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Antimagic labeling in Butterfly and Benes networks

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Abstract

This paper focus on the dynamic and rapidly evolving field of Graph Labeling research. In this paper, Antimagic Labeling for Benes and Butterfly networks has been determined. The findings can be utilized to determine the Antimagic Labeling in Interconnection networks where the routing servers acting as its vertices and the connection between them acts as

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edges to get the best possible solution for building a network with computational power and the ability to send and receive messages over cabled or wireless communication links.

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Keywords: *Antimagic labeling, Benes network, Butterfly network.*

1. Introduction and Background

The graphs taken for this study are simple, finite, undirected, and connected graphs of order $|V(G)|$ and size $|E(G)|$ for a graph $G = (V, E)$. Antimagic Labeling in graphs has been introduced by N. Hartsfield and G. Ringel [7] in the year 1990, From then many concepts in Antimagic label ing has been widely studied, For detailed study in Labeling and Antimagic Labeling, a reader can refer to [1, 2, 3, 9, 10, 11]. For Further study in Interconnection networks a reader can refer to [4, 5, 6, 8]. In this paper, Antimagic Labeling for Benes and Butterfly networks has been determined.

Definition 1.1: [12] Let $G = (V, E)$ is a graph. the bijective function $f: E \to \{1, 2, \cdots, |E|\}$ in which for each vertex u belongs to vertex set of G, the vertex function $\phi_f(u)$ is determined as $\phi_f(u)$ equals $\sum_{e \in \text{Edge of } u} f(e)$, Thus Edge of u are the edges which are incident to u. Thus $\phi_f(u) \neq \phi_f(v)$, $\forall\, u, v \in V(G)$, then f is known as Antimagic (labeling) of G. The Graph admits Antimagic Labeling is defined as Antimagic.

Definition 1.2: [8] An $\mathcal{O}$-dimensional Butterfly network has $\mathcal{O}, \mathcal{O} \geq 1$ Levels. Each of the levels has $\mathscr{b}=2^{\mathcal{O}}$ nodes. The level 0 to level $\mathcal{O}$ nodes contribute an $\mathcal{O}$-dimensional Butterfly Network denoted by $BR(\mathcal{O})$.

Definition 1.3: [8] An $\mathcal{O}$- dim Benes network has $2\mathcal{O} + 1, \mathcal{O} \geq 1$ levels. Each column of the levels contains $\mathscr{b}=2^{\mathcal{O}}$ nodes, Where 0^{th}level to level $2\mathcal{O}$ level nodes contribute an $\mathcal{O}$-dimensional Benes Network denoted by $B(\mathcal{O})$. Benes (network) are made of back-to-back butterfly (network).

Lemma 1.1: [5] *The Benes and butterfly networks are bipartite.*

2. Outcomes and Results

Theorem 2.1: *Graph $G = B(\mathcal{O})$, $\mathcal{O} \geq 1$ is Antimagic.*

Proof: For Graph $G = B(r)$, $\mathcal{O} \geq 1$, The $V(G)$ and $E(G)$ can be denoted by:

$V(G) = \{b_{nj}, 0 \leq n < 2\mathcal{O}, j = 1, \cdots, \mathscr{b}\}$. $E(G) = \{(b_{nj}b_{(n+1)j},$ $0 \leq n < 2\mathcal{O} - 1, j\ equal 1, \cdots, \mathscr{b}), (b_{0j}b_{1k}, j$ equal $1, 2, \cdots, \frac{\mathscr{b}}{2}, k$ equal $j + 2^{\mathcal{O}-1}$ $(0r)\ j = \frac{\mathscr{b}}{2} + 1, \cdots, \mathscr{b}, k$ equal $j - 2^{\mathcal{O}-1}), (b_{1j}b_{2k}, j = 1, 2, \cdots \frac{\mathscr{b}}{4}, \frac{\mathscr{b}}{2} + 1, \cdots,$ $\frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4}, k$ equal $j + 2^{\mathcal{O}-2} (or)\ j = \frac{\mathscr{b}}{4} + 1, \cdots, \frac{\mathscr{b}}{2}, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} + 1, \cdots, \mathscr{b}, k$ equal $j -$ $2^{\mathcal{O}-2}), (b_{2j}b_{3k}, j = 1, 2, \cdots, \frac{\mathscr{b}}{8}, \frac{\mathscr{b}}{4} + 1, \cdots, \frac{\mathscr{b}}{4} + \frac{\mathscr{b}}{8}, \frac{\mathscr{b}}{2} + 1, \cdots \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{8}, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} +$

