

The bijective soft semigroups and fuzzy parameterized bijective soft sets

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Abstract

In this paper, bijective soft semigroups over semigroups are introduced. We give some properties of bijective soft semigroups over semigroups with supporting examples. Moreover, we defined fuzzy parameterized bijective soft sets over a universe and present an algorithm for solving some decision-making problem based on fuzzy parameterized bijective soft sets over a universe.

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1. Introduction

The most of real-world problems in environmental, science, sociology, engineering, economics, medical science, and many other fields involve the use of mathematical tools. Some classical tools, such as fuzzy sets [1], rough sets [2], intuitionistic fuzzy sets [3], are often useful approaches to explaining uncertainty. In 1999, Molodtsov [4] purposed the concept of soft set theory which was free from the parameterization of inadequacy condition of fuzzy set theory, probability theory and rough set theory. Later, Maji et al. [5, 6] defined soft subsets and soft super sets, equality of two soft sets. They presented soft binary operations and discussed the application of soft sets in decision-making problems based on the weighted table in the algorithm. The soft set theory has been developed by many researchers [7, 8].

In 2010, Gong et al. [9] proposed the concept of bijective soft sets and some of its operations such as the restricted AND and the relax AND operations on a bijective soft set. Later, Çağman et al. [10] introduced fuzzy parameterized soft sets (FP-soft sets) and presented some of its properties. They discussed the application of the algorithm of fuzzy parameterized soft sets in decision-making problems. The soft sets are extended to fuzzy soft sets by Maji et al. [11]. Next, the fuzzy soft set is also a new notion applying to abstract algebraic structures. In 2011, Yang [12] developed fuzzy soft sets to fuzzy soft semigroups. He defined a soft semigroup and soft [left, right] ideal and a fuzzy soft [left, right] ideal and a fuzzy soft semigroup over a semigroup. Next, Suebsan [13] defined relative fuzzy soft sets over some semigroups and investigated their properties. Moreover, he discussed the new model for solving decision-making problems based on relative fuzzy soft sets over some semigroups.

When it comes to existing researches of soft sets in decision-making problems, they are limited to some extent. The algorithm of Maji et al. [6] and Çağman et al. [10] are designed for each parameter has a single parameter. The fuzzy parameterized bijective soft sets over common universe plays a key role in dealing with problem above. Motivated and inspired by the works above, we are interested in the bijective soft semigroup over semigroups.

In this paper, we define bijective soft semigroups over semigroups and give some properties of bijective soft semigroups over semigroups with supporting examples. Moreover, we defined fuzzy parameterized bijective soft sets over a universe and present an algorithm for solving

some decision-making problem based on fuzzy parameterized bijective soft sets over a universe.

2. Preliminaries

In this section, we shall give some basic definitions and results that will be used

The first concept of soft sets over a universe was introduced by Molodtsov [4].

Definition 2.1: [4] Let $\mathcal{U}$ be an initial universal set and Ξ be a set of parameters. Let $\mathcal{P}(\mathcal{U})$ denotes the set of all subsets of $\mathcal{U}$ and $\emptyset \neq \Lambda \subseteq \Xi$. A pair (Ψ, Λ) is called a *soft set* over $\mathcal{U}$, where Ψ is a mapping from Λ into the power set $\mathcal{P}(\mathcal{U})$ ($\Psi: \Lambda \rightarrow \mathcal{P}(\mathcal{U})$).

Note that a soft set (Ψ, Λ) can be represented by the set of ordered pairs

$$(\Psi, \Lambda) = \{(p, \Psi(p)) : p \in \Lambda, \Psi(p) \in \mathcal{P}(\mathcal{U})\}.$$

The some operations of soft sets over a universe were studied by Maji et al. [5].

