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A complex of Lascoux for the skew shape (11,10,3;1,1)

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Abstract

The consideration of the complex of Lascoux for (11,10,3;1,1) by usage the divided power (d.p.) of the place polarization (p.p.)., diagrams (dig.) and Capelli identities. The k^{th} d.p. of p.p. from place 1 to 2 is ($\partial_{21}^{(k)}$), e^{th} d.p. of p.p.from place 2 to place 3 ($\partial_{32}^{(e)}$), j^{th} d.p. of p.p.from place 1 to place 3 ($\partial_{31}^{(j)}$).

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1. Introduction

The following Capelli identities savvy by [1]

$$\partial_{32}^{(2)}\partial_{21} = \partial_{21}\partial_{32}^{(2)} + \partial_{32}\partial_{31} \qquad \ldots(1)$$

$$\partial_{32}\partial_{21}^{(2)} = \partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31} \qquad \ldots(2)$$

$$\partial_{21}^{(2)}\partial_{32} = \partial_{32}\partial_{21}^{(2)} - \partial_{21}\partial_{31} \qquad \ldots(3)$$

$$\partial_{32}\partial_{31} = \partial_{31}\partial_{32} \qquad \ldots(4)$$

The d.p. $\mathcal{DF} = \sum_{i\geq 0} \mathcal{D}_i\mathcal{F}$. For the partition λ/μ, $\mathcal{K}_{\lambda/\mu}\mathcal{F} = \mathrm{Im}(\mathcal{DF} \to \Lambda\mathcal{F})$, [2]. We applied the same idea in [3,4] for (11,10,3;1,1). Also can applied that for $\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U}$ =81, 729, 125,3125,121,196, [5-7], PG(3,8) [8-10], $\mathcal{PSL}(2,\mathfrak{U})$, and $\mathcal{SL}(2,\mathfrak{U})$ where $\mathfrak{U}$ = 23,29,31,37, [11-14]. Also the studies by authors in [15-18].

2. Complex of Lascoux

For (11,10,3;1,1) the terms of the complex are

$$0 \to \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \to \begin{matrix} \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \end{matrix} \to \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

See the Figure.

$$\begin{tikzcd}
\mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "d_1"] \arrow[d, "\hbar_1"'] \arrow[dr, phantom, "\mathcal{T}"] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "\hbar_2"] \\
\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "d_2"'] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
\end{tikzcd}$$

Figure 1
The commutative of dig. $\mathcal{T}$.

Savvy the maps $d_2(v) = \partial_{32}^{(2)}(v)$, $\hbar_1(v) = \partial_{21}^{(4)}(v)$, $\hbar_2(v) = \partial_{21}^{(2)}(v)$.

Proposition 2.1: The commutative of dig. $\mathcal{T}$ in Figure 1.

Proof: $d_2 \circ \hbar_1 = \hbar_2 \circ d_1$

$$d_2 \circ \hbar_1 = \partial_{32}^{(2)}\partial_{21}^{(4)} = \partial_{21}^{(4)}\partial_{32}^{(2)} + \partial_{21}^{(3)}\partial_{32}\partial_{31} + \partial_{21}^{(2)}\partial_{31}^{2}$$

$$= \frac{1}{6}\partial_{21}^{(4)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}^{(3)}\partial_{32}\partial_{31} + \partial_{21}^{(2)}\partial_{31}^{2}$$

$$= \frac{1}{6}\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}^{(2)}\partial_{21}\partial_{32}\partial_{31} + \partial_{21}^{(2)}\partial_{31}^{2}$$

$$d_2 \circ \hbar_1 = =\partial_{21}^{(2)}\left(\frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{2}\right)$$

Thus, $$d_1 = \left(\frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{2}\right)$$

Take the pursue dig.

$$\begin{tikzcd}
\mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "d_1"] \arrow[d, "h_1"'] \arrow[dr, "g", "\mathcal{H}"'] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "h_2"] \arrow[dl, phantom, "\mathcal{N}"] \\
\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "d_2"'] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}
\end{tikzcd}$$

Figure 2
The commutative of dig. $\mathcal{N}$.

Proposition 2.2: The commutative of dig. $\mathcal{N}$ in Figure 2.

Proof: To prove $h_2 \circ d_1 = g$

$$h_2 \circ d_1 = \partial_{21}^{(2)}\left(\frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{2}\right)$$
$$= \partial_{21}^{(4)}\partial_{32}^{(2)} + \partial_{21}^{(3)}\partial_{32}\partial_{31} + \partial_{21}^{(2)}\partial_{31}^{2}.$$

Proposition 2.3: The commutative of dig. $\mathcal{H}$.in Figure 1.

Proof: To prove $h_2 \circ d_1 = g$

$$d_2 \circ h_1 = \partial_{32}^{(2)}\partial_{21}^{(4)}$$
$$= \partial_{21}^{(4)}\partial_{32}^{(2)} + \partial_{21}^{(3)}\partial_{32}\partial_{31} + \partial_{21}^{(2)}\partial_{31}^{2}$$

Proposition 2.4: The complex of the pursue seq.

$$0 \to \mathcal{D}_{14}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\sigma_2} \begin{matrix} \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \end{matrix} \xrightarrow{\sigma_1} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

Where, $\sigma_2(x) = \big(d_1(x), h_1(x)\big)$, and $\sigma_1((x_1, x_2)) = \big(h_2(x_2) + d_2(x_1)\big)$.

Proof: By (1)-(4) and the injective of ∂_{21} and ∂_{32} [1] we gain σ_3 is also injective, so

$$(\sigma_1 \circ \sigma_2)(x) = \sigma_1\big(-d_1(x), h_1(x)\big)$$
$$= \sigma_1\left(-\left(\frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{2}\right)(x), \partial_{21}^{(4)}(x)\right)$$
$$= \left(\partial_{21}^{(2)}(x_1) + \partial_{32}^{(2)}(x_2)\right)\left(-\left(\frac{1}{6}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{3}\partial_{21}\partial_{32}\partial_{31} + \partial_{31}^{2}\right)(x), \partial_{21}^{(4)}(x)\right)$$
$$= \left(-\frac{1}{6}\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{32}^{(2)} - \frac{1}{3}\partial_{21}^{(2)}\partial_{21}\partial_{32}\partial_{31} - \partial_{21}^{(2)}\partial_{31}^{2}\right)(x_1) + \partial_{32}^{(2)}\partial_{21}^{(4)}(x_2)$$

$$= -\partial_{21}^{(4)}\partial_{32}^{(2)} - \partial_{21}^{3}\partial_{32}\partial_{31} - \partial_{21}^{(2)}\partial_{31}^{2} + \partial_{21}^{(4)}\partial_{32}^{(2)} + \partial_{21}^{(3)}\partial_{32}^{(1)}\partial_{31} + \partial_{21}^{(2)}\partial_{31}^{2} = 0$$

Thus $\sigma_1 \circ \sigma_2 = 0$. So the sequence is complex.

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