$1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}, k$ equal $j+2^{\mathcal{O}-3}$ (or) $j=2^{\mathcal{O}-3}+1,\cdots,\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}+1,\cdots,\mathscr{b}, k$ equal $j-2^{\mathcal{O}-3}),\cdots,$ $(b_{\mathcal{O}j}b_{(\mathcal{O}+1)k}, j$ equal$1,3,\cdots,\mathscr{b}-1, k$ equal $j+1$ (or) $j=2,4,\cdots,\mathscr{b}, k$ equal $j-1), (b_{(\mathcal{O}+1)j}b_{(\mathcal{O}+2)k}, j=1,2,4,5,\cdots,\mathscr{b}-3,\mathscr{b}-2, k$ equal $j+2$ (or) $j=3,4,6,7,\cdots,\mathscr{b}-1,\mathscr{b}, k$ equal $j-2),\cdots,(b_{(2\mathcal{O}-2)j}b_{(2\mathcal{O}-1)k}, j$ equal $1,2,\cdots,\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}, k$ equal $j+2^{\mathcal{O}-2}$ (or) $j=\frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+1,\cdots,\mathscr{b}, k$ equal $j-2^{\mathcal{O}-2}), (b_{(2\mathcal{O}-1)j}b_{(2\mathcal{O})k}, j$ equal $1,2,\cdots,\frac{\mathscr{b}}{2}, k$ equal $j+\frac{\mathscr{b}}{2}$ (or) j equal $\frac{\mathscr{b}}{2}+1,\cdots,\mathscr{b}, k$ equal $j-2^{\mathcal{O}-1})\}$. Figure 1 denotes Graph $G=B(1)$, where vertices and edges are been marked also B(1) is Antimagic Labeled.

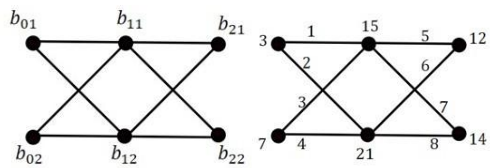

Figure 1
***G* = *B*(1), *G* is Antimagic labeled**

For G = B($\mathcal{O}$), $\mathcal{O}$ > 1, We have the edge function f as

$$f(b_{0j}b_{1j})=\begin{cases}2j-1, j=1,2,\cdots,\frac{\mathscr{b}}{2}\\ 2j, j=\frac{\mathscr{b}}{2}+1,\cdots,\mathscr{b}\end{cases}$$

$$f(b_{1j}b_{2j})=\begin{cases}2j-1+\mathscr{b}^2, \text{Where } j \text{ equal } 1,\cdots\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}\\ 2j+\mathscr{b}^2, \text{Where } j \text{ equal } \frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+1,\cdots,\mathscr{b}\end{cases}$$

$$f(b_{2j}b_{3j})=\begin{cases}2j-1+2\mathscr{b}^2, j \text{ equal} 1,2,\cdots,\frac{\mathscr{b}}{8},\frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8},\\ \quad\frac{\mathscr{b}}{2}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{8},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}\\ 2j+2\mathscr{b}^2, j \text{ equal} \frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+\\ \quad\frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}+\cdots,\mathscr{b}\end{cases}$$

$$\cdots$$

$$f(b_{(\mathcal{O}-1)j}b_{\mathcal{O}j})=\begin{cases}2j-1+(r-1)\mathscr{b}^2, j \text{ equal } 1,3,\cdots,\mathscr{b}-1\\ 2j+(r-1)\mathscr{b}^2, j \text{ equal } 2,4,\cdots,\mathscr{b}\end{cases}$$

$$f(b_{\mathcal{O}j}b_{(\mathcal{O}+1)j})=\begin{cases}2j-1+\mathcal{O}\mathscr{b}^2, j \text{ equal } 1,3,\cdots,\mathscr{b}-1\\ 2j+\mathcal{O}\mathscr{b}^2, j \text{ equal } 2,4,\cdots,\mathscr{b}\end{cases}$$

$$f(b_{(\mathcal{O}+1)j}b_{(\mathcal{O}+2)j})=\begin{cases}2j-1+(\mathcal{O}+1)\mathscr{b}^2, j=1,3,\cdots,\frac{\mathscr{b}}{2}\\ 2j+(\mathcal{O}+1)\mathscr{b}^2, j=2,4,\cdots,\mathscr{b}\end{cases}$$

