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Lucky edge geometric mean labeling of graphs and applications in electrical networks

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Abstract

A Lucky Edge Geometric Mean Labeling (LEGML) is a function μ that assigns the integers to the vertices of graph G such that for every edge as '{E}', $uv \in E(G)$ there exists a function $\mu^*: E(G) \rightarrow N$ is defined by $\mu^*(uv) = \lceil \sqrt{\mu(u)\mu(v)} \rceil$ (or) $\lfloor \sqrt{\mu(u)\mu(v)} \rfloor$ with the state that $\mu^*(uv) \neq \mu^*(vw)$ whenever $\{u\}$ and $\{vw\}$ have a common vertex as $\{v\}$. A minor number '$k \rightarrow E = \{1, 2, 3 \ldots k\}$ is the LEGML of G and is signified by $h'_{GM}(G)$. A result that accepts LEGML is the Lucky Edge Geometric Mean Graph (LEGMG). The idea of the Graph Theory (GT) concept is fused into the Electric Circuit (EC), and the related complicated problems are transformed into graph model representation problems; as a result, the valuation method is simplified and optimized. Network analysis means finding the current or voltage in each

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branch. One application of GT is the representation of EC. An 'E' in a diagram can signify anything in an EC. Each part of the diagram can represent a port or a terminal in the EC. In this article, we have to investigate the Middle Graph (MG) of the star, the Total Graph (TG) of the star, and the central graph (CG) of the star, which accepts LEGML and LEGMG. This method also computes the power consumed by all the resistors of the EC by LEGML.

Subject Classification: *05C78.*

Keywords: *Central graph, Electrical networks, Lucky edge geometric mean, Middle graph, Star graph, Total graph.*

1. Introduction

In [1], Graph Labelling (GL) is obtainable. For example, GL is the task for vertices (V), edges (E), or both. If the list is a V (E) collection, the label is called a V (E); in the intervening years, many symbols have been studied in more than 1100 papers. GL is a model in many fields, including X-ray, Macromolecular, sensor, astronomy, and coding theory. [2] proposed the theorem of Lucky Edge Labeling (LEL) as a function $'\mu'$ is a task of integers to V such that $\mu(u)+\mu(v)$ is assigned to the E =$\underline{uv}$ with the state that $\mu^*(uv) \neq \mu^*(vw)$ The E= $\{uv, vw\}$ have a standard 'V'. A trivial number 'k' for which 'G' has an LEL from the edge set is the Lucky Edge (LE) number of 'G', denoted by $\eta(G)$. The Lucky Edge Labeled Graph (LELG) is a graph that accepts LEL. In [3-5], they Proved P_n, C_n are LELG. In [6], they proved that the 2-D grid graph has LEL. In [7], they proved that triangular graphs are LELG and some unique graphs are LELG. In [8], they proved the subdivision of star and wheel graphs, H – super subdivision of graphs is LELG. In [9], they proved star related graphs are LELG. Motivated by these notions, introduce the Lucky Edge Geometric Mean Labeling (LEGML). This article studies the star's Middle Graph (MG), Total Graph (TG), and Central Graph (CG), which admits to LEGML. This work obtains the number of LEGML. To find the rule captivated by the Electric Circuit (EC) resistors by using the LEGML [10-15].

2. Preliminaries

Definition 2.1: A LEGML is a function $\{\mu: V(G) \to N\}$ such that for all E=$\{uv \in E(G)\}$ there exists a function $\{\mu^*: E(G) \to N\}$ is defined by $\mu^*(uv) = \left\lceil\sqrt{\mu(u)\mu(v)}\right\rceil (or) \left\lfloor\sqrt{\mu(u)\mu(v)}\right\rfloor$ with the state that $\{\mu^*(uv) \neq \mu^*(vw)\}$ whenever *'uv'* and *'vw'* have a standard V. The less number $k \rightarrow$ E={1, 2, 3,.., k} is the LEGML of G and is signified by $\eta'_{GM}(G)$—a result that accepts LEGML.

Definition 2.2: MG of Star M($K_{1,n}$) [6]: Assume $V(K_{1,n}) = \{u, u_1, u_2, \ldots, u_n\}$ and $E(K_{1,n}) = \{uu_1, uu_2, \ldots, uu_n\}$. $M[K_{1,n}]$ is find by updating V=$v_1, v_2, \ldots, v_n$ fetching the E = uu_1, uu_2, ..., uu_n of $K_{1,n}$.

Two vertices *x, y* $V[M[K_{1,n}]]$ are adjacent $\Leftrightarrow$ any of the resulting scenario fixes,

(i) $x, y \in E[K_{1,n}]$ and $\{x, y\}$ have a Standard Vertex (SV) in $K_{1,n}$.

(ii) $\{x\} \rightarrow V[K_{1,n}]$, $\{y\}$ is in $E[K_{1,n}]$, and $'x'$ is incident on $'y'$ in $K_{1,n}$.