Definition 2.2: [5] The *union* of two soft sets (Ψ, Λ) and (Ω, Σ) over a common universe $\mathcal{U}$. Define $(\Psi, \Lambda) \tilde{\cup} (\Omega, \Sigma) := (\Psi \cup \Omega, \Lambda \cup \Sigma)$, where for all $p \in \Lambda \cup \Sigma$,

$$(\Psi \cup \Omega)(p) = \begin{cases} \Psi(p) & \text{if } p \in \Lambda - \Sigma, \\ \Omega(p) & \text{if } p \in \Sigma - \Lambda, \\ \Psi(p) \cup \Omega(p) & \text{if } p \in \Lambda \cap \Sigma. \end{cases}$$

Then $(\Psi, \Lambda) \tilde{\cup} (\Omega, \Sigma)$ is a soft set over $\mathcal{U}$.

Definition 2.3: [5] Let (Ψ, Λ) and (Ω, Σ) be two soft sets over a common universe $\mathcal{U}$. The *AND* operation on two soft sets (Ψ, Λ) and (Ω, Σ) , “ (Ψ, Λ) AND (Ω, Σ) ”, denoted by $(\Psi, \Lambda) \wedge (\Omega, \Sigma)$, is defined by

$$(\Psi, \Lambda) \wedge (\Omega, \Sigma) := (\Upsilon, \Lambda \times \Sigma),$$

where $\Upsilon(\alpha, \beta) = \Psi(\alpha) \cap \Omega(\beta)$, for all $(\alpha, \beta) \in \Lambda \times \Sigma$.

Definition 2.4: [5] A soft set (Ψ, Λ) over a universe $\mathcal{U}$ is said to be a *null soft set* denoted by Φ , if $\Psi(p) = \emptyset$ for all $p \in \Lambda$.

The concept of bijective soft sets over a universe was defined by Gong et al. [9].

Definition 2.5: [9] Let Λ be a non-empty parameter set and let (Ψ, Λ) be a soft set over a common universe $\mathcal{U}$. We say that (Ψ, Λ) is a *bijective soft set over $\mathcal{U}$* if it satisfies

- (i) $\bigcup_{p \in \Lambda} \Psi(p) = \mathcal{U}$ and
- (ii) for any two distinct parameters $p_i, p_j \in \Lambda$, $\Psi(p_i) \cap \Psi(p_j) = \emptyset$.

Example 2.6: [9] Suppose that $\mathcal{U} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ is a universe and (Ψ, Ξ) is a soft set over $\mathcal{U}$, where $\Xi = \{e_1, e_2, e_3, e_4\}$. The mapping of (Ψ, Ξ) is given below:

$$\Psi(e_1) = \{x_1, x_2, x_3\}, \Psi(e_2) = \{x_4, x_5, x_6\}, \Psi(e_3) = \{x_7\}, \Psi(e_4) = \{x_4, x_5, x_6, x_7\}.$$

Let $\Lambda = \{e_1, e_2, e_3\}$, $\Sigma = \{e_1, e_4\}$, $\diamond = \{e_1, e_2\}$ and $\heartsuit = \{e_2, e_3\}$. From Definition 2.5, (Ψ, Λ) and (Ψ, Σ) are bijective soft sets but $(\Psi, \diamond)$ and $(\Psi, \heartsuit)$ are not bijective soft sets.

The following theorems were presented by Gong et al. [9].

Theorem 2.7: [9] Suppose that (Ψ, Λ) and (Ω, Σ) are two bijective soft sets over a common universe $\mathcal{U}$. Then $(\Upsilon, \Theta) = (\Psi, \Lambda) \wedge (\Omega, \Sigma)$ is a bijective soft set.

Theorem 2.8: [9] Suppose that (Ψ, Λ) is a bijective soft set over a universe $\mathcal{U}$ and (Ω, Σ) is a null soft set over $\mathcal{U}$. Then $(\Gamma, \Delta) = (\Psi, \Lambda) \widetilde{\cup} (\Omega, \Sigma)$ is a bijective soft set.

Fuzzy parameterized soft set (FP-soft set) theory was proposed by Çagman et al. [10].