$$\cdots$$

$$f\left(b_{(2\mathcal{O}-2)j}b_{(2\mathcal{O}-1)j}\right) = \begin{cases} 2j-1+(2\mathcal{O}-2)\mathscr{b}^2, j=1,2,\cdots\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4} \\ 2j+(2\mathcal{O}-2)\mathscr{b}^2, j=\frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+1,\cdots,\mathscr{b} \end{cases}$$

$$f\left(b_{(2\mathcal{O}-1)j}b_{(2\mathcal{O})j}\right) = \begin{cases} 2j-1+(2\mathcal{O}-1)\mathscr{b}^2, j \text{ equal } 1,2,\cdots,\frac{\mathscr{b}}{2} \\ 2j+(2\mathcal{O}-1)\mathscr{b}^2, j \text{ equal } \frac{\mathscr{b}}{2}+1,\cdots,\mathscr{b} \end{cases}$$

$$f\left(b_{0j}b_{1k}\right) = \begin{cases} 2j, j \text{ equal } 1,2,\cdots,\frac{\mathscr{b}}{2}, k \text{ equal } j+2^{\mathcal{O}-1} \\ 2j-1, j \text{ equal } \frac{\mathscr{b}}{2}+1,\cdots,\mathscr{b}, k \text{ equal } j-2^{\mathcal{O}-1} \end{cases}$$

$$f\left(b_{1j}b_{2k}\right) = \begin{cases} 2j+\mathscr{b}^2, j \text{ equal} 1,2,\cdots\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}, k \text{ equal } j+2^{\mathcal{O}-2} \\ 2j-1+\mathscr{b}^2, j \text{ equal} \frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+1,\cdots,\mathscr{b}, k \text{ equal } j-2^{\mathcal{O}-2} \end{cases}$$

$$f\left(b_{2j}b_{3k}\right) = \begin{cases} 2j+2.\mathscr{b}^2, j \text{ equal} 1,2,\cdots,\frac{\mathscr{b}}{8},\frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8},\frac{\mathscr{b}}{2}+1, \\ \cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{8},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}, k \text{ equal } j+2^{r-3} \\ 2j-1+2.\mathscr{b}^2, j \text{ equal} \frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+ \\ \frac{\mathscr{b}}{8}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+\frac{\mathscr{b}}{8}+\cdots,\mathscr{b}, k \text{ equal } j-2^{r-3} \end{cases}$$

$$\cdots$$

$$f\left(b_{(\mathcal{O}-1)j}b_{\mathcal{O}k}\right) = \begin{cases} 2j+(r-1)\mathscr{b}^2, j \text{ equal} 1,3,\cdots,\mathscr{b}-1, k \text{ equal } j+1 \\ 2j-1+(r-1)\mathscr{b}^2, j \text{ equal } 2,4,\cdots,\mathscr{b}, k \text{ equal } j-1 \end{cases}$$

$$f\left(b_{\mathcal{O}j}b_{(\mathcal{O}+1)k}\right) = \begin{cases} 2j+\mathcal{O}\mathscr{b}^2, j \text{ equal} 1,3,\cdots,\mathscr{b}-1, k=j+1 \\ 2j-1+\mathcal{O}\mathscr{b}^2, j \text{ equal } 2,4,\cdots,\mathscr{b}, k=j-1 \end{cases}$$

$$f\left(b_{(\mathcal{O}+1)j}b_{(\mathcal{O}+2)k}\right) = \begin{cases} 2j+(r\mathscr{b}^2+\mathscr{b}^2), j \text{ equal } 1,2,5,6,\cdots,\mathscr{b}-3,2^{\mathcal{O}}-2, k \text{ equal } j+2 \\ 2j-1+(r\mathscr{b}^2+\mathscr{b}^2), j \text{ equal } 3,4,7,8,\cdots,2^{\mathcal{O}}-1,2^{\mathcal{O}}, k \text{ equal } j-2 \end{cases}$$

