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Complex of characteristic zero for (13,10,3)/(0,0)

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Abstract

The elements of the complex of characteristic-zero resolution for the partition (13,10,3)/(0,0) survey in this work with prove that the sequence (seq.) of these term is complex, for any commutative ring has 1 and a free module.

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Keywords: *Weyl module, Characteristic-zero, Complex terms, Resolution, Place polarization.*

1. Introduction

Searchers in [1-3] study different cases. We survey the resolution of the Weyl module in this work for the partition (13,10,3)/(0,0) as in [4]. We gain the elements of the characteristic-zero resolution, and from the diagrams of these terms prove that the sequence of them is complex.

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We can apply the same idea for $\mathcal{SL}(2,\mathfrak{U})$ where $\mathfrak{U}$ = 81, 729, 125,3125,121, 196, [5-7], also the ideas in [8-11], Mxn-open sets and LC-Spaces, [12-15, 17, 18].

2. The Results

As in [2] the seq. of the elements of characteristic zero is

$$0 \to D_{15}\mathcal{F}\otimes \mathrm{D}_{10}\,\mathcal{F}\otimes \mathrm{D}_1\mathcal{F} \to \mathrm{D}_{15}\mathcal{F}\otimes \mathrm{D}_9\mathcal{F} \otimes \mathrm{D}_2\mathcal{F}\oplus$$
$$\mathrm{D}_{14}\mathcal{F}\otimes \mathrm{D}_{11}\mathcal{F}\otimes \mathrm{D}_1\mathcal{F} \to \mathrm{D}_{13}\mathcal{F}\otimes \mathrm{D}_{11}\mathcal{F}\otimes \mathrm{D}_2\mathcal{F}\oplus \mathrm{D}_{14}\mathcal{F}\otimes \mathrm{D}_9\,\mathcal{F}\otimes \mathrm{D}_3\mathcal{F}$$
$$\to \mathrm{D}_{13}\mathcal{F}\otimes \mathrm{D}_{10}\,\mathcal{F}\otimes \mathrm{D}_3\mathcal{F} \to 0$$

Consider the following Diagram:

$$\begin{tikzcd}
D_{15}\mathcal{F}\otimes \mathrm{D}_{10}\,\mathcal{F}\otimes \mathrm{D}_1\mathcal{F} \arrow[r, "r_1"] \arrow[d, "\ell_1"'] \arrow[dr, phantom, "\mathrm{H}"] & \mathrm{D}_{15}\mathcal{F}\otimes \mathrm{D}_9\mathcal{F} \otimes \mathrm{D}_2\mathcal{F} \arrow[r, "r_2"] \arrow[d, "\ell_2"] \arrow[dr, phantom, "\mathrm{Z}"] & \mathrm{D}_{14}\mathcal{F}\otimes \mathrm{D}_9\,\mathcal{F}\otimes \mathrm{D}_3\mathcal{F} \arrow[d, "\ell_3"] \\
\mathrm{D}_{14}\mathcal{F}\otimes \mathrm{D}_{11}\mathcal{F}\otimes \mathrm{D}_1\mathcal{F} \arrow[r, "\mathrm{e}_1"'] & \mathrm{D}_{13}\mathcal{F}\otimes \mathrm{D}_{11}\mathcal{F}\otimes \mathrm{D}_2\mathcal{F} \arrow[r, "\mathrm{e}_2"'] & \mathrm{D}_{13}\mathcal{F}\otimes \mathrm{D}_{10}\,\mathcal{F}\otimes \mathrm{D}_3\mathcal{F}
\end{tikzcd}$$

We specify $r_1(\mathrm{v}) = \partial_{32}(\mathrm{v})$, $\ell_1(\mathrm{v}) = \partial_{21}(\mathrm{v})$, and $\ell_2(\mathrm{v}) = \partial_{21}^{(2)}(\mathrm{v})$

Proposition 2.1: The commute of $\mathcal{H}$ Diagram.

Proof: We prove $\mathrm{e}_1 \circ \ell_1 = \ell_2 \circ r_1$

$$\mathrm{e}_1 \circ \partial_{21} = \partial_{21}^{(2)} \circ \partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31} = \frac{1}{2}\partial_{32}\,\partial_{21}\,\partial_{21} - \partial_{31}\,\partial_{21}$$
$= \left(\frac{1}{2}\partial_{32}\,\partial_{21} - \partial_{31}\right) \circ \partial_{21}$. Thus, $\mathrm{e}_1 = \frac{1}{2}\partial_{32}\,\partial_{21} - \partial_{31}$.

