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Consequence for the resolution of the complex terms in the case (9,8,3)

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Abstract

The target of this treatise is to prove that the concatenation of the terms of characteristic zero (cha.zer.) resolution (reso.) is complex in the case (9,8,3) by ply the injective of maps, Capelli identities (Cap.ide.), planner and place polarization (pl.pol.).

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1. Introduction

Anywhere this treatise $\mathcal{R}$ commutative ring and has 1, $\mathcal{F}$ free $\mathcal{R}$-moduleand $\mathcal{D}_i\mathcal{F}$ be the divided power (div.pow.) algebra of degree i, $\Lambda\mathcal{F}$ is the exterior algebra of finitely generated free $\mathcal{R}$-module $\mathcal{F}$ and for the partition λ/μ, $\mathcal{K}_{\lambda/\mu}\mathcal{F} = \mathrm{Im}(d'_{\lambda/\mu})$ where $d'_{\lambda/\mu}$ is the Weyl map $d'_{\lambda/\mu}: \mathcal{D}\mathcal{F} \to \Lambda\mathcal{F}$, [1]. The maps $\sum_{k>0} \mathcal{D}_{p+t_1+k} \otimes \mathcal{D}_{q-t_1-k} \otimes \mathcal{D}_r \to \mathcal{D}_p \otimes \mathcal{D}_q \otimes \mathcal{D}_r$ is k^{th} div.pow.of the pl.pol. from place 1 to place 2 ($\partial_{21}^{(k)}$), and the maps $\sum_{e>0} \mathcal{D}_p \otimes \mathcal{D}_{q+t_2+e} \otimes \mathcal{D}_{r-t_2-e} \to \mathcal{D}_p \otimes \mathcal{D}_q \otimes \mathcal{D}_r$ explicated as e^{th} div.pow. of the pl.pol. from place 2 to place 3 ($\partial_{32}^{(e)}$), and as in two-rowed case, where the resolution of the Weyl module with the three-rowed $(p, q, r; t_1, t_2)$, [2,3]. Anywhere this treatise we deficiency these Cap.ide.

$$\partial_{32}^{(2)}\,\partial_{21} = \partial_{21}\,\partial_{32}^{(2)} + \partial_{32}\,\partial_{31} \quad \ldots(1)$$

$$\partial_{32}\,\partial_{21}^{(2)} = \partial_{21}^{(2)}\,\partial_{32} + \partial_{21}\,\partial_{31} \quad \ldots(2)$$

$$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31} \quad \ldots(3)$$

$$\partial_{32}\,\partial_{31} = \partial_{31}\,\partial_{32} \quad \ldots(4)$$

The cases (3,3,2), (8,7,3), (6,6,4;0,0) and (8,8)/(1,0) studied respectively in [4-7]. We stratify the same idea in [8] for the case (9,8,3). Also we can employ that for 2-visible submodules and modules, $\mathcal{PSL}(2,\mathfrak{U})$, where $\mathfrak{U}$ = 23, 29, 31, 37, [9-11]$\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U}$ = 23, 29, 31, 37, 125, 3125, [12-14], $\mathcal{S.U.T.}(2,p)$, for a prime $p \leq 9$ and rad-quasi-prime submodules which are study in [15-16].

2. The treatise and the Results

The cha. ze. reso. in the case (9,8,3) has the formulation:-

$$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$
$$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \to \oplus \to \oplus \to\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$
$$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$$

Behold the press planner:

$$\begin{array}{ccccccc}
\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & \overset{t_1}{\to} & \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} & \overset{t_2}{\to} & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\
\downarrow f_1 & \mathcal{O} & \downarrow f_2 & \mathcal{J} & \downarrow f_3 \\
\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & \underset{u_1}{\longrightarrow} & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} & \underset{u_2}{\longrightarrow} & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
\end{array}$$

Define the maps $t_1(k) = \partial_{32}(k)$, $f_1(k) = \partial_{21}(k)$, and $f_2(k) = \partial_{21}^{(2)}(k)$.

We requirement to acquaint the map u_1 and by (1) which manufacture the planner $\mathcal{O}$ commute. Thus $u_1 \circ f_1 = f_2 \circ d_1$

$$\mathcal{u}_1 \circ \partial_{21} = \partial_{21}^{(2)}\partial_{32} = \partial_{32}\partial_{21}^{(2)} - \partial_{31}\partial_{21} = (\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31})\partial_{21}.$$

Hence, $\mathcal{u}_1 = \frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}$.

