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Pre – supra v – open and some related concepts

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Abstract

The main goals of the paper is presenting a comprehensive study about the concept of pre-supra v – open set and giving many new results. Studying the properties of some concepts relative to the supra v – open set such as semi – supra v – open, regular – supra v – open, α – supra v – open and β – supra v – open as well as these concepts are generalizations of a supra v – open set.

Subject Classification: *00A05, 00A06.*

Keywords: *Algebra, Topology, Cryptography, Open set.*

1. Introduction

Originally coming from questions in analysis [1] and differential geometry by now topology permeates mostly every field of math including algebra [2],[3], combinatorics, logic and plays a fundamental role in algebraic geometry as we know it today.

The concept of semi-open set was studied by Levine [4] in 1963 which is a generalization of an open set.

Njastad in 1965 [5] was first introduced α – open set concept as a generalization of open set.

The concept of supra topological space was proposed by Mashhour [6] in 1983 as a generalization of the concept of topological space.

There are many authors are studied some concepts related to topological spaces for example Abdalbaqi, see [7] and [8] .

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In 2023 Raoof and Abdalbaqi are introduced the concept of supra $\mathcal{v}-$ open set where $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa \geq 2$, supra topologies on $\mathcal{X}$ and $\mathcal{V} \subseteq \mathcal{X}$. Then $\mathcal{V}$ is supra $\mathcal{v}$ $-$open set in $\mathcal{X}$ if $\exists T \in \bigcap_{\kappa\in J} \mathfrak{F}_\kappa \ni \phi \neq T \subseteq \mathcal{V}$ whenever $\mathcal{V} \neq \phi$ & $T = \phi$ when $\mathcal{V} = \phi$. $\mathcal{S}\mathcal{v}O_\mathcal{X}$ denotes to the set of all supra $\mathcal{v}$ $-$open sets.

2. The main results

Definition 2.1: Assume $(\mathcal{X}, \mathcal{S}\mathcal{v}O_\mathcal{X})$ is a $supra\ \mathcal{v}-space$. Then a subset $\mathcal{V}$ of $\mathcal{X}$ is a $pre-supra\ \mathcal{v}-open$ if $\mathcal{V} \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))$.

$\mathcal{P}(\mathcal{S}\mathcal{v}O_\mathcal{X})$ isa set of all $pre-supra\ \mathcal{v}-opensets$.

The complement of an $pre-supra\ \mathcal{v}-open$ is called $pre-supra\ \mathcal{v}-closed$. $\mathcal{P}(\mathcal{S}\mathcal{v}C_\mathcal{X})$ is a set of all $pre-supra\ \mathcal{v}-closed$.

Proposition 2.2: Every supra $\mathcal{v}-$ open is a pre $-$ supra $\mathcal{v}-$ open.

Definition 2.3: Assume $(\mathcal{X}, \mathcal{S}\mathcal{v}O_\mathcal{X})$ is a $supra\ \mathcal{v}-space$& $\mathcal{V} \subseteq \mathcal{X}$, then $\mathcal{V}$ is a $regular-supra\ \mathcal{v}-open$ if $\mathcal{V} = lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))$.

- the set of all $regular-supra\ \mathcal{v}-open$ is known as $\mathcal{R}(\mathcal{S}\mathcal{v}O_\mathcal{X})$.
- the complement of $regular-supra\ \mathcal{v}-open$ is called $regular-supra\ \mathcal{v}-closed$.
- the set of all $regular-supra\ \mathcal{v}-closed$ is known as $\mathcal{R}(\mathcal{S}\mathcal{v}C_\mathcal{X})$.

Proposition 2.4: $Every\ regular-supra\ \mathcal{v}-open\ is\ a\ supra\ \mathcal{v}-open.$

Proposition 2.5: $Every\ regular-\mathcal{v}-open\ is\ a\ pre-supra\ \mathcal{v}-open.$

Proof: Assume that $\mathcal{V}$ is a regular $-\ \mathcal{v}-$ open, then by Proposition (2.4), we have $\mathcal{V}$ is a supra $\mathcal{v}-$ open, hence by Proposition (2.2), we get $\mathcal{V}$ is a pre $-$ supra $\mathcal{v}-$ open.

Definition 2.6: Assume $(\mathcal{X}, \mathcal{S}\mathcal{v}O_\mathcal{X})$ is a $supra\ \mathcal{v}-space$& $\mathcal{V} \subseteq \mathcal{X}$, then $\mathcal{V}$ is a $\beta-supra\ \mathcal{v}-open$ if $\mathcal{V} \subseteq cl_{\mathcal{S}\mathcal{v}}\big(lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))\big)$.

- the set of all $\beta-supra\ \mathcal{v}-open$ is denoted by $\beta(\mathcal{S}\mathcal{v}O_\mathcal{X})$.
- the complement of an $\beta-supra\ \mathcal{v}-open$ is said to be $\beta-supra\ \mathcal{v}-closed$.
- the set of all $\beta-supra\ \mathcal{v}-closed$ is denoted by $\beta(\mathcal{S}\mathcal{v}C_\mathcal{X})$.

Proposition 2.7: Every supra $\mathcal{v}-$ open is a $\beta-$ supra $\mathcal{v}-$ open.

Proof: Assume that $\mathcal{V}$ is a supra $\mathcal{v}-$ open. then $\mathcal{V} \subseteq cl_{\mathcal{S}\mathcal{v}}(\mathcal{V})$, hence $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))$, but $\mathcal{V}$ is a supra $\mathcal{v}-$ open, then $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) = \mathcal{V}$, Thus $\mathcal{V} \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))$. Hence, $cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}) \subseteq cl_{\mathcal{S}\mathcal{v}}\big(lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))\big)$ by Theorem (1.39):(1). So, we have $\mathcal{V} \subseteq cl_{\mathcal{S}\mathcal{v}}\big(lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))\big)$.