$$\cdots$$

$$f\left(b_{(2\mathcal{O}-2)j}b_{(2\mathcal{O}-1)k}\right) = \begin{cases} 2j+(2\mathcal{O}-2)\mathscr{b}^2, j \text{ equal } 1,2,\cdots\frac{\mathscr{b}}{4},\frac{\mathscr{b}}{2}+1,\cdots,\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}, k \text{ equal } j+2^{\mathcal{O}-2} \\ 2j-1+(2\mathcal{O}-2)\mathscr{b}^2, j \text{ equal} \frac{\mathscr{b}}{4}+1,\cdots,\frac{\mathscr{b}}{2},\frac{\mathscr{b}}{2}+\frac{\mathscr{b}}{4}+1,\cdots,\mathscr{b}, k \text{ equal } j-2^{\mathcal{O}-2} \end{cases}$$

$$f\left(b_{(2\mathcal{O}-1)j}b_{(2\mathcal{O})k}\right) = \begin{cases} 2j-1+(2\mathcal{O}-2)\mathscr{b}^2, j \text{ equal } 1,2,\cdots,\frac{\mathscr{b}}{2}, k \text{ equal } j+2^{\mathcal{O}-1} \\ 2j+(2\mathcal{O}-2)\mathscr{b}^2, j \text{ equal } \frac{\mathscr{b}}{2}+1,\cdots,\mathscr{b}, k \text{ equal } j-2^{\mathcal{O}-1} \end{cases}$$

Thus the induced vertex function is: $\phi_f\left(v_{0j}\right) = 4j - 1, j = 1,2,\cdots,\mathscr{b}$

$$\phi_f\left(v_{1j}\right) = \begin{cases} \phi_f\left(v_{0j}\right) + K_1 + 4j - 2, j = 1,2,\cdots,\frac{\mathscr{b}}{2} \\ \phi_f\left(v_{0j}\right) + K_1 + 4(j - 2^{\mathcal{O}-1}), j = \frac{\mathscr{b}}{2} + 1,\cdots,\mathscr{b} \end{cases}$$

$$\phi_f\left(v_{2j}\right) \begin{cases} \phi_f\left(v_{0j}\right) + K_2 + 4j - 2, j = 1, 2,\cdots \frac{\mathscr{b}}{4}, \frac{\mathscr{b}}{2} + 1,\cdots, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} \\ \phi_f\left(v_{0j}\right) + K_2 + 4(j - 2^{r-2}), j = \frac{\mathscr{b}}{4} + 1,\cdots, \frac{\mathscr{b}}{2}, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} + 1,\cdots,\mathscr{b} \end{cases}$$

$$\cdots$$

$$\phi_f\left(v_{\mathcal{O}j}\right) = \begin{cases} \phi_f\left(v_{0j}\right) + K_{\mathcal{O}} + 4j - 2, j = 1,3,\cdots,\mathscr{b} - 1 \\ \phi_f\left(v_{0j}\right) + K_{\mathcal{O}} + 4\left(j - 2^{\mathcal{O}-(\mathcal{O}-1)}\right), j = 2,4,\cdots,\mathscr{b} \end{cases}$$

$$\phi_f\left(v_{(\mathcal{O}+1)j}\right) = \begin{cases} \phi_f\left(v_{0j}\right) + K_{\mathcal{O}+1} + 4j - 2, j\ equal\ 1,3,\cdots,\mathscr{b} - 1 \\ \phi_f\left(v_{0j}\right) + K_{\mathcal{O}+1} + 4\left(j - 2^{\mathcal{O}-(\mathcal{O}-1)}\right), j\ equal\ 2,4,\cdots,\mathscr{b} \end{cases}$$

$$\cdots$$

$$\phi_f\left(v_{(2\mathcal{O}-1)j}\right) =$$

$$\begin{cases} \phi_f\left(v_{0j}\right) + K_{2\mathcal{O}-1} + 4j - 2, j \text{ equal } 1, 2,\cdots \frac{\mathscr{b}}{4}, \frac{\mathscr{b}}{2} + 1,\cdots, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} \\ \phi_f\left(v_{0j}\right) + K_{2\mathcal{O}-1} + 4(j - 2^{\mathcal{O}-1}), j \text{ equal } \frac{\mathscr{b}}{4} + 1,\cdots, \mathscr{b}/2, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} + 1,\cdots,\mathscr{b} \end{cases}$$

$$\phi_f\left(v_{(2\mathcal{O})j}\right) = \begin{cases} \phi_f\left(v_{0j}\right) + K_{\mathcal{O}r} + 4(j - 1), j \text{ equal } 1,2,\cdots,\frac{\mathscr{b}}{2} \\ \phi_f\left(v_{0j}\right) + K_{\mathcal{O}r} + 4(j - 2) - 2, j \text{ equal } \frac{\mathscr{b}}{2} + 1,\cdots,\mathscr{b} \end{cases}$$

