

Admixture vertex edges system and A-space

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Abstract

The admixture vertex edges system is the central idea of this paper, which are used serve to present, the new space called A-space. We present the essential and required foundation for the ideas of A-space. And we investigate the notions and some properties of admixture vertex edges system associated with some certain graphs.

Subject Classification: 04A05, 05C20.

Keywords: Graph, Incidence vertex edges, Non-incidence vertex edges, Admixture vertex edges system.

1. Introduction

There are two main reasons that graph theory is an important concept in many different academic domains and a useful tool in many other mathematical subjects. Some concepts are easier to understand when presented in a graph format. turn into a more sensible person. Regarding the connection between topology and graph theory, one of the instruments utilized to represent the graph is topological ideas, such as converting a collection of vertices or edges into topological space. Here are some previous works on the topic of topological graph s. Evans and Harary originally proposed topology on digr in 1967. [1, 9]. They discovered one relationship between the collection of all topologies with n

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vertices and the collection of all topologies with n vertices. additional connections between polynomial and topological indices can be found in [1-2]. In [3-8] and [10-17] a revolutionary concept about undigs and digrs generated by a novel open set was developed by Al'Dzhabri et al. They investigated many elements of this concept using the closed relationship between topology and graph." For all terminology and notation. We refer to Hdrary [7] in graph theory. Aundirected graph $\mathbb{G} = (\mathbb{W}(\mathbb{G}), \mathbb{E}(\mathbb{G}))$ is a mathematical structure consisting of two sets: the first $\mathbb{W}(\mathbb{G})$ is a non-empty set represents pooints (called vertices). The second set $\mathbb{E}(\mathbb{G})$ represents binary relations between these points (called edges). An edge that starts and ends at the same vertex is called loop. $\mathbb{G}$ is called complete graph if any vertex in $\mathbb{W}(\mathbb{G})$ is connected to all others vertices in $\mathbb{W}(\mathbb{G})$.

2. Admixture vertex edges system and A-space

In this section, we introduces new structure called A-space by using admixture vertex edges system. Some properties and notions are investigated about this structure.

Definition 2.1: Let $\mathbb{G} = (\mathbb{W}(\mathbb{G}), \mathbb{E}(\mathbb{G}))$ be a graph, where $\check{\omega}$ is a vertex in $\mathbb{W}(\mathbb{G})$ then :

- (a) The incidence vertex edges set of $\check{\omega}$ is represented by $\text{INVE}(\check{\omega})$ and characterized by:

$$\text{INVE}(\check{\omega}) = \{\check{\tau} \in \mathbb{E}(\mathbb{G}) : \check{\tau} = (\check{\omega}, u) \text{ for some } u \in \mathbb{W}(\mathbb{G})\}$$

- (b) The non-incidence vertex edges set of $\check{\omega}$ is represented by $\text{NINVE}(\check{\omega})$ and characterized by:

$$\text{NINVE}(\check{\omega}) = \{\check{\tau} \in \mathbb{E}(\mathbb{G}) : \check{\tau} = (u, r) \text{ and } u, r \neq \check{\omega} \text{ for some } u, r \in \mathbb{W}(\mathbb{G})\}.$$

Definition 2.2: Let $\mathbb{G} = (\mathbb{W}(\mathbb{G}), \mathbb{E}(\mathbb{G}))$ be a graph the admixture vertex edges system of a vertex $\check{\omega} \in \mathbb{W}(\mathbb{G})$ is represented by $\text{AVES}(\check{\omega})$ and described by: $\text{AVES}(\check{\omega}) = \{\text{INVE}(\check{\omega}), \text{NINVE}(\check{\omega})\}$.

Note: $\text{AVE}(\check{\omega}) \in \text{AVES}(\check{\omega})$

Definition 2.3: Let $\mathbb{G} = (\mathbb{W}(\mathbb{G}), \mathbb{E}(\mathbb{G}))$ be a graph. and suppose that $\mathfrak{h}_a : \mathbb{W}(\mathbb{G}) \rightarrow \mathcal{P}(\mathbb{E}(\mathbb{G}))$ is a mapping such that designates for all $\check{\omega}$ in $\mathbb{W}(\mathbb{G})$ belonging to it admixture vertex edges system in $\mathcal{P}(\mathbb{E}(\mathbb{G}))$. The pair $(\mathbb{G}, \mathfrak{h}_a)$ is referred to as the A-space.

