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Several types of derivations on prime and semiprime gamma rings

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Abstract

In this paper the notion of asymmetric skew 4-derivation of prime and semiprime Γ-ring is presented and studied. Therefore we proved that; presume Ň be a 3,2-torsion free non commutative prime Γ-rings accomplishing a certain assumption and L be admissible Lie ideal of Ň; Let : Ň4→Ň is a symmetric 4-derivation. If f is a trace of such that $[f(s),s]_\sigma = 0$; for each one s ∈ L, σ ∈ Γ therefore Ꝑ= 0,also we prove that; if Ň be a 3,2–torsion free non commutative semiprime Γ-rings accomplishing a certain assumption and L be admissible Lie ideal of Ň. Presume α is an automorphism of Ň and : $Ň^4$→Ň is a symmetric skew 4-derivation associated with α. Presume that the trace function *f* is commuting on L such that $[f(s), \alpha(s)]_\sigma \in Z$ for each s ∈ L, σ ∈ Γ, where f(L) ⊆ L, therefore $[f(s), \alpha(s)]_\sigma = 0$ for eachs ∈ L, σ ∈

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Γ,the quantity demand and investment in research and development, while the other model focuses on a more realistic relationship between the quantity demand and the price.

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1. Introduction

The first gamma rings were proposed. by Nabusawa [14] and therefore Barnes [15] generalized the definition of Γ-rings. Let M and Γ be two additive abelian groups. It there is a mapping (a $\boldsymbol{\alpha}$ b)→ a $\boldsymbol{\alpha}$ b of M × Γ × M →M, accomplishing the conditions listed below, for all a, b, c ∈ M and $\boldsymbol{\alpha}$, β ∈ Γ.

1. (a + b) $\boldsymbol{\alpha}$ c = a $\boldsymbol{\alpha}$ c + b $\boldsymbol{\alpha}$ c,
   a ($\boldsymbol{\alpha}$ + β) b = a $\boldsymbol{\alpha}$ b +a β b,
   a (b + c) = a $\boldsymbol{\alpha}$ b +a $\boldsymbol{\alpha}$ c,
2. (a + b) β c = a $\boldsymbol{\alpha}$ (b β c), therefore M is said to be Γ-ring.

Throughout this paper $\check{N}$ will denote a Γ-ring with centre $Z(\check{N})$. A ring $\check{N}$ is ɳ-torsion free where ɳ is a positive integer, ɳs = 0, s ∈ $\check{N}$ implies s = 0, a Γ-rings $\check{N}$ is said to be prime if $s\Gamma\check{N}\Gamma t = (0)$ implies that either $s = 0$ or $t = 0$, and semi-prime if $s\Gamma\check{N}\Gamma s = (0)$ implies that $s = 0$. the set Ẓ($\check{N}$) = {$s \in \check{N}$: $s\beta t = t\beta s$, for each one $s, t \in \check{N}$ and $\beta \in \Gamma$ } is named the center of the Γ-ring $\check{N}$, $\check{N}$ is named a commutative if $s\beta t$ = tβs, for each one $s, t \in \check{N}$ and $\beta \in \Gamma$, a map f: $\check{N} \to \check{N}$ is said to be commuting if $[f(s), s]_\sigma = 0$ for each one $s \in \check{N}$ and σ ∈ Γ, and f centralizing if $[f(s), s]_\sigma \in Z(\check{N})$ for each one $s \in \check{N}$ and σ ∈ Γ, An additive subgroup L of $\check{N}$ is named Lie ideal of $\check{N}$ if when aver $u \in L$, $r \in \check{N}$ therefore $[u, r]_\sigma \in L$, and L is named square close Lie ideal if $u^2 \in L$ for each one $u \in L$, an admissible Lie ideal L is square close lie ideal of $\check{N}$ such that $L \subseteq Z(\check{N})$, the commutator $s\sigma t$ - $t\sigma s$ will be symbolized by $[s, t]_\sigma$. We know that $[s\beta t, z]_\sigma = s\beta[t, z]_\sigma + [s, z]_\sigma \beta t + s[\beta, \sigma]zt$ and $[s, t\beta z]_\sigma = t\beta[s, z]_\sigma + [s, t]_\sigma \beta z + t[\beta, \sigma]_s z$ for each one $s, t, z \in \check{N}$, β, σ ∈ Γ. We shall take an assumption (*) $s\sigma t\beta z = s\beta t\sigma z$ for each one $s, t, z \in \check{N}$, β, σ ∈ Γ. Presuming (*) the above identities reduce to $[s\beta t, z]_\sigma = s\beta[t, z]_\sigma + [s, z]_\sigma \beta t$ and $[s, t\beta z]_\sigma = t\beta[s, z]_\sigma + [s, t]_\sigma \beta z$, for each one $s, t, z \in \check{N}$ and for each one σ, β ∈ Γ, an additive mapping $\mathcal{P}:\check{N}\to\check{N}$ is named aderivation if $\mathcal{P}(s\beta t) = \mathcal{P}(s)\beta t + s\beta\mathcal{P}(t)$, for each one $s, t \in \check{N}$, β $\in \Gamma$ and named a α-derivation of $\check{N}$ associated with an automorphism α if $\mathcal{P}(s\beta t) = \mathcal{P}(s)\beta t + \alpha(s)\beta\mathcal{P}(t)$, for each one $s, t \in \check{N}$, β $\in \Gamma$ [17].