If we acquaint the map $\mathrm{e}_2\,(\mathrm{v}) = \partial_{32}(\mathrm{v})$ and $\ell_3(\mathrm{v}) = \partial_{21}(\mathrm{v})$.

Proposition 2.2: The commute of Z Diagram.

Proof: We prove $\ell_3 \circ r_2 = \mathrm{e}_2 \circ \ell_2$

$$\partial_{21} \circ r_2 = \partial_{32} \circ \partial_{21}^{(2)} = \partial_{21}^{(2)}\,\partial_{32} + \partial_{21}\ \partial_{31} = \frac{1}{2}\partial_{21}\,\partial_{21}\,\partial_{32} + \partial_{21}\ \partial_{31}$$
$= \left(\frac{1}{2}\partial_{21}\,\partial_{32} + \partial_{31}\right) \circ \partial_{21}$. Thus, $r_2 = \frac{1}{2}\partial_{21}\,\partial_{32} + \partial_{31}$.

Consider the following diagram:

$$\begin{tikzcd}
D_{15}\mathcal{F}\otimes \mathrm{D}_{10}\,\mathcal{F}\otimes \mathrm{D}_1\mathcal{F} \arrow[r, "r_1"] \arrow[d, "\ell_1"'] \arrow[dr, phantom, "\mathcal{X}"] & \mathrm{D}_{15}\mathcal{F}\otimes \mathrm{D}_9\mathcal{F} \otimes \mathrm{D}_2\mathcal{F} \arrow[r, "r_2"] \arrow[d, phantom, "\mathcal{G}"] \arrow[dr, phantom, "\mathcal{Y}"] & \mathrm{D}_{14}\mathcal{F}\otimes \mathrm{D}_9\,\mathcal{F}\otimes \mathrm{D}_3\mathcal{F} \arrow[d, "\ell_3"] \\
\mathrm{D}_{14}\mathcal{F}\otimes \mathrm{D}_{11}\mathcal{F}\otimes \mathrm{D}_1\mathcal{F} \arrow[r, "\mathrm{e}_1"'] \arrow[urr] & \mathrm{D}_{13}\mathcal{F}\otimes \mathrm{D}_{11}\mathcal{F}\otimes \mathrm{D}_2\mathcal{F} \arrow[r, "\mathrm{e}_2"'] & \mathrm{D}_{13}\mathcal{F}\otimes \mathrm{D}_{10}\,\mathcal{F}\otimes \mathrm{D}_3\mathcal{F}
\end{tikzcd}$$

Define $\mathcal{G} : \mathrm{D}_{14}\mathcal{F}\otimes\mathrm{D}_{11}\mathcal{F}\otimes\mathrm{D}_{1}\mathcal{F} \longrightarrow \mathrm{D}_{14}\mathcal{F}\otimes\mathrm{D}_{9}\mathcal{F}\otimes\mathrm{D}_{3}\mathcal{F}$ by

$$\mathcal{G}(v) = \partial_{32}^{(2)}(v);\ \text{where } v \in \mathrm{D}_{14}\mathcal{F}\otimes\mathrm{D}_{11}\mathcal{F}\otimes\mathrm{D}_{1}\mathcal{F}$$

Proposition 2.3: The commutative of $\boldsymbol{\mathcal{X}}$ diagram.

Proof: To prove $r_2 \circ r_1 = \mathcal{G} \circ \ell_1$

$$r_2 \circ r_1 = \left(\frac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right) \circ \partial_{32} = \partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31} + \partial_{31}\partial_{32}$$

$$= \partial_{32}^{(2)}\partial_{21} = \mathcal{G} \circ \ell_1.$$

Proposition 2.4: The commutative of $\mathcal{Y}$ diagram.

Proof: To prove $e_2 \circ e_1 = \ell_3 \circ \mathcal{G}$

$$e_2 \circ e_1 = \partial_{32} \circ \left(\frac{1}{2}\partial_{21}\partial_{32} - \partial_{31}\right) = \partial_{21}\partial_{32}^{(2)} + \partial_{32}\partial_{31} - \partial_{31}\partial_{32}$$

$$= \partial_{21}^{(2)}\partial_{32}^{(3)} = \ell_3 \circ \mathcal{G}.$$

Finally, we define the maps σ_1, σ_2 *and* σ_3 in the following sequence as: $\sigma_3(\varkappa) = \left(r_1(\varkappa), \ell_1(\varkappa)\right)$, $\sigma_2(\varkappa_1,\varkappa_2) = \left(r_2(\varkappa_1) - \mathcal{G}(\varkappa_2), e_1(\varkappa_2) - \ell_2(\varkappa_1)\right)$ and $\sigma_1(\varkappa_1,\varkappa_2) = (\ell_3(\varkappa_1), e_2(\varkappa_2))$