Remark 2.4: Let $(\mathbb{G}, \mathfrak{h}_a)$ be an A-space, $\mathfrak{h}_a$ induced of connected simple graph $\mathbb{G} = (\mathbb{W}(\mathbb{G}), \mathbb{E}(\mathbb{G}))$ such that $n = |\mathbb{W}(\mathbb{G})|$ $m = |\mathbb{E}(\mathbb{G})|$. Then the A-space $(\mathbb{G}, \mathfrak{h}_a)$ possesses the following properties :

P1: $\mathbb{G}$ is cycle graph if and only if $|\text{INVE}(\check{\omega})| = 2$ for all $\check{\omega} \in \mathbb{W}(\mathbb{G})$.

P2: $\mathbb{G}$ is star graph if and only if $\exists! \check{\omega} \in \mathbb{W}(\mathbb{G})$ such that $\text{NINVE}(\check{\omega}) = \emptyset$.

P3: $\mathbb{G}$ is complete graph if and only if $|\text{INVE}(\check{\omega})| = n - 1$ for all $\check{\omega} \in \mathbb{W}(\mathbb{G})$.

- P4:** $\mathbb{G}$ is κ – regular graph if and only if $|\text{INVE}(\check{\omega})| = \kappa$ for all $\check{\omega} \in \mathbb{W}(\mathbb{G})$.
- P5:** $\mathbb{G}$ is $(m - \kappa)$ – regular graph if and only if $|\text{NINVE}(\check{\omega})| = \kappa$ for all $\check{\omega} \in \mathbb{W}(\mathbb{G})$.
- P6:** If $|\text{INVE}(\check{\omega})| = |\text{NINVE}(\check{\omega})| = \kappa$ for all $\check{\omega} \in \mathbb{W}(\mathbb{G})$ then $m = 2\kappa$ in $\mathbb{G}$.
- P7:** The κ – regular $\mathbb{G}$ is either K_4 , $K_{2,2}$ or C_4 if and only if $m = 2\kappa$.
- P8:** $\sum_1^n |\text{INVE}(\check{\omega}_i)| = 2m$.
- P9:** If $|\text{INVE}(\check{\omega}) \cup \text{NINVE}(\check{\omega})| < n - 1$ for each $\check{\omega} \in \mathbb{W}(\mathbb{G})$ then $\mathbb{G}$ is disconnected graph.

Theorem 3.5: Let $(\mathbb{G}_1, \mathfrak{h}_{a_1})$ and $(\mathbb{G}_2, \mathfrak{h}_{a_2})$ be two A-space if $\mathbb{G}_1 \cong \mathbb{G}_2$ then $\mathfrak{h}_{a_1} = \mathfrak{h}_{a_2}$.

Proof: Let $\mathbb{G}_1 = (\mathbb{W}(\mathbb{G}_1), \mathbb{E}(\mathbb{G}_1))$ $\mathbb{G}_2 = (\mathbb{W}(\mathbb{G}_2), \mathbb{E}(\mathbb{G}_2))$ be two a graphs. Since $\mathbb{G}_1 \cong \mathbb{G}_2$ then $|\mathbb{W}(\mathbb{G}_1)| = |\mathbb{W}(\mathbb{G}_2)|$, $|\mathbb{E}(\mathbb{G}_1)| = |\mathbb{E}(\mathbb{G}_2)|$ and there exist bijective mapping $\mu: \mathbb{W}(\mathbb{G}_1) \rightarrow \mathbb{W}(\mathbb{G}_2)$ such that for all $\omega, \upsilon \in \mathbb{W}(\mathbb{G}_1)$, $\mathfrak{t} = (\omega, \upsilon) \in \mathbb{E}(\mathbb{G}_1)$ if and only if $\mathfrak{t} = (\mu(\omega), \mu(\upsilon)) \in \mathbb{E}(\mathbb{G}_2)$, $\forall \mathfrak{t} \in \mathbb{E}(\mathbb{G}_1)$ incidence on ω in $\mathbb{G}_1 \exists$ itself $\mathfrak{t} \in \mathbb{E}(\mathbb{G}_2)$ incidence on $\mu(\omega)$ in $\mathbb{G}_2$ thus $\text{INVE}(\omega)$ in $\mathbb{G}_1 = \text{INVE}(\mu(\omega))$ in $\mathbb{G}_2$ also $\text{NINVE}(\omega)$ in $\mathbb{G}_1 = \text{NINVE}(\mu(\omega))$ in $\mathbb{G}_2$. Since $\mathfrak{h}_{a_1}(\omega) = \{\text{INVE}(\omega), \text{NINVE}(\omega)\}$ and $\mathfrak{h}_{a_2}(\mu(\omega)) = \{\text{INVE}(\mu(\omega)), \text{NINVE}(\mu(\omega))\}$ thus $\mathfrak{h}_{a_1} = \mathfrak{h}_{a_2}$.