A mapping $\mathcal{P}:\check{N}\times\check{N}\to\check{N}$ is a symmetric if $\mathcal{P}(s, t) = \mathcal{P}(s, t)$, for each one $s, t \in \check{N}$, . A mapping $f:\check{N}\to\check{N}$ defined by $f(s) = \mathcal{P}(s, s)$, is named the trace of $\mathcal{P}$ where $\mathcal{P}:\check{N}\times\check{N}\to\check{N}$ is asymmetric mapping. The trace f of $\mathcal{P}$ accomplishes the relation $f(s+t) = f(s)+ f(t)+2\ \mathcal{P}(s, t)$, for each one $s, t \in \check{N}$, bi-derivation we mean a bi-additive map $Ƥ:\check{N}\times\check{N}\rightarrow\check{N}$ (i.e., $Ƥ$ is additive For eachargument), which satisfies the relations $Ƥ(s\beta t, v) = Ƥ(s, v)\beta t + s\beta Ƥ(t, v)$ and $Ƥ(s, t\beta v) = Ƥ(s, t)\beta v + t\beta Ƥ(s, v)$ for each one $s, t \in \check{N}$, $\beta \in \Gamma$, A symmetric bi derivation was first described by G. Maksa [6]. For an example, see [7]. In [6], it was shown that symmetric bi derivations are linked to the general answer to several functional equations. There are several results on symmetric bi derivation in prime and semiprime rings in [1, 2, 3, 4, 8, 9, 12]. Ozturk et al. [18] looked into symmetric bi-derivations on prime Γ-rings [17, 19, 20].

A mapping $Ƥ$: $\check{N}^3 \rightarrow\check{N}$ is said to be 3-additive if its additive For each argument and it is named symmetric if $Ƥ(s_1, s_2, s_3) = Ƥ(s_{\pi(1)}, s_{\pi(2)}, s_{\pi(3)})$ for each one $s_1, ..., s_3 \in\check{N}$ and for each changing about $\{\pi_{(1)}, \pi_{(2)}, \pi_{(3)}\}$. The map $f:\check{N}\rightarrow\check{N}$ defined by $f(s) = Ƥ(s, s, s,)$, $s \in \check{N}$ is named trace of Ƥ where $Ƥ:\check{N}^3 \rightarrow\check{N}$ is asymmetric mapping. We have $f(s+t) = [f(s) + 3Ƥ(s, s, t) + 3Ƥ(s, t, t) + f(t)]$, a 3-additive map $Ƥ:\check{N}^3 \rightarrow\check{N}$ is named skew 3−derivation associated with an automorphism α if $Ƥ(s\beta u, t, v) = Ƥ(s, t, v,)\beta u+\alpha(s)\beta Ƥ(u, t, v,)$, $Ƥ(s, t\beta u, v) = Ƥ(s, t, v)\beta u +\alpha(t)\beta Ƥ(s, u, v)$, $Ƥ(s, t, v\beta u) = Ƥ(s, t, v)\beta u + \alpha(v)\beta Ƥ(s, t, u)$, it is derivation in each argument. Jung and Park [5] recently looked at permuting 3-derivations on prime and semiprime rings and found the following: Let R be a semiprime ring that doesn't commutate and doesn't have any twisting. Let I be a nonzero two-sided ideal of R. It's possible that there is a permuting 3-derivation D: $R^3\rightarrow R$ such that f centers on I. So f is traveling on I. Posner proved an interesting result in 1957 [10]. It says that if a prime ring has a nonzero centralizing derivation, therefore the ring must be commutative. These writers were able to get several useful results with prime Γ-rings. Ozturk [18] and got several information about the mapping permuting tri-derivations on prime and semiprime Γ-rings. Dey and Paul [16] looked into permuting tri-derivations of semiprime gamma rings.

A mapping $Ƥ: \check{N}^4\rightarrow\check{N}$ is named 4-additive if it additive In eachcase, and it's defined as symmetric if $Ƥ(s_1, s_2, s_3, s_4) = Ƥ(s_{\pi(1)},, s_{\pi(2)}, s_{\pi(3)}, s_{\pi(4)})$ for each one $s_1, ..., s_4 \in \check{N}$ and eachpermutation$\{\pi_{(1)}, \pi_{(2)}, \pi_{(3)}, \pi_{(4)}\}$. The map $f:M\rightarrow M$ defined by $f(s) = Ƥ(s, s, s, s) \in \check{N}$ is named trace of $Ƥ$ where $Ƥ:\check{N}^4 \rightarrow\check{N}$ is asymmetric mapping. We have $f(s + t) = [f(s) + 4Ƥ(s, s, s, t) + 6Ƥ(s, s, t, t) + 4Ƥ(s, t, t, t) + (t)]$, 4-additive map $Ƥ:\check{N}^4 \rightarrow\check{N}$ is named 4−derivation if satisfy