Definition 2.9: [10] Let $\mathcal{U}$ be an initial universal set, $\mathcal{P}(\mathcal{U})$ be a set of all subsets of $\mathcal{U}$, Ξ be a set of all parameters and X be a fuzzy set over Ξ with the membership function $\mu_X : \Xi \rightarrow [0, 1]$. An *FP-soft set* Ψ_X over $\mathcal{U}$ is a set defined by a function ψ_X representing a mapping

$$\psi_X : \Xi \rightarrow \mathcal{P}(\mathcal{U}) \text{ such that } \psi_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

Here, ψ_X is called an *approximate function* of the FP-soft set Ψ_X over $\mathcal{U}$ can be represented by the set of ordered pairs

$$\Psi_X = \{(\mu_X(x) / x, \psi_X(x)) : x \in \Xi, \psi_X(x) \in \mathcal{P}(\mathcal{U}), \mu_X(x) \in [0, 1]\}.$$

Example 2.10: [10] Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ be an initial universal set and $\Xi = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $X = \{0.2 / x_2, 0.5 / x_3, 1 / x_4\}$ and $\psi_X(x_2) = \{u_2, u_4\}$, $\psi_X(x_3) = \{u_1, u_2\}$, $\psi_X(x_4) = \mathcal{U}$, then the FP-soft set Ψ_X is written by

$$\Psi_x = \{(0.2/x_2, \{u_2, u_4\}), (0.5/x_3, \{u_1, u_2\}), (1/x_4, \mathcal{U})\}.$$

Definiton 2.11: [14] A non-empty subset T of a semigroup S is called a *subsemigroup* if $T^2 \subseteq T$.

The first concept of soft sets over a semigroup was introduced by Yang [12].

Definition 2.12: [12] A soft set (Ψ, Λ) over a semigroup S is said to be a *soft semigroup* over S if $\Psi(p)$ is a subsemigroup of S for all $p \in \Lambda$.

The following example is an example of a soft semigroup over a semigroup.

Example 2.13: Let $S = \{a_1, a_2, \dots, a_n\}$ and define a binary operation $*$ on S by $a_s * a_t = a_{\max\{s, t\}}$ for every $a_s, a_t \in S$. Then $(S, *)$ is a semigroup. Let $\Xi = \{e_1, e_2, \dots, e_m\}$ be a set of parameters and let $\Lambda = \{e_1, e_2, \dots, e_k\}$ where $k \leq m$. Let (Ψ, Λ) be a soft set over S as follows:

$$(\Psi, \Lambda) = \{(e_1, \{a_1\}), (e_2, \{a_1, a_2\}), (e_3, \{a_1, a_2, a_3\}), \dots, (e_k, \{a_1, a_2, \dots, a_k\})\},$$

where $\Psi(e_1) = \{a_1\}$, $\Psi(e_2) = \{a_1, a_2\}$, ..., $\Psi(e_k) = \{a_1, a_2, \dots, a_k\}$.

It is to verify that $\Psi(e_1), \Psi(e_2), \dots, \Psi(e_k)$ are subsemigroups of S . Hence (Ψ, Λ) is a soft semigroup over S .

3. The bijective soft semigroups over semigroups

In this section, we study a bijective soft set over semigroups and give some operations, and define a bijective soft semigroup over semigroups and obtain some operations with supporting examples.

Definition 3.14: Let Λ be a non-empty parameter set and let (Ψ, Λ) be a soft set over a semigroup S . We say that (Ψ, Λ) is a *bijective soft set over S* if (Ψ, Λ) satisfies

- (i) $\bigcup_{p \in \Lambda} \Psi(p) = S$ and
- (ii) for any two distinct parameters $p, q \in \Lambda$, $\Psi(p) \cap \Psi(q) = \emptyset$.

Example 3.15: Let $S = \{x_1, x_2, x_3, x_4\}$ be a semigroup having the following multiplication in Table 1.

Table 1
Multiplication table of a semigroup S

$\bullet$	x_1	x_2	x_3	x_4
x_1	x_1	x_2	x_3	x_4
x_2	x_2	x_2	x_3	x_4
x_3	x_3	x_3	x_3	x_4
x_4	x_4	x_4	x_4	x_4

Let $\Lambda = \{e_1, e_2, e_3\}$ be a set of parameters. The mapping of (Ψ, Λ) is given below:

$$\Psi(e_1) = \{x_1, x_2\}, \Psi(e_2) = \{x_3\}, \Psi(e_3) = \{x_4\}, \Psi(e_4) = \{x_3, x_4\}.$$

Let $\Lambda = \{e_1, e_2, e_3\}$, $\Sigma = \{e_1, e_4\}$, $\diamond = \{e_1, e_2\}$ and $\heartsuit = \{e_2, e_3\}$. From Definition 3.14, (Ψ, Λ) and (Ψ, Σ) are bijective soft sets but $(\Psi, \diamond)$ and $(\Psi, \heartsuit)$ are not bijective soft sets.