3. Limit vertex edges system

In this section, we introduced new concepts called limit vertex edges system dependent on our structure A-space and proved new properties of this concepts.

Definition 3.1: Let $(\mathbb{G}, \mathfrak{h}_a)$ be an A-space. every edge $\mathfrak{t}$ in $\mathbb{E}(\mathbb{G})$ is termed a limit vertex edge of a sub- graph $\mathcal{J}_b \subseteq \mathbb{G}$ if and only if $\mathfrak{t} \in [\mathbb{E}(\mathcal{J}_b)]_a$ such that :

$$[\mathbb{E}(\mathcal{J}_b)]_a = \{\mathfrak{t} \in \mathbb{E}(\mathbb{G}); \forall \text{AVE}(\check{\omega}) \text{ where } \check{\omega} \text{ incidence on } \mathfrak{t}, \text{AVE}(\check{\omega}) \cap (\mathbb{E}(\mathcal{J}_b) - \{\mathfrak{t}\}) \neq \emptyset\}.$$

Proposition 3.2: Let $(\mathbb{G}, \mathfrak{h}_a)$ be an A-space then: (i) $[\emptyset]_a = \emptyset$. (ii) If $\mathcal{J}_b \subseteq \mathcal{J}_c$ subsequently $[\mathbb{E}(\mathcal{J}_b)]_a \subseteq [\mathbb{E}(\mathcal{J}_c)]_a$ for each $\mathcal{J}_b, \mathcal{J}_c \subseteq \mathbb{G}$. (iii) If $\mathfrak{t} \in [\mathbb{E}(\mathcal{J}_b)]_a$ then $\mathfrak{t} \in [\mathbb{E}(\mathcal{J}_b - \mathfrak{t})]_a$ for all $\mathcal{J}_b \subseteq \mathbb{G}$.

Proof: (i) From definition (4.1) we note that $\forall \mathfrak{t} \in \mathbb{E}(\mathbb{G})$ and for all $\check{\omega} \in \mathbb{W}(\mathbb{G})$ incidence on $\mathfrak{t}$ then $\text{AVE}(\check{\omega}) \cap (\emptyset - \{\mathfrak{t}\}) = \emptyset$. Thus $[\emptyset]_a = \emptyset$. (ii) Let $\mathcal{J}_b, \mathcal{J}_c \subseteq \mathbb{G}$ and $\mathcal{J}_b \subseteq \mathcal{J}_c$, to demonstrate that $[\mathbb{E}(\mathcal{J}_b)]_a \subseteq [\mathbb{E}(\mathcal{J}_c)]_a$. Let $\mathfrak{t}_1 \in [\mathbb{E}(\mathcal{J}_b)]_a \Rightarrow \forall \text{AVE}(\check{\omega})$ where $\check{\omega}$ incidence on $\mathfrak{t}_1$, $\text{AVE}(\check{\omega}) \cap (\mathbb{E}(\mathcal{J}_b) - \{\mathfrak{t}_1\}) \neq \emptyset$. Since $\mathbb{E}(\mathcal{J}_b) \subseteq \mathbb{E}(\mathcal{J}_c)$ hence $\text{AVE}(\check{\omega}) \cap (\mathbb{E}(\mathcal{J}_c) - \{\mathfrak{t}_1\}) \neq \emptyset \Rightarrow \mathfrak{t}_1 \in [\mathbb{E}(\mathcal{J}_c)]_a$ thus $[\mathbb{E}(\mathcal{J}_b)]_a \subseteq [\mathbb{E}(\mathcal{J}_c)]_a$. (iii) Let $\mathcal{J}_b \subseteq \mathbb{G}$ and $\mathfrak{t} \in [\mathbb{E}(\mathcal{J}_b)]_a$ to demonstrate that $\mathfrak{t} \in [\mathbb{E}(\mathcal{J}_b - \mathfrak{t})]_a$. Since $\mathfrak{t} \in [\mathbb{E}(\mathcal{J}_b)]_a$