$Ƥ(s\beta u, t, v, w) = Ƥ(s, t, v, w)\beta u +s\beta Ƥ(u, t, v, w)$, $Ƥ(s, t\beta u, v, w) = Ƥ(s, t, v, w)\beta u + t\beta Ƥ(s, u, v, w)$, $Ƥ(s, t, v\beta u, w) = Ƥ(s, t, v, w)\beta u + v\beta Ƥ(s, t, u, w)$, $Ƥ(s, t, v, w\beta u) = Ƥ(s, t, v, w)\beta u + w\beta Ƥ(s, t, v, u)$, for each one $s, t, v, w, u \in\check{N}, \beta \in\Gamma$

A 4-additive map $Ƥ:\check{N}^4 \rightarrow\check{N}$ is known as skew 4−derivation related to an automorphism α if for each one $s, t, v, w, u \in\check{N}, \beta \in\Gamma$. if satisfy

$Ƥ(s\beta u, t, v, w) = Ƥ(s, t, v, w)\beta u + \alpha(s)\beta Ƥ(u, t, v, w)$, $Ƥ(s, t\beta u, v, w) = Ƥ(s, t, v, w)\beta u + \alpha(t)\beta Ƥ(s, u, v, w)$, $Ƥ(s, t, v\beta u, w) = Ƥ(s, t, v, w)\beta u + \alpha(v)\beta Ƥ(s, t, u, w)$, $Ƥ(s, t, v, w\beta u) = Ƥ(s, t, v, w)\beta u + \alpha(w)\beta Ƥ(s, t, v, u)$,

The four relations listed above are the same if $Ƥ$ is symmetric. for instance, research on symmetric skew 4-derivations on semi-prime rings was done in [11] and [13].

We establish the following lemmas, which are required for our conclusions.

Lemma 1.1: *Presume Ň be a Γ-ring accomplishing the conditions (*) and* $a, b \in Ň$. *If* $a\beta[s, b]_\sigma = 0$, *for each one* $s \in Ň$, $\sigma \in \Gamma$ *therefore either* $a = 0$ *or* $b \in Z$.

Proof: $0 = a\beta[s\lambda t, b]_\sigma = a\beta s\lambda[t, b]_\sigma + a\beta[s, b]_\sigma \lambda t = a\beta s\lambda [t, b]_\sigma$, for each one s, $t \in Ň$, $\sigma, \beta, \lambda \in \Gamma$.

Thus $a\betaŇ \lambda[t, b]_\sigma = 0$, $t \in Ň$, $\sigma, \beta, \lambda \in \Gamma$, because Ň is prime, therefore $a = 0$ or $b \in Z$.

Lemma 1.2: *Presume Ň 3, 2-torsion free prime Γ-ring and L be a nonzero admissible Lie ideal of Ň, therefore Lcontains a nonzero ideal of Ň .*

Lemma 1.3: *Presume Ň be a 3, 2-torsion free prime Γ -ring and L admissible Lie ideal of Ň, and* $Ƥ : Ň^4 \rightarrow Ň$ *is if Ƥ = 0 on L, therefore Ƥ = 0 on Ň.*

Proof: Since $Ƥ = 0$ on L we get

$$Ƥ(s, t, v, w) = 0 \text{ for each one } s, t, v, w \in L. \quad (1)$$

Since L an admissible lie ideal of Ň and by Lemma(1.2) .That is L contains anon zero ideal of Ň Such that

$$Ƥ(s, t, v, w) = 0$$

Replace $2r\beta s$ for s for each one $s \in I$, $r \in Ň$, $\beta \in \Gamma$ to get

$$0 = Ƥ(2r\beta s, t, v, w) = 2\, Ƥ(r, t, v, w)\beta s+2\, r\beta Ƥ(s, t, v, w) = 2\, Ƥ(r, t, v, w)\, \beta s. \quad (2)$$

Since Ň be 3, 2-torsion free

$$Ƥ(r, t, v, w)\, \beta s = 0, \text{ for each one } s \in I, r \in Ň, \beta \in \Gamma .$$

This means that $Ƥ(r, t, v, w)\, \beta I = 0$ for each one $t, v, w \in I$, $r \in Ň$, $\beta \in \Gamma$

We get because Ň is prime,

$Ƥ(r, t, v, w) = 0$ for each one $t, v, w \in L$, $r \in Ň$.

after this process over and over until we get

$Ƥ(r, s, t, p) = 0$ for each one $r, s, t, p \in Ň$. Therefore $Ƥ = 0$

Lemma 1.4: *Presume Ň be a 3, 2-torsion free non commutative prime Γ-ring accomplishing the conditions (*) and* $Ƥ:Ň^4 \rightarrow Ň$ *is, where L be an admissible lie ideal of Ň if f is a trace of Ƥ such that* $[f(s), s]_\sigma = 0$ *for each one* $s \in L$, *therefore* $f(s)\beta[t, s]_\sigma = 0$ *for each one* $s, t \in L$, $\beta, \sigma \in \Gamma$.