The following theorem shows that the AND operation of two bijective soft sets over a semigroup is a bijective soft set over a semigroup.

Theorem 3.16: Suppose that (Ψ, Λ) and (Ω, Σ) are two bijective soft sets over a semigroup S . If $\Lambda \cap \Sigma \neq \emptyset$ then $(\Upsilon, \Theta) = (\Psi, \Lambda) \wedge (\Omega, \Sigma)$ is a bijective soft set over S .

Proof: It follows from Theorem 2.7. □

The following theorem show that $(\Gamma, \Delta) = (\Psi, \Lambda) \tilde{\cup} (\Omega, \Sigma)$ of a bijective soft set (Ψ, Λ) over a semigroup and a null soft set (Ω, Σ) over a semigroup is a bijective soft set over a semigroup

Theorem 3.17: Suppose that (Ψ, Λ) is a bijective soft set over a semigroup S and (Ω, Σ) is a null soft set over S . Then $(\Gamma, \Delta) = (\Psi, \Lambda) \tilde{\cup} (\Omega, \Sigma)$ is a bijective soft set over S .

Proof. It follows from Theorem 2.8. □

Definition 3.18: A bijective soft set (Ψ, Λ) over a semigroup S is said to be a bijective soft semigroup over S if (Ψ, Λ) is a soft semigroup over S .

The following example is an example of bijective soft semigroups over a semigroup, and we show that a soft semigroup over a semigroup is not a bijective soft semigroup over a semigroup.

Example 3.19: Let $S = \{o_1(3.50), o_2(3.65), o_3(3.75), o_4(3.85), o_5(3.90), o_6(3.95)\}$ be a set of six candidates with grade point average(GPA) under consideration. Let $\bullet$ be a binary operation which defined by Table 2.

Table 2
Multiplication table of a semigroup S

$\bullet$	o_1	o_2	o_3	o_4	o_5	o_6
o_1	o_1	o_2	o_3	o_4	o_5	o_6
o_2	o_2	o_2	o_3	o_4	o_5	o_6
o_3	o_3	o_3	o_3	o_4	o_5	o_6
o_4	o_4	o_4	o_4	o_4	o_5	o_6
o_5	o_5	o_5	o_5	o_5	o_5	o_6
o_6	o_6	o_6	o_6	o_6	o_6	o_6

Let $\Xi = \{e_1\{\text{good computer knowledge}\}, e_2\{\text{good speaking}\}, e_3\{\text{friendly}\}, e_4\{\text{good englishs kill}\}, e_5\{\text{experience}\}\}$ be a set of parameters and let $\Lambda = \{e_1, e_2, e_3, e_5\}$ and $\Sigma = \{e_1, e_2, e_4, e_5\}$.

Let (Ψ, Λ) be a soft set over S defined by

$$(\Psi, \Lambda) = \{(e_1, \{o_1\}), (e_2, \{o_2, o_4\}), (e_3, \{o_3, o_5\}), (e_5, \{o_6\})\},$$

where $\Psi(e_1) = \{o_1\}, \Psi(e_2) = \{o_2, o_4\}, \Psi(e_3) = \{o_3, o_5\}, \Psi(e_5) = \{o_6\}$. It is to verify that $\Psi(e_1), \Psi(e_2), \Psi(e_3), \Psi(e_5)$ are subsemigroups of S . Thus (Ψ, Λ) is a soft semigroup over S . It is easy to see that (Ψ, Λ) is a bijective soft set over S . Hence (Ψ, Λ) is a bijective soft semigroup over S .