hence $\forall AVE(\check{\omega})$ where $\check{\omega}$ incidence on $\check{t}$, $AVE(\check{\omega}) \cap (\Xi(\mathcal{J}_b) - \{\check{t}\}) \neq \emptyset \Rightarrow AVE(\check{\omega}) \cap (\Xi(\mathcal{J}_b) \cap \{\check{t}\}^c) \neq \emptyset \Rightarrow AVE(\check{\omega}) \cap (\Xi(\mathcal{J}_b - \check{t}) \cap \{\check{t}\}^c) \neq \emptyset \Rightarrow AVE(\check{\omega}) \cap (\Xi(\mathcal{J}_b - \check{t}) - \{\check{t}\}) \neq \emptyset$ thus $\check{t} \in [\Xi(\mathcal{J}_b - \check{t})]_a$.

Proposition 3.3: Every subgraph $\mathcal{J}_b$ of $\mathbb{G}$ has just single edge, so the a-derived of $\mathcal{J}_b$ is null set.

Proof: Let $\mathbb{G} = (V(\mathbb{G}), \Xi(\mathbb{G}))$ be a graph, and let $\mathcal{J}_b \subseteq \mathbb{G}$ assume that $\Xi(\mathcal{J}_b) = \{\check{t}\}$. Now to demonstrate that $[\Xi(\mathcal{J}_b)]_a = \emptyset$. Let us assume that $[\{\check{t}\}]_a \neq \emptyset$ hence $\exists \check{t}_1 \in [\{\check{t}\}]_a$ thus $\forall AVE(\check{\omega})$ where $\check{\omega}$ incidence on $\check{t}_1$, $AVE(\check{\omega}) \cap (\{\check{t}\} - \{\check{t}_1\}) \neq \emptyset$ since $\check{t} \neq \check{t}_1 \Rightarrow AVE(\check{\omega}) \cap \{\check{t}\} \neq \emptyset$, hence $\check{t} \in INVE(\check{\omega})$ and $\check{t} \in NINVE(\check{\omega})$ this a contradiction then $\check{t}_1 \notin [\{\check{t}\}]_a \Rightarrow [\Xi(\mathcal{J}_b)]_a = \emptyset$.

Proposition 3.4: Let $(\mathbb{G}, h_a)$ be an A-space and $\mathcal{J}_b$ is every subgraph of $\mathbb{G}$ if $\mathcal{J}_b$ has only two elements then this elements are not limit vertex edges of $\mathcal{J}_b$.

Proof: Let $\mathbb{G} = (V(\mathbb{G}), \Xi(\mathbb{G}))$ be a graph and let $\mathcal{J}_b \subseteq \mathbb{G}$ suppos that $\Xi(\mathcal{J}_b) = \{\check{t}_1, \check{t}_2\} \subseteq \Xi(\mathbb{G})$ to prove $\check{t}_1, \check{t}_2 \notin [\Xi(\mathcal{J}_b)]_a$, we will proceed by contradiction to prove this. Now If $\check{t}_1 \in [\Xi(\mathcal{J}_b)]_a$ then $\forall AVE(\check{\omega})$ where $\check{\omega}$ incidence on $\check{t}_1$, $AVE(\check{\omega}) \cap (\Xi(\mathcal{J}_b) - \{\check{t}_1\}) \neq \emptyset$ but since $\Xi(\mathcal{J}_b) - \{\check{t}_1\} = \{\check{t}_2\}$ so $AVE(\check{\omega}) \cap \{\check{t}_2\} \neq \emptyset$ this means $\check{t}_2 \in INVE(\check{\omega})$ and $\check{t}_2 \in NINVE(\check{\omega})$ this contradiction. Hence $\check{t}_1 \notin [\Xi(\mathcal{J}_b)]_a$. In the same we can prove that $\check{t}_2 \notin [\Xi(\mathcal{J}_b)]_a$.