Proof: Since we have

$$[f(s), s]_\sigma = 0, \text{ for each one } s \in L, \sigma \in \Gamma \quad (3)$$

Linearization of (1) we get

$[f(s+t), s+t]_\sigma = 0$, for each one $s, t \in L,$, $\sigma \in \Gamma$

$[f(s), s]_\sigma +4[Ƥ(s, s, s, t), s]_\sigma +6[Ƥ(s, s, t, t), s]_\sigma +4[Ƥ(s, t, t, t), s]_\sigma +[f(t), t]_\sigma +[f(s), t]_\sigma +4[Ƥ(s, s, s, t), t]_\sigma +6[Ƥ(s, s, t, t), s]_\sigma +4[Ƥ(s, t, t, t), t]_\sigma +[f(t), \alpha(t)]_\sigma = 0$, for each one $s, t \in L$, $\sigma \in \Gamma$. (4)

In view of (3), (4) tields that

$4[\mathcal{P}(s, s, s, t), s]\,]_\sigma + 6[\mathcal{P}(s, s, t, t), s]_\sigma + 4[\mathcal{P}(s, t, t, t), s]_\sigma + [f(t), s]_\sigma + [f(s), t]_\sigma + 4[\mathcal{P}(s, s, s, t), t]_\sigma + 6[\mathcal{P}(s, s, t, t), t]_\sigma + 4[\mathcal{P}(s, t, t, t), s]_\sigma = 0$, for each one $s, t \in L$, $\sigma \in$(5)

Replacing t by $-t$ in (5) we find

$-4[\mathcal{P}(s, s, s, t), s]_\sigma + 6[\mathcal{P}(s, s, t, t), s]_\sigma - 4[\mathcal{P}(s, t, t, t), s]_\sigma + [f(t), s]_\sigma - [f(s), t]_\sigma + 4[\mathcal{P}(s, s, s, t), t]_\sigma - 6[\mathcal{P}(s, s, t, t), t]_\sigma + 4[\mathcal{P}(s, t, t, t), t]_\sigma = 0$, for each one $s, t \in L$, $\sigma \in \Gamma$(6)

By comparing (5) and (6) and using the fact that α is free of two-torsion, we get

$4[\mathcal{P}(s, s, s, t), s]_\sigma + 4[\mathcal{P}(s, t, t, t), s]_\sigma + [f(s), t]_\sigma + 6[\mathcal{P}(s, s, t, t), t]_\sigma = 0$, for each one $s, t \in L$, $\sigma \in \Gamma$. (7) Substitute t+v for t in (7) and use (7) to get

$12[\mathcal{P}(s, t, v, v), s]_\sigma + 12[\mathcal{P}(s, v, t, t), s]_\sigma + [\mathcal{P}(s, s, t, v), t]_\sigma + 6[\mathcal{P}(s, s, v, v), t]_\sigma + 6[\mathcal{P}(s, s, t, t), v]_\sigma + 12[\mathcal{P}(s, s, t, v), v]_\sigma = 0$, for each one $s, t, v \in L$, $\sigma \in \Gamma$. (8)

Putting $-v$ in place of v in (8) and comparing it to (8) gives us $12[\mathcal{P}(s, v, t, t), s]_\sigma + 12[\mathcal{P}(s, s, t, v), t]_\sigma + 6[\mathcal{P}(s, s, t, t), v]_\sigma = 0$, each one $s, t, v \in L$, $\sigma\, \Gamma \in$(9)

Substitute t+u for t in (9) and use (9) we get

$24[\mathcal{P}(s, v, t, u), s]_\sigma + 12[\mathcal{P}(s, s, t, v), u]_\sigma + 12[\mathcal{P}(s, s, u, v), t]_\sigma + 12[\mathcal{P}(s, s, t, u), v]_\sigma = 0$, for each one $u, s, t, v \in L,$, $\sigma \in \Gamma$ (10)

Since $\check{N}$ doesn't have any 2 and 3 -torsion, we can replace t and u with s in (10).

$[\mathcal{P}(s, s, s, v), s]_\sigma + [f(s), v]_\sigma = 0$ for each one $s, v \in L$, $\sigma \in \Gamma$. (11)

Again replace v by $2v\beta t$ in (11) and using (11) we get

$[\mathcal{P}(s, s, s, 2v\beta t), s]_\sigma + [f(s), 2v\beta t]_\sigma = 0$, for each one $s, v \in L$, $\sigma, \beta \in \Gamma$. $2\,[\mathcal{P}(s, s, s, v), s]_\sigma \beta t + 2\mathcal{P}(s, s, s, v)\beta[t, s]_\sigma + [v, s]_\sigma \beta\mathcal{P}(s, s, s, t) + [f(s), v]_\sigma\beta t = 0$ for each one $s, t, v \in L$, $\sigma, \beta \in \Gamma$

When we use 2-torsion free, we get

$[\mathcal{P}(s, s, s, v), s]_\sigma\beta t + \mathcal{P}(s, s, s, v)\beta[t, s]_\sigma\, [v, s]_\sigma\beta\mathcal{P}(s, s, s, t) + [f(s), v]_\sigma\, \beta t = 0$, for each one $s, t, v \in L$ $\sigma, \beta \in \Gamma$. (12)

Substitute s for v in (12) and in view of (12) we find (13) $2f(s)\beta[t, s]_\sigma = 0$ for each one $s, t \in L$, $\sigma, \beta \in \Gamma$.