Definition 2.3: TG of Star T($K_{1,n}$) [6] :

Assume $V(K_{1,n}) = \{u, u_1, u_2, \dots, u_n\}$ and $E(K_{1,n}) = \{uu_1, uu_2, \dots, uu_n\}$. $T[K_{1,n}]$ is attained by updating the V.

$(v_1, v_2, ..., v_n)$ fetching to the edges $uu_1, uu_2, ..., uu_n$; $K_{1,n}$.

Two V as $\{x, y\}$ in $V[T[K_{1,n}]]$ are adjacent $\Leftrightarrow$ any of the following states holds,

(i) $\{x, y\} \rightarrow V[T[K_{1,n}]]$ and $\{x, y\}$have common V in $K_{1,n}$.

(ii) $\{x, y\} \rightarrow E[T[K_{1,n}]]$ and $\{xy\} \in K_{1,n}$.

(iii) $\{x\} \rightarrow V[T[K_{1,n}]]$, y is in $E[T[K_{1,n}]]$, and $'x'$ is test case on $'y'$ in $K_{1,n}$.

Definition 2.4: CG of Star C($\boldsymbol{K_{1,n}}$) [6]:

Let $V(K_{1,n}) = \{u, u_1, u_2, \dots, u_n\}$ and $E(K_{1,n}) = \{uu_1, uu_2, \dots, uu_n\}$. $C[K_{1,n}]$ is found by subdividing the E = $\{uu_1, uu_2, \dots, uu_n\}$; $K_{1,n}$ by the label the V = $\{v1, v2, \dots, vn\}$. $v_1, v_2, ..., v_n$. Then, join all the non-adjacent V of the star $K_{1,n}$.

3. Preliminary Results

Theorem 3.1: *$M[K_{1,n}]$ is a LEGML.*

Proof: Define $\mu: V[M(K_{1,n})] \rightarrow N$

The V labels are distinct by $\mu(u) = (n+1)^2$ and $\mu(u_i) = \mu(v_i) = i^2, 1 \leq i \leq n$

This study has done the function $\mu^*: E(G) \rightarrow N$ is well defined by $\mu^*(uv) = \left\lceil \sqrt{\mu(u)\mu(v)} \right\rceil (or) \left\lfloor \sqrt{\mu(u)\mu(v)} \right\rfloor$ with the state that $\mu^*(uv) \neq \mu^*(vw)$ whenever *{uv}* and *{vw}* have an SV 'v'.

The less integer $n^2 + n$ for which a graph $M[K_{1,n}]$ has a LEGML is $\{1,2,3, \dots, n^2 + n\}$ is the LEGML as G$\rightarrow \eta'_{GM}(M[K_{1,n}])$.

$\therefore \eta'_{GM}(M[K_{1,n}]) = n^2 + n$; Hence $M[K_{1,n}]$ is a LEGML.

Theorem 3.2: *$T[K_{1,n}]$ is a LEGML*

Proof: Define $\mu: V\left[T\left[K_{1,n}\right]\right] \to N$

The V labels are defined by

$$\mu(u) = (n+1)^2$$
$$\mu(u_i) = i^2, 1 \le i \le n$$
$$\mu(v_i) = (n+i+1)^2, 1 \le i \le n$$

This method of function $\mu^*: E(G) \to N$ is defined by $\mu^*(uv) = \left\lceil \sqrt{\mu(u)\mu(v)} \right\rceil (or) \left\lfloor \sqrt{\mu(u)\mu(v)} \right\rfloor$ with the state that $\mu^*(uv) \neq \mu^*(vw)$ whenever *{uv}* and *{vw}* have an SV 'v'.

The less integer $2n^2 + 3n + 1$ for which a graph $T\left[K_{1,n}\right]$ has a LEGML from the set $\{1,2,3, \ldots, 2n^2 + 3n + 1\}$ is the LEGML number of G denoted by $\eta'_{GM}\left(T\left[K_{1,n}\right]\right)$

$$\therefore \eta'_{GM}\left(T\left[K_{1,n}\right]\right) = 2n^2 + 3n + 1$$

Hence $T\left[K_{1,n}\right]$ is a LEGML

Theorem 3.3: *$C[K_{1,n}]$ is a LEGML.*

Proof: Define $\mu: V[C[K_{1,n}]] \to N$

The V labels are defined by

$$\mu(u) = 1$$
$$\mu(u_i) = \mu(v_i) = (i+1)^2, 1 \le i \le n$$

This method of function $\mu^*: E(G) \to N$ is defined by $\mu^*(uv) = \left\lceil \sqrt{\mu(u)\mu(v)} \right\rceil (or) \left\lfloor \sqrt{\mu(u)\mu(v)} \right\rfloor$ with the state that $\mu^*(uv) \neq \mu^*(vw)$ whenever *{uv}* and *{vw}* have an SV 'v'.

