. 1, pp. 48-54,

2022 (2025). [Online]. Available: http://jomardpublishing.com/UploadFiles/Files/journals/AMMAV1N1/V7N1/Al-Asadi_et_al.pdf.

[16] A. Somasundaram and S. Somasundaram, " Domination in fuzzy graphs-I," *Pattern Recognition Letters,* vol. 19, no. 9, pp. 787-791 (1998). [Online]. Available: https://doi.org/10.1016/s0167-8655(98)00064-6.

[17] A. Somasundaram, "Domination in fuzzy graphs-II", *J. Fuzzy Math*, vol. 13, no. 2, pp. 281 (2005).

[18] A. Nagoor Gani and V.T. Chandrasekaran, "Domination in fuzzy graph", *Advances in Fuzzy Sets And Systems,* vol. 1, no. 1, pp. 17-26 (2006).

[19] D.A. Xavior, F. Isido and V. M. Chitra, " On domination in fuzzy graphs", *Int. J. Comput. Algorithm,* vol. 2, no. 2, pp.81-82 (2013).

[20] W.M. Salah and H.B. Shelash, " Structure Graph of the Finite Group," *Journal of Physics Conference Series,* vol. 1818, no. 1, pp. 012126-012126 (2021). [Online]. Available: https://doi.org/10.1088/1742-6596/1818/1/012126.

[21] A.A. Shukur, A. Jahanbani, and H. Shelash, " The Behavior of Weighted Graph's Orbit and Its Energy, "*Math. Probl. Eng.,* vol. 2021, pp. 1-6 (2021). [Online]. Available: https://doi.org/10.1155/2021/9933072.

[22] H.R. Hashim, F. Luca, H.B. Shelash, and A.A. Shukur, "Generalized Lucas graphs", *Afrika Matematika*, vol. 34, no. 1 (2023). [Online]. Available: https://doi.org/10.1007/s13370-023-01048-6.

[23] H.B. Shelash and A.R. Ashrafi, " The Number of Subgroups of a Given Type in Certain Finite Groups", *Iran. J. Math. Sci. Inform.*, vol. 16, no. 2, pp. 73-87 (2021). [Online]. Available: https://doi.org/10.52547/ijmsi.16.2.73.

[24] S. Hussain, F. Afzal, D. Afzal, M. R. Farahani, M.Cancan, and S. Ediz, " Theoretical study of benzene ring embedded in P-type surface in 2d network using some new degree based topological indices via M-polynomial," *Eurasian Chem. Commun.*, vol. 3, no. 3, pp. 180-186 (2021). [Online] Available: https://doi.org/10.22034/ecc.2021.272315.1134.

[25] S. Ahmad, H. M. A. Siddiqui, A. Ali, M. R. Farahani, M. Imran, and I. N. Cangul, "On Wiener index and Wiener polarity index of some polyomino chains," *J. Discrete Math. Sci. Cryptogr.,* vol. 22, no. 7, pp. 1151–1164 (2019). [Online]. Available: https://doi.org/10.1080/09720529.2019.1688965.

[26] A. J. M. Khalaf, A. Q. Baig, M. R. Azhar, M. Imran, and M. R. Farahani, “The Eccentric Based Zagreb Indices of Carbon,” *J. Discrete Math. Sci. Cryptogr.,* vol. 23, no. 6, pp. 1121–1137 (2020). [Online]. Available: https://doi.org/10.1080/09720529.2020.1818448.

[27] M. Imran, A. A. E. Abunamous, D. Adi, S. H. Rafique, A. Q. Baig, and M. R. Farahani, “Eccentricity based topological indices of honeycomb networks,” *J. Discrete Math. Sci. Cryptogr.,* vol. 22, no. 7, pp. 1199–1213 (2019). [Online]. Available: https://doi.org/10.1080/09720529.2019.1691326.

[28] M. Imran, M. K. Siddiqui, S. Ahmad, M. F. Hanif, M. H. Muhammad, and M. R. Farahani, “Topological properties of Benzenoid, phenylenes and nanostar dendrimers,” *J. Discrete Math. Sci. Cryptogr.,* vol. 22, no. 7, pp. 1229–1248 (2019). [Online]. Available: https://doi.org/10.1080/09720529.2019.1701267.

[29] A. J. M. Khalaf, M. F. Hanif, M. K. Siddiqui, and M. R. Farahani, “On degree based topological indices of bridge graphs,” *J. Discrete Math. Sci. Cryptogr.,* vol. 23, no. 6, pp. 1139–1156 (2020). [Online]. Available: https://doi.org/10.1080/09720529.2020.1822040.

Received November, 2024