By using 2-torsion freeness of $\check{N}$ we get

$f(s)\beta[t, s]_\sigma = 0$ for each one $s, t \in L$, $\sigma, \beta \in \Gamma$. (13)

Theorem 2.1*:* *Presume $\check{N}$ be a 3, 2-torsion free non commutative prime Γ-ring accomplishing the conditions (*) and L be admissible Lie ideal of $\check{N}$. Let $\mathcal{P} : \check{N}^4 \rightarrow \check{N}$ is a symmetric 4-derivation. If f is a trace of $\mathcal{P}$ such that $[f(s), s]_\sigma = 0$ for each one $s \in L$, $\sigma \in \Gamma$ therefore $\mathcal{P} = 0$.*

Proof: By Lemma (1.4)

$f(s)\beta[t, s]_\sigma = 0$ for each one $s, t \in L$, $\sigma, \beta \in \Gamma$.

Substitute $2t\lambda v$ for t and use 2- tortion free to get

$f(s)\beta t\lambda[v, s]_\sigma = 0$ for each one $s, t, v \in L$, $\sigma, \beta, \lambda \in \Gamma$. (14)

Since $\check{N}$ is prime, this means that either

$f(s) = 0$ or $[v, s]_\sigma = 0$ for each one $s \in L \setminus Z(\check{N})$, $v \in L$, $\sigma \in \Gamma$.

After that, we'll show that $f(s) = 0$ for each one $s \in L$. Let $v \in L \cap Z(\check{N})$ and $s \in L \setminus Z(\check{N})$.

Therefore $s+v,\ s-v \in L \setminus Z(\check{N})$ and we have

$$f(s+v) = f(v)+4\,\mathcal{P}(s, s, s, v)+4\,\mathcal{P}(s, v, v, v)+6\mathcal{P}(s, s, v, v) = 0 \quad (15)$$

$$f(s-v) = f(v)-4\,\mathcal{P}(s, s, s, v)-4\,\mathcal{P}(s, v, v, v)+6\,\mathcal{P}(s, s, v, v) = 0 \quad (16)$$

By using the torsion condition and comparing the last two relations, we get

$f(v)+6\mathcal{P}(s, s, v, v) = 0$.

Using the right linearization and (16), we get $f(s) = 0$ for all $s \in L$ So, we have $\mathcal{P}(s, t, v, w) = 0$ for each one $s, t, v, w \in L$.

Theorem 2.2: *Presume $\check{N}$ be a 3, 2-torsion free non commutative semi prime Γ-ring accomplishing the conditions (*) and L be admissible lie ideal of $\check{N}$. Presume α is an automorphism of $\check{N}$ and $\mathcal{P}$: $\check{N}^4 \to \check{N}$ is a symmetric skew-4derivation associated with α. Presume that the trace function f is commuting on L such that $[f(s), (s)]_\sigma \in Z$ for each one $s \in L$, $\sigma \in \Gamma$, therefore $[f(s), \alpha(s)]_\sigma = 0$ for each one $s \in L$, $\sigma \in \Gamma$, where $f(L) \subseteq L$.*

Proof: $[f(s), \alpha(s)]_\sigma \in Z$ for each one $s \in L$, $\sigma \in \Gamma$. (17)

Linearization of (17) yields that

$[f(s), \alpha(s)]_\sigma+4[\mathcal{P}(s, s, s, t), \alpha(s)]_\sigma +6[\mathcal{P}(s, s, t, t), \alpha(s)]_\sigma +4[\mathcal{P}(s, t, t, t), \alpha(s)]_\sigma + [f(t), \alpha(t)]_\sigma +[f(s), \alpha(t)]_\sigma +4[\mathcal{P}(s, s, s, t), \alpha(t)]_\sigma +6[\mathcal{P}(s, s, t, t), \alpha(s)]_\sigma +4[\mathcal{P}(s, t, t, t), \alpha(t)]_\sigma +[f(t), \alpha(t)]_\sigma \in Z$ for each one $s, t \in L$, $\sigma \in \Gamma$. (18)

In view of (17), (18) yields that

$4[\mathcal{P}(s, s, s, t), \alpha(s)]_\sigma +6[\mathcal{P}(s, s, t, t), \alpha(s)]_\sigma +4[\mathcal{P}(s, t, t, t), \alpha(s)]_\sigma + [f(t), \alpha(t)]_\sigma +[f(s), \alpha(t)]_\sigma +4[\mathcal{P}(s, s, s, t), \alpha(t)]_\sigma +6[\mathcal{P}(s, s, t, t), \alpha(s)]_\sigma +4[\mathcal{P}(s, t, t, t), \alpha(t)]_\sigma \in Z$ for each one $s, t \in L$, $\sigma \in \Gamma$. (19)