If we acquaint the maps $\mathcal{u}_2(\mathcal{k}) = \partial_{32}(\mathcal{k})$ and $\mathcal{f}_3(\mathcal{k}) = \partial_{21}(\mathcal{k})$.

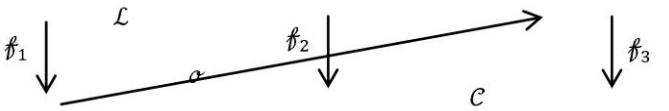
We requirement to acquaint the map $\mathcal{t}_2$ and by (2) which manufacture the planner $\mathcal{J}$ commute. Thus $\mathcal{f}_3 \circ \mathcal{t}_2 = \mathcal{u}_2 \circ \mathcal{f}_2$

$$\partial_{21} \circ \mathcal{t}_2 = \partial_{32}\partial_{21}^{(2)} = \partial_{32}\partial_{21}^{(2)} = \partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31} = \partial_{21}(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31})$$

Hence, $\mathcal{t}_2 = \frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}$.

Behold the press planner and acquaint $\sigma(\mathcal{k}) = \partial_{32}^{(2)}(\mathcal{k})$

$$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\mathcal{t}_1} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \xrightarrow{\mathcal{t}_2} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$



$$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \underset{\mathcal{u}_1}{\longrightarrow} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \underset{\mathcal{u}_2}{\longrightarrow} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Proposition 2.1: The planner $\mathcal{L}$ is commute.

Proof: $\mathcal{t}_2 \circ \mathcal{t}_1 = (\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31})\partial_{32} = \partial_{21}\partial_{32}^{(2)} + \partial_{31}\partial_{32} = \partial_{32}^{(2)}\partial_{21} = \sigma \circ \mathcal{f}_1$.

Proposition 2.2: The planner $\mathcal{C}$ is commute.

Proof: $\mathcal{u}_2 \circ \mathcal{u}_1 = \partial_{32}(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}) = \partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31} = \partial_{21}\partial_{32}^{(2)} = \mathcal{f}_3 \circ \sigma$.

Proposition 2.3: The press concatenation is complex

$$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$
$$0 \to \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\ell_3} \oplus \xrightarrow{\ell_2} \oplus \xrightarrow{\ell_1} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$
$$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$$

Where $\ell_3(\mathcal{k}) = \big(\mathcal{t}_1(\mathcal{k}), \mathcal{f}_1(\mathcal{k})\big)$, $\ell_2((\mathcal{k}_1, \mathcal{k}_2)) = (\mathcal{t}_2(\mathcal{k}_1) - \sigma(\mathcal{k}_2), \mathcal{u}_1(\mathcal{k}_2) - \mathcal{f}_2(\mathcal{k}_1))$ and $\ell_1((\mathcal{k}_1, \mathcal{k}_2)) = \big(\mathcal{f}_3(\mathcal{k}_1) + \mathcal{u}_2(\mathcal{k}_2)\big)$.

Proof: By (1)-(4) and the injective of ∂_{21} and ∂_{32} [3] we gain ℓ_3 is also injective

$$(\ell_2 \circ \ell_3)(\mathcal{k}) = \ell_2\big(\mathcal{t}_1(\mathcal{k}), \mathcal{f}_1(\mathcal{k})\big)$$
$$= \Big(\mathcal{t}_2\big(\partial_{32}(\mathcal{k})\big) - \sigma\big(\partial_{21}(\mathcal{k})\big), \mathcal{u}_1\big(\partial_{21}(\mathcal{k})\big) - \mathcal{f}_2\big(\partial_{32}(\mathcal{k})\big)\Big)$$
$$\mathcal{t}_2\big(\partial_{32}(\mathcal{k})\big) - \sigma\big(\partial_{21}(\mathcal{k})\big) = (\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31} + \partial_{31}\partial_{32} - \partial_{32}^{(2)}\partial_{21})(\mathcal{k}) = 0$$
$$\mathcal{u}_1\big(\partial_{21}(\mathcal{k})\big) - \mathcal{f}_2\big(\partial_{32}(\mathcal{k})\big) = (\partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31} - \partial_{31}\partial_{21} - \partial_{21}^{(2)}\partial_{32})(\mathcal{k}) = 0$$
$$(\ell_2 \circ \sigma\ell_3)(\mathcal{k}) = 0.$$

By the same tactic $\ell_1 \circ \ell_2 = 0$.

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