The less integer $(n+1)^2$ from the E= $\{1,2, \ldots, (n+1)^2\}$ is a LEGML number of G denoted by $\eta'_{GM}(C[K_{1,n}])$

$$\therefore \eta'_{GM}(C[K_{1,n}]) = (n+1)^2.$$

Hence $C[K_{1,n}]$ is a LEGML

4. Applications in Electrical Networks

An electrical network is a collection of interconnected e-devices. Network elements are the best Physical devices, and to represent many things, they must follow Kirchhoff's law of current and voltage. The diagram represents an electrical network in lines or arcs (E) and points (V).

Problem 4.1: Investigate the energy threshold of all resistors from the below EC using LEGML (Figure 1).

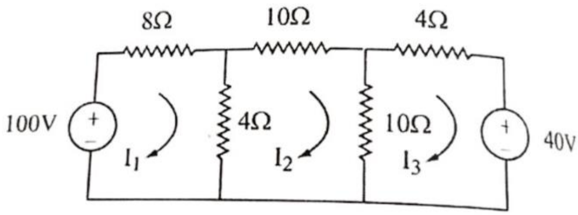

Figure 1
Test case

Solution: Electrical networks can be epitomized using diagrams because each part of the network is experimental as a V of the figure, and each network division is practical as an E.

GT represents networks using vertical lines and E that span the complete network (Figure 2).

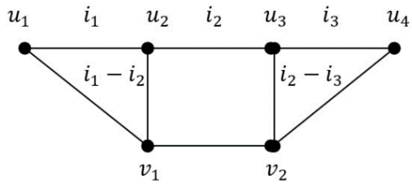

Figure 2
A GT representation and ordinary labelling

Assume G be a given network, $V(G) = \{u_i, v_j: 1 \leq i \leq 4, 1 \leq j \leq 2\}$ and $E(G) = \{u_1u_2, u_2u_3, u_3u_4, u_1v_1, u_2v_1, u_3v_2, u_4v_2, v_1v_2\}$

Assume the resistors $R_A = 8\Omega, R_B = 4\Omega, R_C = 10\Omega, R_D = 10\Omega, R_E = 4\Omega$

Define $\mu: V[G] \to N$

The V labels are defined by

$$\mu(u_1) = 5R_A$$
$$\mu(u_2) = \frac{R_B}{2}$$
$$\mu(u_3) = \frac{R_C}{10}$$
$$\mu(u_4) = 2R_E$$
$$\mu(v_1) = 5R_B$$
$$\mu(v_2) = R_E$$

Induced function $\mu^*: E(G) \to N$ is defined by $\mu^*(uv) = \left\lceil\sqrt{\mu(u)\mu(v)}\right\rceil$ $(or)\left\lfloor\sqrt{\mu(u)\mu(v)}\right\rfloor$ with the state that $\mu^*(uv) \neq \mu^*(vw)$ whenever *{uv}* and *{vw}* have an SV '*v*' (Figure 3).

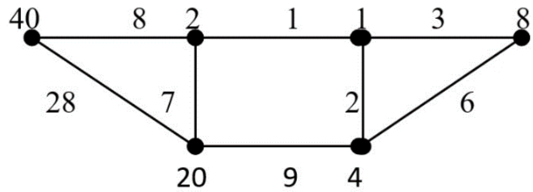

Figure 3
LEGML representation of EC network

Assume, $i_1 = 8, i_2 = 1, i_3 = 3, i_1 - i_2 = 7, i_2 - i_3 = 2$, Hence, the value of the power consumed by the resistor $R_A(i_1) = 8$, $R_C(i_2) = 1$, $R_E(i_3) = 3$, $R_{\boldsymbol{B}}(i_1 - i_2) = 7$, $R_D(i_2 - i_3) = 2$.

5. Conclusion and Future Work

LEGML is a GT concept that is especially relevant to EC networks. This labelling process involves assigning a number to the edges of the LEGML representation of the EC. In summary, LEGML has many advantages and applications in the field of power grids.

(a) **Analysis:** This labelling process allows the calculation of the LEGML number between two nodes

(b) **Flexibility:** LEGML helps to evaluate the flexibility of the network

(c) **Optimization:** LEGML helps to improve grid efficiency. Some parameters, such as signal dissipation or power conversion efficiency, can be optimized by considering the geometric mean.

(d) **Research and Development:** The application of LEGML to EC networks contributes to research and continuous development in the field of research graphics, network analysis and electrical engineering. It provides researchers and practitioners with new methods to solve problems related to network design, analysis and optimization.

In this article, we have computed the LEGML of the star's MG, the star's total, and the star's CG. We have also found the power consumed by the resistors of the electric circuit using the LEGML.

In the future, we can compute the power absorbed by the resistors of various III-Phase EC using LEGML.

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