Let (Ω, Σ) be a soft set over S defined by

$$(\Omega, \Sigma) = \{(e_1, \{o_1\}), (e_2, \{o_1, o_2, o_3\}), (e_4, \{o_1, o_2, o_3, o_4, o_5\}), (e_5, \{o_1, o_2, o_3, o_4, o_5, o_6\})\},$$

where $\Omega(e_1) = \{o_1\}, \Omega(e_2) = \{o_1, o_2, o_3\}, \Omega(e_4) = \{o_1, o_2, o_3, o_4, o_5\}, \Omega(e_5) = \{o_1, o_2, o_3, o_4, o_5, o_6\}$.

It is to see that (Ω, Σ) is a soft semigroup over S , but (Ω, Σ) is not a bijective soft set over S .

Hence (Ω, Σ) is not a bijective soft semigroup over S .

The following theorem show that the AND operation of two bijective soft semigroup over a semigroup is a bijective soft set over a semigroup under some conditions.

Theorem 3.20: Suppose that (Ψ, Λ) and (Ω, Σ) are two bijective soft semigroup over a semigroup S . If $\Lambda \cap \Sigma \neq \emptyset$ then $(\Psi, \Lambda) \wedge (\Omega, \Sigma)$ is a bijective soft set over S .

Proof: Assume that $\Lambda \cap \Sigma \neq \emptyset$. Let $(\Upsilon, \Lambda \times \Sigma) := (\Psi, \Lambda) \wedge (\Omega, \Sigma)$. Thus

$$\bigcup_{(\alpha, \beta) \in \Lambda \times \Sigma} \Upsilon(\alpha, \beta) = \bigcup_{\alpha \in \Lambda} \bigcup_{\beta \in \Sigma} \Psi(\alpha) \cap \Omega(\beta) = \left(\bigcup_{\alpha \in \Lambda} \Psi(\alpha) \right) \cap \left(\bigcup_{\beta \in \Sigma} \Omega(\beta) \right) = S \cap S = S.$$

Let $(a, c) \neq (b, d)$. Then $\Psi(a) \cap \Omega(c) = \emptyset$ or $\Psi(b) \cap \Omega(d) = \emptyset$. That is $\Upsilon(a, c) \cap \Upsilon(b, d) = \emptyset$.

Hence $(\Upsilon, \Lambda \times \Sigma) = (\Psi, \Lambda) \wedge (\Omega, \Sigma)$ is a bijective soft set over S . $\square$

The following theorem show that $(\Gamma, \Delta) = (\Psi, \Lambda) \widetilde{\cup} (\Omega, \Sigma)$ of a bijective soft semigroup (Ψ, Λ) over a semigroup and a null soft set (Ω, Σ) over a semigroup is a bijective soft set over a semigroup.

Theorem 3.21: Suppose that (Ψ, Λ) is a bijective soft semigroup over a semigroup S and (Ω, Σ) is a null soft set over S . $(\Gamma, \Delta) = (\Psi, \Lambda) \widetilde{\cup} (\Omega, \Sigma)$ is a bijective soft set over S .

Proof: From the Definition 2.2 and 2.4, we have

$$(\Psi, \Lambda) \widetilde{\cup} (\Omega, \Sigma) = \begin{cases} \Psi(p) & \text{if } p \in \Lambda - \Sigma \text{ and } \Lambda \cap \Sigma \\ \emptyset & \text{if } p \in \Sigma - \Lambda. \end{cases}$$

That is $(\Gamma, \Delta) = (\Psi, \Lambda \cup \Sigma)$ is a null soft set. Hence $(\Gamma, \Delta) = (\Psi, \Lambda \cup \Sigma)$ is a bijective soft set over S . Therefore $(\Gamma, \Delta) = (\Psi, \Lambda) \widetilde{\cup} (\Omega, \Sigma)$ is a bijective soft set over S . $\square$

4. Fuzzy parameterized bijective soft sets

In this section, we define a fuzzy parameterized bijective soft sets and give some operations with supporting examples.

