

Application for the skew shape $(11,10,3;1,1)$

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Abstract

Many applications can be apply for the skew shape $(11,10,3;1,1)$ in this search we emply the mapping Cone (map.Co.) to gain the sequence (sequ.) of the elements of the Weyl module (We.mo.) resolution (reso.) for Lascoux (L.) is exact by inusage the diagrams (diag.) of map.Co. and employing the Capelli identities.

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1. Introduction

The We.mo. is $\mathcal{K}_{\lambda/\mu}\mathcal{F} = \text{Im}(\mathcal{D}\mathcal{F} \rightarrow \wedge\mathcal{F})$ for the partition λ/μ , we need in these work Capelli identities which are illustrated in [1].

In this survey we applied the same idea in [2] for (11,10,3;1,1). Also can applied that for $\mathcal{SL}(2, \mathcal{U})$, $\mathcal{U}=81, 729, 125, 3125, 121, 196, [3-5]$, (α, β) Derivation, Asymptotic Behavior, $\text{PG}(3,8)$ [6-9], Also the studies by authors in [10-13] and $\mathcal{PSL}(2, \mathcal{U})$, $\mathcal{SL}(2, \mathcal{U})$, $\mathcal{U}=23, 29, 31, 37, [14-17]$. Also [18-20].

2. Result for the sequ. of terms of reso. of L.

For (11,10,3;1,1) the terms of reso. of L.

$$\begin{aligned} & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ 0 \rightarrow & \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \rightarrow \oplus \\ & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \rightarrow & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \rightarrow 0 \end{aligned}$$

Take the dig.

$$\begin{array}{ccccc} 0 \rightarrow \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & \xrightarrow{h_1} & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & & \\ & \downarrow f_1 & \downarrow G & \downarrow f_2 & \\ 0 \rightarrow \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & \xrightarrow{h_2} & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & & \\ & & & \downarrow d' & \\ & & & \mathcal{K}_{(10,9,3)}(\mathcal{F}) & \\ & & & \downarrow & \\ & & & 0 & \end{array}$$

Diag. (1)

Savvy the maps $h_1(s) = \partial_{21}^{(4)}(s)$; $s \in \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$,

$h_2(s) = \partial_{21}^{(2)}(s)$; $s \in \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$, and $f_2(s) = \partial_{32}^{(2)}(s)$; $s \in \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$, We need to define f_1 which make diagram G in diag. (1) is commute that is gain in the following proposition.

Proposition 2.1: The diag. G in diag. (1) is commute.

Proof: The result follows by

$$\begin{aligned} (f_2 \circ h_1)(s) &= (h_2 \circ f_1)(s), \quad s \in \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ (f_2 \circ h_1)(s) &= (\partial_{32}^{(2)} \partial_{21}^{(4)})(s) = (\partial_{21}^{(4)} \partial_{32}^{(2)} + \partial_{21}^{(3)} \partial_{32} \partial_{31} + \partial_{21}^{(2)} \partial_{31}^{(2)})(s) \\ (f_2 \circ h_1)(s) &= \partial_{21}^{(2)} \left(\frac{1}{6} \partial_{21}^{(2)} \partial_{32}^{(2)} + \frac{1}{3} \partial_{21} \partial_{32} \partial_{31} + \partial_{31}^{(2)} \right) = (h_2 \circ f_1)(s) \\ \text{So, } f_1(s) &= \left(\frac{1}{6} \partial_{21}^{(2)} \partial_{32}^{(2)} + \frac{1}{3} \partial_{21} \partial_{32} \partial_{31} + \partial_{31}^{(2)} \right)(s). \end{aligned}$$

Now, by using the map.Co. to the following diag.

$$\begin{array}{ccc}
 0 \rightarrow \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & \xrightarrow{h_1} & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\
 f_1 \downarrow & & \downarrow f_2 \\
 0 \rightarrow \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} & \xrightarrow{h_2} & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
 \end{array}$$

Diag (2)

We gain the sequ. $\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$

$$0 \rightarrow \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\alpha_2} \oplus \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \xrightarrow{\alpha_1} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}, \text{ where}$$

$$\begin{aligned}
 \alpha_2(s) &= \left(-\partial_{21}^{(4)}(s), \frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{(2)}(s) \right) \text{ and} \\
 \alpha_1(s_1, s_2) &= \partial_{32}^{(2)}(s_1) + \partial_{21}^{(2)}(s_2)
 \end{aligned}$$

Proposition 2.2: The following sequ. is exact

$$\begin{aligned}
 &\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\
 0 \rightarrow \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} &\xrightarrow{\alpha_2} \oplus \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\
 \xrightarrow{\alpha_1} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} &\xrightarrow{d'_{(10,9,3)}(\mathcal{F})} \mathcal{K}_{(10,9,3)}(\mathcal{F}) \rightarrow 0
 \end{aligned}$$

Proof: Since the diag. G is commute and the maps of place polarization are injective then the map $h_1(s) = \partial_{21}^{(4)}(s)$ is injective so we have a commuting diag. with exact rows. But

$$\begin{aligned}
 (\alpha_1 \circ \alpha_2)(s) &= \alpha_1 \left(-\partial_{21}^{(4)}(s), \frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{(2)}(s) \right) \\
 &= -\partial_{32}^{(2)}\partial_{21}^{(4)}(s) + \partial_{21}^{(2)} \left(\frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{(2)}(s) \right) \\
 &= -\partial_{21}^{(4)}\partial_{32}^{(2)} - \partial_{21}^{(3)}\partial_{32}\partial_{31} - \partial_{21}^{(2)}\partial_{31}^{(2)} + \partial_{21}^{(4)}\partial_{32}^{(2)} + \partial_{21}^{(3)}\partial_{32}\partial_{31} + \partial_{21}^{(2)}\partial_{31}^{(2)}
 \end{aligned}$$

By using Capelli identities we get, $(\alpha_1 \circ \alpha_2)(s) = 0$. So, the map.Co. provisions are redeem then the sequ. is exact.

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