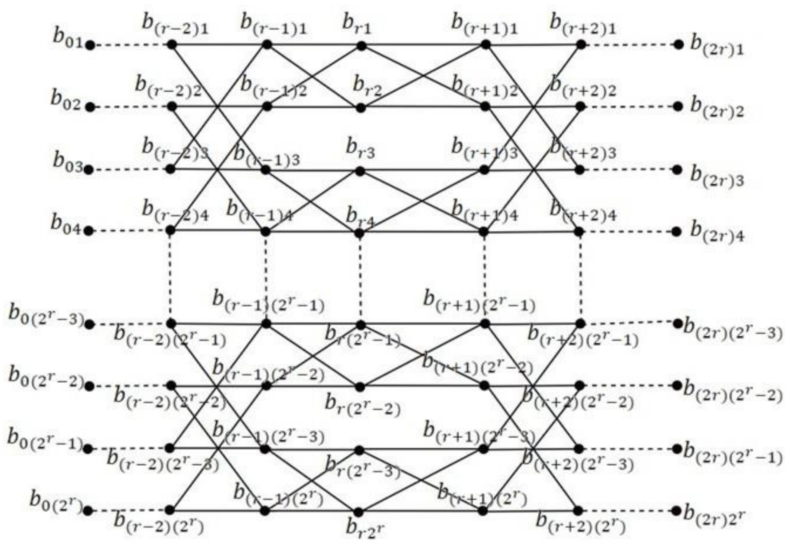

Figure 2
Graph $G = B(\mathcal{O})$, $\mathcal{O} = r$, $r \geq 1$. Vertices of G has been Antimagic Labeled.

Where $K_1, K_2, \cdots, K_{2\mathcal{O}}$ are constants, $K_1 < K_2 < \cdots < K_{2\mathcal{O}}$. Figure 2 denotes the general case where graph $G = B(r), r \geq 1$ is Antimagic Labeled

Theorem 2.2: *Graph $G = BF(\mathcal{O})$, $\mathcal{O} \geq 1$ is Antimagic.*

Proof: For Graph $G = BF\,(\mathcal{O}), \mathcal{O} \geq 1$, The V(G) and E(G) can be denoted by: $V\,(G) = \{b_{nj}, n = 0, 1, \cdots, \mathcal{O}, j \text{ equal } 1, 2, \cdots, \mathscr{b}\}$. $E(G) = \{(b_{nj}b_{(n+1)j}\,, n = 0, 1, \cdots, \mathcal{O} - 1, j \text{ equal } 1, 2, \cdots, \mathscr{b}), (b_{0j}b_{1k}, j \text{ equal } 1, 2, \cdots, \frac{\mathscr{b}}{2}, k \text{ equal } j + 2^{\mathcal{O}-1}$ (or) $j = \frac{\mathscr{b}}{2} + 1, \cdots, \mathscr{b}, k = j - 2^{\mathcal{O}-1}), (b_{1j}b_{2k}, j = 1, 2, \cdots \frac{\mathscr{b}}{4}, \frac{\mathscr{b}}{2} + 1, \cdots, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4}, k \text{ equal } j + 2^{\mathcal{O}-2}$ (or) $j = \frac{\mathscr{b}}{4} + 1, \cdots, \frac{\mathscr{b}}{2}, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} + 1, \cdots, \mathscr{b}, k = j - 2^{\mathcal{O}-2}), (b_{2j}b_{3k}, j = 1, 2, \cdots, \frac{\mathscr{b}}{8}, \frac{\mathscr{b}}{4} + 1, \cdots, \frac{\mathscr{b}}{4} + \frac{\mathscr{b}}{8}, \frac{\mathscr{b}}{2} + 1, \cdots, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{8}, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} + 1, \cdots, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} + \frac{\mathscr{b}}{8}, k = j + 2^{\mathcal{O}-3}$ (or) $j = \frac{\mathscr{b}}{8} + 1, \cdots, \frac{\mathscr{b}}{4}, \frac{\mathscr{b}}{4} + \frac{\mathscr{b}}{8} + 1, \cdots, \frac{\mathscr{b}}{2}, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{8} + 1, \cdots, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4}, \frac{\mathscr{b}}{2} + \frac{\mathscr{b}}{4} + \frac{\mathscr{b}}{8} + 1, \cdots, 2^{\mathcal{O}}, k = j - 2^{\mathcal{O}-3}), \cdots, (b_{(\mathcal{O}-1)j}\, b_{\mathcal{O}k}, j = 1, 3, \cdots, \mathscr{b} - 1, k \text{ equal } j + 1$ (or) $j = 2, 4, \cdots, \mathscr{b}\; k \text{ equal } j - 1)$.