Definiton 4.22: Let $\mathcal{U}$ be an initial universal set, Ξ be a set of parameters and let $\emptyset \neq \Lambda \subseteq \Xi$ be a fuzzy set over Ξ with the membership function $\mu_\Lambda : \Lambda \rightarrow [0, 1]$ and let (Ψ, Λ) be a fuzzy parameterized soft set over $\mathcal{U}$. An ordered pair (Ψ, Λ) is called a *fuzzy parameterized bijective soft sets over $\mathcal{U}$* if (Ψ, Λ) is a bijective soft set over $\mathcal{U}$.

Note that a fuzzy parameterized bijective soft set (Ψ, Λ) can be represented by the set of ordered pairs

$$(\Psi, \Lambda) = \{(\mu_\Lambda(p) / p, \Psi(p)) : p \in \Lambda, \Psi(p) \in \mathcal{P}(\mathcal{U}), \mu_\Lambda(p) \in [0, 1]\}.$$

Example 4.23: Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a set of six stores, $\Xi = \Lambda \cup \Sigma \cup \diamond \cup \heartsuit$ be a set of parameters, where Λ describes the empowerment of sale personnel, Σ describes the perceived quality of merchandise, $\diamond$ describes the high traffic location and $\heartsuit$ describes store profit or loss. Suppose that the sets of these parameters are $\Lambda = \{1/a_1\{\text{high}\}, 0.5/a_2\{\text{medium}\}, 0.1/a_3\{\text{low}\}\}$, $\Sigma = \{0.9/b_1\{\text{good}\}, 0.5/b_2\{\text{average}\}\}$, $\diamond = \{0.8/c_1\{\text{no}\}, 0.2/c_2\{\text{yes}\}\}$ and $\heartsuit = \{1/d_1\{\text{profit}\}, 0.1/d_2\{\text{loss}\}\}$. According to the data collected, the fuzzy parameterized bijective soft sets $(\Psi, \Lambda), (\Omega, \Sigma), (\nabla, \diamond)$ and $(\perp, \heartsuit)$ are given by

$$\begin{aligned}(\Psi, \Lambda) &= \{(1/a_1, \{u_1, u_6\}), (0.5/a_2, \{u_2, u_3, u_5\}), (0.1/a_3, \{u_4\})\} \\(\Omega, \Sigma) &= \{(0.9/b_1, \{u_1, u_2, u_3\}), (0.5/b_2, \{u_4, u_5, u_6\})\} \\(\nabla, \diamond) &= \{(0.8/c_1, \{u_1, u_2, u_3, u_4\}), (0.2/c_2, \{u_5, u_6\})\} \text{ and} \\(\perp, \heartsuit) &= \{(1/d_1, \{u_1, u_3, u_6\}), (0.1/d_2, \{u_2, u_4, u_5\})\}.\end{aligned}$$

The following definitions are some relations of the fuzzy parameterized bijective soft set over a universe.

Definiton 4.24: Let (Ψ, Λ) and (Ω, Σ) be two fuzzy parameterized bijective soft sets over a common universe $\mathcal{U}$. (Ψ, Λ) is a *fuzzy parameterized bijective soft subset* of (Ω, Σ) , denoted by $(\Psi, \Lambda) \subseteq (\Omega, \Sigma)$, if

- (i) $\Lambda \subseteq \Sigma$ and
- (ii) $\mu_\Lambda(p) \leq \mu_\Sigma(p)$ for all $p \in \Lambda$ and
- (iii) $\Psi(p)$ and $\Omega(p)$ are identical approximations for all $p \in \Lambda$.

Definiton 4.25: Let (Ψ, Λ) and (Ω, Σ) be two fuzzy parameterized bijective soft sets over a common universe $\mathcal{U}$. (Ψ, Λ) and (Ω, Σ) are *equal*, denoted by $(\Psi, \Lambda) = (\Omega, \Sigma)$, if $(\Psi, \Lambda) \subseteq (\Omega, \Sigma)$ and $(\Omega, \Sigma) \subseteq (\Psi, \Lambda)$.