Replacing t by $-t$ in (19) we find

$-4[\mathcal{P}(s, s, s, t), \alpha(s)]_\sigma + 6[\mathcal{P}(s, s, t, t), \alpha(s)]_\sigma - 4[\mathcal{P}(s, t, t, t), \alpha(s]_\sigma + [f(t), \alpha(s)]_\sigma - [f(s), \alpha(t)]_\sigma + 4[\mathcal{P}(s, s, s, t), \alpha(t)]_\sigma - 6[\mathcal{P}(s, s, t, t), \alpha(t)]_\sigma + 4[\mathcal{P}(s, t, t, t), \alpha(t)]_\sigma \in Z$, for each one $s \in L$, $\sigma \in \Gamma$. (20)

Applying the fact that χ is free of two-torsion and comparing (19) and (20), we obtain

$4[\mathcal{P}(s, s, s, t), \alpha(s)]_\sigma +4[\mathcal{P}(s, t, t, t), \alpha(s)]_\sigma +[f(s), \alpha(t)]_\sigma +6[\mathcal{P}(s, s, t, t), \alpha(t)]_\sigma \in Z$, for each one $s \in L$ $\sigma \in \Gamma$. (21)

Substitute $t + v$ for t in (21) and use (21), we get

$4[\mathcal{P}(s, t, t, v), \alpha(s)]_\sigma + 4[\mathcal{P}(s, t, v, t), \alpha(s)]_\sigma + 4[\mathcal{P}(s, t, v, v), \alpha(s)]_\sigma + 4[\mathcal{P}(s, v, t, t), \alpha(s)]_\sigma + 4[\mathcal{P}(s, v, t, v), \alpha(s)]_\sigma + 4[\mathcal{P}(s, v, v, t), \alpha(s)]_\sigma + 6[\mathcal{P}(s, s, t, v), \alpha(t)]_\sigma + 6[\mathcal{P}(s, s, v, t), \alpha(t)]_\sigma + 6[\mathcal{P}(s, s, v, v), \alpha(t)]_\sigma +6[\mathcal{P}(s, s, t, t), \alpha(v)]_\sigma +6[\mathcal{P}(s, s, t, v), \alpha(v)]_\sigma +6[\mathcal{P}(s, s, v, t), \alpha(v)]_\sigma \in Z$

$12[\mathcal{P}(s, t, t, v), \alpha(s)]_\sigma + 12[\mathcal{P}(s, t, v, v), \alpha(s)]_\sigma + 12[\mathcal{P}(s, s, t, v), \alpha(t)]_\sigma + 6[\mathcal{P}(s, s, v, v), \alpha(t)]_\sigma + 6[\mathcal{P}(s, s, t, t), \alpha(v)]_\sigma + 12[\mathcal{P}(s, s, t, v), \alpha(v)]_\sigma \in Z$, for each one $s, t, v \in L$, $\sigma \in \Gamma$. (22)

After changing v in –v in (22) and comparing it to (22), we get

$-12[\mathcal{P}(s, t, t, v), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, t, v, v), \alpha(s)]_{\sigma} - 12[\mathcal{P}(s, s, t, v), \alpha(t)]_{\sigma} + 6[\mathcal{P}(s, s, v, v), \alpha(t)]_{\sigma} - 6[\mathcal{P}(s, s, t, t), \alpha(v)]_{\sigma} + 12[\mathcal{P}(s, s, t, v), \alpha(v)]_{\sigma} \in Z$

$2(12[\mathcal{P}(s, v, t, t), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, s, t, v), \alpha(t)]_{\sigma} + 6[\mathcal{P}(s, s, t, t), \alpha(v)]_{\sigma}) \in Z$

Using of two torsion free ring, we have $12[\mathcal{P}(s, v, t, t), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, s, t, v), \alpha(t)]_{\sigma} + 6[\mathcal{P}(s, s, t, t), \alpha(v)]_{\sigma} \in Z$, for each one $s, t, v \in L$, $\sigma \in \Gamma$. (23) Substitute $t + u$ for t in (24) and use (24) we get

$24[\mathcal{P}(s, v, t, u), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, s, t, v), \alpha(u)]_{\sigma} + 12[\mathcal{P}(s, s, u, v), \alpha(t)]_{\sigma} + 12[\mathcal{P}(s, s, t, u), \alpha(v)]_{\sigma} \in Z$, for each one s, t, $v, u \in L$, $\sigma \in \Gamma$. (24)

If we change t, u to s in (24) and remember that $\check{N}$ is free of 2 and 3 twists, we have

$24[\mathcal{P}(s, v, s, s), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, s, s, s), \alpha(v)]_{\sigma} \in Z$ $48[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma} + 12[\mathcal{P}(s, s, s, s), \alpha(v)]_{\sigma} \in Z$

$4[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma} + [f(s), \alpha(v)]_{\sigma} \in Z$, for each one $s, v \in L$, $\sigma \in \Gamma$. (25)

Again replaced v by $2s\lambda v$ in (25) and using (25) we obtain

$4[\mathcal{P}(s, s, s, 2s\lambda v), \alpha(s)]_{\sigma} + [f(s), \alpha(2s\lambda v)]_{\sigma} \in Z$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