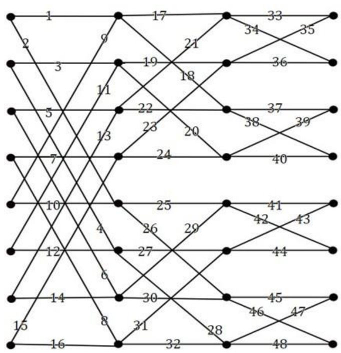

Figure 3
Graph $G = BF\,(3)$, G is Antimagic Labeled.

From Lemma 1.1, we get $B(\mathcal{O})$ and $BF\,(\mathcal{O})$ follows the same edge function f and thus follow same vertex function ϕ_f except for the level $\mathcal{O}$ of $BF\,(\mathcal{O})$. Such that, From Lemma 1.1 and Theorem 1.1 the function f for benes network and butterfly network remains same for level o to r. Thus the vertex function ϕ_f of $BF(\mathcal{O})$ in level $\mathscr{b}$ is:

$$\phi_f\,(v_{\mathcal{O}j}) = \begin{cases} \phi_f\,(v_{0j}) + F_{\mathcal{O}} + 4j - 2, j = 1, 2, \cdots \frac{\mathscr{b}}{2} \\ \phi_f\,(v_{0j}) + F_{\mathcal{O}} + 4(j-1), j = \frac{\mathscr{b}}{2} + 1, \cdots, \mathscr{b} \end{cases}$$

Where F_O is a constant, $K_1 < K_2 < \cdots < K_{r-1} < F_O$. Figure 3 denotes the graph $G = BF(3)$ is Antimagic Labeled.

3. Application Idea

- Graph representation: A graph with nodes (or vertices) representing entities and edges representing the connections or paths between them can be used to illustrate a parallel network. numerous edges can exist between the same pair of nodes in a parallel network, suggesting that there are numerous unique pathways or connections available.
- Assume list of elements to be labeled: In order to more easily recognize and refer to a list of elements (nodes or edges) that need to be labeled in the context of a parallel network, you might name each element individually. A graph is a collection of vertices and edges which are tend to be labeled. In this case, processors and connecting servers acts as vertices and edges and thus are labeled.
- distribution of function among the processors and the connections: Say, Graph $G = B(3)$ has been antimagic labeled is been illustrated in Figure 4.
- Creating a poll of process: In order to acquire the network's Antimagic labeling, the edge and vertex functions were developed specifically for it and applied to it.

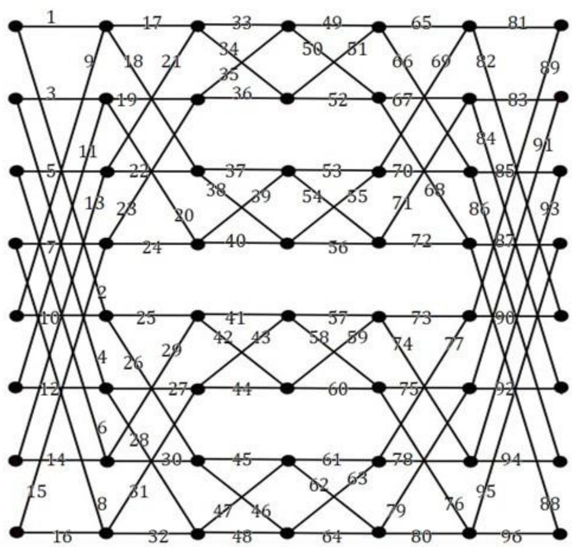

Figure 4
Graph $G = B(3)$ has been antimagic labeled

5. Discussion and Conclusion

One of the primary applications of parallel computing is Databases and Data mining. Another application of parallel computing is the simulation of systems in real time. In this paper, Antimagic Labeling for Butterfly and Benes Networks has been discussed. Further, Benes and Butterfly networks has been proved as Antimagic.

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