Proposition 3.5: Let $(\mathbb{G}, h_a)$ be an A-space for each $\mathcal{J}_b, \mathcal{J}_c \subseteq \mathbb{G}$ then: (i) $[\Xi(\mathcal{J}_b)]_a \cup [\Xi(\mathcal{J}_c)]_a \subseteq [\Xi(\mathcal{J}_b \cup \mathcal{J}_c)]_a$. (ii) $[\Xi(\mathcal{J}_b \cap \mathcal{J}_c)]_a \subseteq [\Xi(\mathcal{J}_b)]_a \cap [\Xi(\mathcal{J}_c)]_a$.

Proof: (i) Since $\Xi(\mathcal{J}_b) \subseteq \Xi(\mathcal{J}_b \cup \mathcal{J}_c)$ by using proposition (4.1(ii)) we obtain $[\Xi(\mathcal{J}_b)]_a \subseteq [\Xi(\mathcal{J}_b \cup \mathcal{J}_c)]_a$ (1). And $\Xi(\mathcal{J}_c) \subseteq \Xi(\mathcal{J}_b \cup \mathcal{J}_c)$ by using Proposition (4.1(ii)) we obtain $[\Xi(\mathcal{J}_c)]_a \subseteq [\Xi(\mathcal{J}_b \cup \mathcal{J}_c)]_a$ (2). Based on (1) and (2) as result, we find $[\Xi(\mathcal{J}_b)]_a \cup [\Xi(\mathcal{J}_c)]_a \subseteq [\Xi(\mathcal{J}_b \cup \mathcal{J}_c)]_a$.

(ii) Since $\Xi(\mathcal{J}_b \cap \mathcal{J}_c) \subseteq \Xi(\mathcal{J}_b)$ by using proposition (4.1(ii)) we obtain $[\Xi(\mathcal{J}_b \cap \mathcal{J}_c)]_a \subseteq [\Xi(\mathcal{J}_b)]_a$ (1). And $\Xi(\mathcal{J}_b \cap \mathcal{J}_c) \subseteq \Xi(\mathcal{J}_c)$ by using proposition (4.1(ii)) we obtain $[\Xi(\mathcal{J}_b \cap \mathcal{J}_c)]_a \subseteq [\Xi(\mathcal{J}_c)]_a$ (2). Based on (1) and (2) as result, we find $[\Xi(\mathcal{J}_b \cap \mathcal{J}_c)]_a \subseteq [\Xi(\mathcal{J}_b)]_a \cap [\Xi(\mathcal{J}_c)]_a$.

Remark 3.6: The A - space $(\mathbb{G}, h_a)$ possesses the following properties :

P1: If $\mathbb{G} = K_{m,n}$ and $m, n \geq 2$ is complete bipartite graph then $[\Xi(K_{m,n})]_a = \Xi(K_{m,n})$.

P2: If $\mathbb{G} = C_n$ is cycle graph then $[\Xi(C_n)]_a = \Xi(C_n)$.

P3: If $\mathbb{G} = S_n$ is star graph then for every subgraph $\mathcal{J}_b$ of $\mathbb{G}$, $[\Xi(\mathcal{J}_b)]_a = \emptyset$

P4: If $\mathbb{G} = P_n$ is path graph then then $[\Xi(P_n)]_a = \Xi(P_n) - \{\check{t}_1, \check{t}_{n-1}\}$

Conclusion

In this work, we introduced new structure called admixture vertex edges system and associated it with certain graphs to get more results and new notions.

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Received September, 2024