$4[\mathcal{P}(s, s, s, 2s\lambda v), \alpha(s)]_{\sigma} + [f(s), 2(\alpha(s) \lambda \alpha(v))]_{\sigma} \in Z$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

$2(4[\mathcal{P}(s, s, s, s) \lambda v + \alpha(s) \lambda \mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma} + [f(s), \alpha(s)]_{\sigma} \lambda \alpha(v) + \alpha(s) \lambda [f(s), \alpha(v)]_{\sigma}) \in Z$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

As $\check{N}$ 2-torsion free, we obtain

$4[\mathcal{P}(s, s, s, s) \lambda v, \alpha(s)]_{\sigma} + 4 [\alpha(s) \lambda \mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma} + [f(s), \alpha(s)] \lambda \alpha(v) + \alpha(s) \lambda [f(s), \alpha(v)]_{\sigma} \in Z$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

$4f(s) \lambda [v, \alpha(s)]_{\sigma} + 4[f(s), \alpha(s)]_{\sigma} \lambda v + 4\alpha(s) \lambda [\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma} + [f(s), \alpha(s)]_{\sigma} \lambda \alpha(v) + \alpha(s) \lambda [f(s), \alpha(v)]_{\sigma} \in Z$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

$\alpha(s) \lambda([f(s), \alpha(v)]_{\sigma} + 4[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma}) + (\alpha(v) + 4v) \lambda [f(s), \alpha(s)]_{\sigma} + 4f(s) \lambda [v, \alpha(s)]_{\sigma} \in Z$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$. (26)

$[\alpha(s) \lambda([f(s), \alpha(v)]_{\sigma} + 4[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma}), \alpha(s)]_{\sigma} + [(\alpha(v) + 4v) f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} + 4[f(s) \lambda [v, \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} = 0$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$. (27)

$\alpha(s) \lambda[([f(s), \alpha(v)]_{\sigma} + 4[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma}), \alpha(s)]_{\sigma} + (\alpha(v) + 4v) \lambda[[f(s), \alpha(s)], \alpha(s)]_{\sigma} + [\alpha(v) + 4v, \alpha(s)]_{\sigma} \lambda [f(s), \alpha(s)]_{\sigma} + 4f(s) \lambda[[v, \alpha(s)]_{\sigma}, \alpha(s)] + 4[f(s), \alpha(s)]_{\sigma} \lambda [v, \alpha(s)]_{\sigma} = 0$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

$\alpha(s) \lambda[[f(s), \alpha(v)]_{\sigma}, \alpha(s)]_{\sigma} + 4\alpha(s) \lambda[[\mathcal{P}(s, s, s, v), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} + (\alpha(v) + 4v) \lambda[[f(s), \alpha(s)]_{\sigma}, \alpha(s)] + [\alpha(v), \alpha(s)]_{\sigma} \lambda [f(s), \alpha(s)]_{\sigma} + 4[v, \alpha(s)]_{\sigma} \lambda [f(s), \alpha(s)]_{\sigma} + 4f(s) \lambda [[v, \alpha(s)], \alpha(s)]_{\sigma} + 4[f(s), \alpha(s)]_{\sigma} \lambda [v, \alpha(s)]_{\sigma} = 0$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

$[\alpha(v), \alpha(s)] \lambda [f(s), \alpha(s)] + [4v, \alpha(s)] \lambda [f(s), \alpha(s)] + [f(s), \alpha(s)] \lambda [4v, \alpha(s)] + 4f(s) \lambda [[v, \alpha(s)], \alpha(s)] = 0$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$ (28)

$[(\alpha(v) + 8v), \alpha(s)]_{\sigma} \lambda [f(s), \alpha(s)]_{\sigma} + 4f(s) \lambda [[v, \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} = 0$, for each one $s, v \in L$, $\sigma, \lambda \in \Gamma$.

Replacing v by $2f(s) \gamma [f(s), \alpha(s)]_{\sigma}$ in (28), we get

$2([(\alpha(f(s) \gamma [f(s), \alpha(s)]_{\sigma}) + 8f(s) \gamma [f(s), \alpha(s)], \alpha(s)]_{\sigma} \lambda [f(s), \alpha(s)]_{\sigma} + 4f(s) \lambda [[f(s) \gamma [f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma}) = 0$, for each one $s \in L$, $\sigma, \lambda, \gamma \in \Gamma$.

As $\check{N}$ 2-torsion free, we obtain

$[(\alpha(f(s)\ \gamma[f(s), \alpha(s)]_{\sigma}) + 8f(s)\ \gamma[f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma}\ \lambda[f(s), \alpha(s)]_{\sigma} + 4f(s)\ \lambda[[f(s)\ \gamma\ [f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} = 0$, for each one $s \in L$, $\sigma, \lambda, \gamma \in \Gamma$.

$[(\alpha(f(s)\gamma\alpha([f(s), \alpha(s)]_{\sigma}), \alpha(s)]_{\sigma}\ \lambda[f(s), \alpha(s)]_{\sigma} + 8[f(s)\gamma[f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma}\ \lambda[f(s), \alpha(s)]_{\sigma} + 4f(s)\ \lambda\ [[f(s), \alpha(s)]_{\sigma}\gamma[f(s), \alpha(s)]_{\sigma} + f(s)\gamma[[f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} = 0$, for each one $s \in L$, $\sigma, \lambda, \gamma \in \Gamma$.