Theorem 2.5: The following sequence is complex

$$0 \to D_{15}\mathcal{F}\otimes\mathrm{D}_{10}\mathcal{F}\otimes\mathrm{D}_{1}\mathcal{F} \xrightarrow{\sigma_3} \mathrm{D}_{15}\mathcal{F}\otimes\mathrm{D}_{9}\mathcal{F}\otimes\mathrm{D}_{2}\mathcal{F}\oplus$$
$$\mathrm{D}_{14}\mathcal{F}\otimes\mathrm{D}_{11}\mathcal{F}\otimes\mathrm{D}_{1}\mathcal{F} \xrightarrow{\sigma_2} \mathrm{D}_{13}\mathcal{F}\otimes\mathrm{D}_{11}\mathcal{F}\otimes\mathrm{D}_{2}\mathcal{F}\oplus\mathrm{D}_{14}\mathcal{F}\otimes\mathrm{D}_{9}\mathcal{F}\otimes\mathrm{D}_{3}\mathcal{F}$$
$$\xrightarrow{\sigma_1} \mathrm{D}_{13}\mathcal{F}\otimes\mathrm{D}_{10}\mathcal{F}\otimes\mathrm{D}_{3}\mathcal{F} \to 0$$

Proof: From [16] ∂_{21} and ∂_{32}an injection, we have

$$(\sigma_2 \circ \sigma_3)(\varkappa) = \sigma_2 \circ \left(r_1(\varkappa), \ell_1(\varkappa)\right) = \sigma_2(\partial_{32}(\varkappa), \partial_{21}(\varkappa))$$

$$= \left(r_2\left(\partial_{32}(\varkappa)\right) - \mathcal{G}\left(\partial_{21}(\varkappa)\right), e_1(\partial_{21}(\varkappa)) - \ell_2\left(\partial_{32}(\varkappa)\right)\right),\ \text{also}$$

$$r_2\left(\partial_{32}(\varkappa)\right) - \mathcal{G}\left(\partial_{21}(\varkappa)\right) = \left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)\partial_{32}(\varkappa) - \partial_{32}^{(2)}\partial_{21}(\varkappa)$$

$$= (\partial_{21}\partial_{32}^{(2)} + \partial_{31}\partial_{32} - \partial_{32}^{(2)}\partial_{21})(\varkappa)$$

$$= (\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31} + \partial_{31}\partial_{32} - \partial_{32}^{(2)}\partial_{21})(\varkappa) = 0$$

$$e_1\left(\partial_{21}(\varkappa)\right) - \ell_2\left(\partial_{32}(\varkappa)\right) = \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\partial_{21}(\varkappa) - \partial_{21}^{(2)}\partial_{32}(\varkappa)$$

$$= \left(\partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31} - \partial_{31}\partial_{21} - \partial_{21}^{(2)}\partial_{32}\right)(\varkappa) = 0$$

Thus, $(\sigma_2 \circ \sigma_3)(\varkappa) = 0$. And

$$(\sigma_1 \circ \sigma_2)(\varkappa_1,\varkappa_2) = \sigma_1 \circ \left(r_2(\varkappa_1) - \mathcal{G}(\varkappa_2), e_1(\varkappa_2) - \ell_2(\varkappa_1)\right)$$

$$= \sigma_1 \circ \left(\left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(\varkappa_1) - \partial_{32}^{(2)}(\varkappa_2),\right.$$

$$(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31})(\varkappa_2) - \partial_{21}^{(2)}(\varkappa_1)\Big)$$

$$= \big(\partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31} - \partial_{32}\partial_{21}^{(2)}\big)(\varkappa_1) + \big(\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31} + \partial_{21}\partial_{32}^{(2)}\big)(\varkappa_2)$$

$$= \big(\partial_{32}\partial_{21}^{(2)} - \partial_{21}\partial_{31} + \partial_{21}\partial_{31} - \partial_{32}\partial_{21}^{(2)}\big)(\varkappa_1) +$$

$$\big(\partial_{21}\partial_{32}^{(2)} - \partial_{32}\partial_{31} + \partial_{32}\partial_{31} - \partial_{21}\partial_{32}^{(2)}\big)(\varkappa_2) = 0$$

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