Example 4.26: Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ be a common universe, $\Xi = \{e_1, e_2, e_3, e_4\}$ be a set of parameters and let $\Lambda = \{0.2/e_1, 0.7/e_2, 0.5/e_3\}$ and $\Sigma = \{0.4/e_1, 0.8/e_2, 0.6/e_3\}$. According to the data collected, the fuzzy parameterized bijective soft sets (Ψ, Λ) and (Ω, Σ) are given by

$$\begin{aligned}(\Psi, \Lambda) &= \{(0.2/e_1, \{u_1, u_2\}), (0.7/e_2, \{u_3\}), (0.5/e_3, \{u_4, u_5\})\} \text{ and} \\(\Omega, \Sigma) &= \{(0.4/e_1, \{u_1, u_2\}), (0.8/e_2, \{u_3\}), (0.6/e_3, \{u_4, u_5\})\}.\end{aligned}$$

$$\text{Hence } (\Psi, \Lambda) \subseteq (\Omega, \Sigma).$$

Now, we present an application of fuzzy parameterized bijective soft sets in a decision making problem.

Algorithm

Step 1. Input the fuzzy parameterized bijective soft sets (Ψ_i, Λ_i) .

Step 2. Compute the score value S_{Λ_i} . S_{Λ_i} is defined by

$$S_{\Lambda_i} = \frac{1}{|\Lambda_i|} \sum_{x \in \Lambda_i} \mu_{\Lambda_i}(p) \chi_{\Psi_i(p)}(u),$$

where $|\Lambda_i|$ is the number of elements of Λ_i , $\Psi_i(p)$ is the subset determined by the parameter p and

$$\chi_{\Psi_i(p)}(u) = \begin{cases} 1 & \text{if } u \in \Psi_i(p), \\ 0 & \text{if } u \notin \Psi_i(p). \end{cases}$$

Step 3. Computation of $C_i = \sum_i S_{\Lambda_i}$.

Step 4. If $C_k = \max_i S_{\Lambda_i}$ then we choose u_k corresponding to C_k .

The following example is the using the new algorithm in a decision making problem. The concept is similar to the Example 4.23.

Example 4.27: Assume that a company is evaluating the decision of a purchase store. We consider the Example 4.23. **Step 1.** Let $(\Psi, \Lambda), (\Omega, \Sigma), (\nabla, \diamond)$ and $(\perp, \heartsuit)$ be four fuzzy parameterized bijective soft sets over $\mathcal{U}$. Suppose the following:

$$(\Psi, \Lambda) = \{(1/a_1, \{u_1, u_6\}), (0.5/a_2, \{u_2, u_3, u_5\}), (0.1/a_3, \{u_4\})\}$$

$$(\Omega, \Sigma) = \{(0.9/b_1, \{u_1, u_2, u_3\}), (0.5/b_2, \{u_4, u_5, u_6\})\}$$

$$(\nabla, \diamond) = \{(0.8/c_1, \{u_1, u_2, u_3, u_4\}), (0.2/c_2, \{u_5, u_6\})\} \text{ and}$$

$$(\perp, \heartsuit) = \{(1/d_1, \{u_1, u_3, u_6\}), (0.1/d_2, \{u_2, u_4, u_5\})\}.$$

Step 2. Computation of the score value S_{Λ_i} of fuzzy parameterized bijective soft sets $(\Psi, \Lambda), (\Omega, \Sigma), (\nabla, \diamond)$ and $(\perp, \heartsuit)$ over $\mathcal{U}$. Which are shown in Table 3, Table 4, Table 5 and Table 6, respectively.

Table 3**The score value of fuzzy parameterized bijective soft set (Ψ, Λ)**

$\mathcal{U}$	u_1	u_2	u_3	u_4	u_5	u_6
$1/a_1$	1	0	0	0	0	1
$0.5/a_2$	0	1	1	0	1	0
$0.1/a_3$	0	0	0	1	0	0
S_Λ	0.33	0.16	0.16	0.03	0.16	0.33

Table 4**The score value of fuzzy parameterized bijective soft set (Ω, Σ)**

$\mathcal{U}$	u_1	u_2	u_3	u_4	u_5	u_6
$0.9/b_1$	1	1	1	0	0	0
$0.5/b_2$	0	0	0	1	1	1
S_Σ	0.45	0.45	0.45	0.25	0.25	0.25