$\alpha(f(s))\gamma[\alpha([f(s), \alpha(s)]_{\sigma}), \alpha(s)]\ \lambda[f(s), \alpha(s)]_{\sigma} + [\alpha(f(s)), \alpha(s)]_{\sigma}\ \gamma\alpha([f(s), \alpha(s)]_{\sigma})\ \lambda[f(s), \alpha(s)]_{\sigma} + 8f(s)\gamma[[f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma}\ \lambda[f(s), \alpha(s)]_{\sigma} + 8[f(s), \alpha(s)]_{\sigma}\ \gamma[f(s), \alpha(s)]_{\sigma}\lambda\ [f(s), \alpha(s)]_{\sigma} + 4f(s)\ \lambda\ [[f(s), \alpha(s)]_{\sigma}\ \gamma[f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} = 0$, for each one $s \in L$, $\sigma, \lambda, \gamma \in \Gamma$,

$[\alpha(f(s)), \alpha(s)]_{\sigma}\ \gamma\alpha([f(s), \alpha(s)]_{\sigma})\ \lambda[f(s), \alpha(s)]_{\sigma} + 8[f(s), \alpha(s)]_{\sigma}\ \gamma[f(s), \alpha(s)]_{\sigma}\ \lambda[f(s), \alpha(s)]_{\sigma} + 4f(s)\ \lambda[f(s), \alpha(s)]_{\sigma}\ \gamma[[f(s), \alpha(s)]_{\sigma}, \alpha(s)]_{\sigma} + 4f(s)[[f(s), \alpha(s)]_{\sigma}, \alpha(s)]\ \gamma[f(s), \alpha(s)]_{\sigma} = 0$, for each one $s \in L$, $\sigma, \lambda, \gamma \in \Gamma$.

$[\alpha(f(s)), \alpha(s)]_{\sigma}\ \gamma\alpha([f(s), \alpha(s)]_{\sigma})\ \lambda[f(s), \alpha(s)]_{\sigma} + 8[f(s), \alpha(s)]_{\sigma}\ \gamma[f(s), \alpha(s)]_{\sigma}\ \lambda[f(s), \alpha(s)]_{\sigma} = 0$, for each one $s \in L$ $\sigma, \lambda, \gamma \in \Gamma$.

$[\alpha(f(s)), \alpha(s)]_{\sigma}\ \gamma\alpha[f(s), \alpha(s)]_{\sigma}\ \lambda[f(s), \alpha(s)]_{\sigma} + 8[f(s), \alpha(s)]_{\sigma}{}^{3} = 0$, for each one $s \in L$, $\sigma, \lambda, \gamma \in \Gamma$.

Since f is commuting on L and we have 2* 3-torsion freeness, $2[f(s), (s)]_{\sigma}{}^{3} = 0$.

It follows that $(2[f(s), (s)]_{\sigma}{}^{2})\ \check{N}\ 2([f(s), \alpha(s)]_{\sigma}{}^{2}) = 0$.Since Ň is semi prime, we have $2[f(s), (s)]_{\sigma}{}^{2} = 0$, for each one $s \in L$, $\sigma \in \Gamma$.

It follows that $(2[f(s), (s)]_{\sigma})\ \check{N}\ 2([f(s), \alpha(s)]_{\sigma}) = 0$.Since $\check{N}$ is semi prime, we have

$2[f(s), (s)]_{\sigma} = 0$, for each one $s \in L$, $\sigma \in \Gamma$.

Using of 2- torsion free ring, we have

$[f(s), (s)]_{\sigma} = 0$, for each one $s \in L$, $\sigma \in \Gamma$.

3. Discussion and Conclusion

In our paper we presented and studied the notion of asymmetric skew 4-derivation of prime and semiprime Γ-ring. Therefore we proved that;

i) assume that Ň be a 3, 2-torsion free non commutative prime Γ-rings accomplishing a certain assumption and L be admissible Lie ideal of Ň; Let $\text{Ƥ} : \check{N}^4 \to \check{N}$ is a symmetric 4-derivation. If f is a trace of Ƥ such that $[f(s), s]\sigma = 0$; for each one $s \in L$, $\sigma \in \Gamma$ therefore Ƥ = 0,

ii) if Ň *be a* 3, 2−torsion free non commutative semiprime Γ-rings accomplishing a certain assumption and L be admissible Lie ideal of Ň. Presume α is an automorphism of Ň and $\text{Ƥ}: \check{N}^4 \to \check{N}$ is a symmetric skew 4-derivation associated with α. Presume that the trace function f is commuting on L such that $[f(s), \alpha(s)]\sigma \in Z$ for each $s \in L$, $\sigma \in \Gamma$, where $f(L) \subseteq L$, therefore $[f(s), \alpha(s)]\sigma = 0$ for eachs $\in L$, $\sigma \in \Gamma$,

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