Table 5**The score value of fuzzy parameterized bijective soft set $(\nabla, \spadesuit)$**

$\mathcal{U}$	u_1	u_2	u_3	u_4	u_5	u_6
$0.8/c_1$	1	1	1	1	0	0
$0.2/c_2$	0	0	0	0	1	1
$S_\spadesuit$	0.4	0.4	0.4	0.4	0.1	0.1

Table 6**The score value of fuzzy parameterized bijective soft set $(\perp, \heartsuit)$**

$\mathcal{U}$	u_1	u_2	u_3	u_4	u_5	u_6
$1/d_1$	1	0	1	0	0	1
$0.2/d_2$	0	1	0	1	1	0
$S_\heartsuit$	0.5	0.05	0.5	0.05	0.05	0.5

Step 3. Computation of $C_i = \sum_i S_{\Lambda_i}$. The core value are shown in Table 7.

Table 7
The score value C_i

$\mathcal{U}$	u_1	u_2	u_3	u_4	u_5	u_6
$S_\wedge$	0.33	0.16	0.16	0.03	0.16	0.33
S_Σ	0.45	0.45	0.45	0.25	0.25	0.25
$S_\bullet$	0.4	0.4	0.4	0.4	0.1	0.1
$S_\perp$	0.5	0.05	0.5	0.05	0.05	0.5
C_i	1.68	1.06	1.51	0.73	0.56	1.18

Step 4. Hence the company is selected u_1 .

5. Conclusions

In this paper, bijective soft semigroups over semigroups are introduced. We obtained some properties of bijective soft semigroups over semigroups. We defined fuzzy parameterized bijective soft sets over the universe. Moreover, the application of fuzzy parameterized bijective soft sets over the universe in decision-making problems is very interesting and useful for solving day-to-day problems. It helps making decisions in critical situation.

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References

- [1] Zadeh, L.A., 1965. Fuzzy sets, *Information and Control*, 8, pp. 338-353.
- [2] Pawlak, Z., 1982. Rough sets. *International Journal of Information Technology and Computer Sciences*, 11, pp. 341-356.
- [3] Atanassov, K., 1986. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, pp. 87-96.
- [4] Molodtsov, D., 1999. Soft set theory-First results. *Computer and Mathematics with Applications*, 37, pp. 19-31.

- [5] Maji, P.K., Biswas, R. and Roy, A.R., 2003. Soft set theory. *Computer and Mathematics with Applications*, 45, pp. 555-562.
- [6] Maji, P.K., Biswas, R. and Roy, A.R., 2002. An application of soft sets in a decision making problem. *Computer and Mathematics with Applications*, 44, pp. 1077-1083.
- [7] Çağman, N. and Enginoglu, S., 2010. Soft set theory and uni-int decision making, *European Journal of Operational Research*, 207(2), pp. 848-855.
- [8] Çağman N. and Enginoğlu, S., 2010. Soft matrix theory and its decision making, *Computer and Mathematics with Applications*, 59, pp. 3308-3314.
- [9] Gong, K., Xiao Z. and Zhang X., 2010. The bijective soft set with its operations, *Computer and Mathematics with Applications*, 60, pp. 2270-2278.
- [10] N. Çağman, F. Çitak and S. Enginoglu, FP-soft set theory and its applications, *Annals of Fuzzy Mathematics and Informatics* **2(2)** (2011), 219--226.
- [11] Maji, P.K., Biswas, R. and Roy, A.R., 2001. Fuzzy soft set. *The Journal of Fuzzy Mathematics*, 9(3), pp. 589-602.
- [12] Yang, C.F., 2011. Fuzzy soft semigroups and fuzzy soft ideals. *Computer and Mathematics with Applications*, 44, pp. 255-261.
- [13] Suebsan, P., 2021. On relative fuzzy soft sets over some semigroups in decision-making problems. *Journal of Discrete Mathematical Sciences and Cryptography*, 24(1), pp. 209-222. Doi:10.1080/09720529.2020.1845466
- [14] Howie, M.J., *An introduction to semigroup theory*, Academic Press, London, 1976.

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