Therefore, $\mathcal{V}$ is a $\beta-$ supra $\mathcal{v}-$ open.

Proposition 2.8: Every regular $-\ v\ -$ open is a $\beta\ -$ supra $v\ -$ open.

Theorem 2.9: Assume $(\mathcal{X}, \mathcal{S}v\mathcal{O}_\mathcal{X})$ is a supra $v\ -$ space & $\phi \neq \mathcal{V} \subseteq \mathcal{X}$. If $lnt_{\mathcal{S}v}(\mathcal{V}) \neq \phi$,then $lnt_{\mathcal{S}v}(\mathcal{V}) = \mathcal{V}$.

Proof: When $\mathcal{V} = \mathcal{X}$, we done. Thus, suppose $\mathcal{V} \neq \mathcal{X}$.

Now, we have, $lnt_{\mathcal{S}v}(\mathcal{V})$ is the largest supra $v\ -$ open set contained in $\mathcal{V}$. That is $\mathcal{V} \supseteq lnt_{\mathcal{S}v}(\mathcal{V}) \in \mathcal{S}v\mathcal{O}_\mathcal{X}$. Hence, there exist $T \in \bigcap_{\kappa\in J} \mathfrak{F}_\kappa$ such that $\phi \neq T \subseteq lnt_{\mathcal{S}v}(\mathcal{V})$, thus $T \subseteq \mathcal{V}$, which is implies that $\mathcal{V} \in \mathcal{S}v\mathcal{O}_\mathcal{X}$.

Therefore, $lnt_{\mathcal{S}v}(\mathcal{V}) = \mathcal{V}$.

Theorem 2.10: Suppose $(\mathcal{X}, \mathcal{S}v\mathcal{O}_\mathcal{X})$ is a $supra\ v - space$ & $\mathcal{V} \subset \mathcal{X}$. If $cl_{\mathcal{S}v}(\mathcal{V}) \neq \mathcal{X}$, then $cl_{\mathcal{S}v}(\mathcal{V}) = \mathcal{V}$.

Proposition 2.11: If $(\mathcal{X}, \mathcal{S}v\mathcal{O}_\mathcal{X})$ is a $supra\ v - space$, then every pre $-$ supra $v\ -$ open is a $\beta\ -$ supra $v\ -$ open.

Theorem 2.12: Assume $(\mathcal{X}, \mathcal{S}v\mathcal{O}_\mathcal{X})$ is a supra $v\ -$ space, then every $\beta\ -$ supra $v\ -$ open is a pre $-$ supra $v\ -$ open.

Proof: If $\mathcal{V}$ is $\beta\ -$ supra $v\ -$ open, so$\mathcal{V} \subseteq cl_{\mathcal{S}v}\big(lnt_{\mathcal{S}v}(cl_{\mathcal{S}v}(\mathcal{V}))\big)$. If $\mathcal{V} = \phi\ or\ \mathcal{V} = \mathcal{X}$, then $\mathcal{V}$ is a pre $-$ supra $v\ -$ open. Thus, let $\phi \neq \mathcal{V} \neq \mathcal{X}$.

Now, either $cl_{\mathcal{S}v}(\mathcal{V}) = \mathcal{X}$ or $cl_{\mathcal{S}v}(\mathcal{V}) \neq \mathcal{X}$.

If $cl_{\mathcal{S}v}(\mathcal{V}) = \mathcal{X}$, then $lnt_{\mathcal{S}v}\big(cl_{\mathcal{S}v}(\mathcal{V})\big) = \mathcal{X}$, hence $\mathcal{V} \subset lnt_{\mathcal{S}v}\big(cl_{\mathcal{S}v}(\mathcal{V})\big)$, therefore $\mathcal{V}$ is a pre $-$ supra $v\ -$ open. If $cl_{\mathcal{S}v}(\mathcal{V}) \neq \mathcal{X}$, since $\mathcal{V} \subset \mathcal{X}$, then $cl_{\mathcal{S}v}(\mathcal{V}) = \mathcal{V}$ by Theorem (2.10). hence $lnt_{\mathcal{S}v}\big(cl_{\mathcal{S}v}(\mathcal{V})\big) = lnt_{\mathcal{S}v}(\mathcal{V})$.

Now, either $lnt_{\mathcal{S}v}(\mathcal{V}) = \phi$ or $lnt_{\mathcal{S}v}(\mathcal{V}) \neq \phi$.

If $lnt_{\mathcal{S}v}(\mathcal{V}) = \phi$, then $cl_{\mathcal{S}v}\big(lnt_{\mathcal{S}v}(\mathcal{V})\big) = \phi$, that is $cl_{\mathcal{S}v}\big(lnt_{\mathcal{S}v}\big(cl_{\mathcal{S}v}(\mathcal{V})\big)\big) = \phi$, but $\mathcal{V} \subseteq cl_{\mathcal{S}v}\big(lnt_{\mathcal{S}v}(cl_{\mathcal{S}v}(\mathcal{V}))\big)$ by hypothesis , then we get $\mathcal{V} \subseteq \phi$ this is contradiction. So, $lnt_{\mathcal{S}v}(\mathcal{V}) \neq \phi$, hence by Theorem (2.9) we get $lnt_{\mathcal{S}v}(\mathcal{V}) = \mathcal{V}$. Now, since $\mathcal{V} \subseteq cl_{\mathcal{S}v}\big(lnt_{\mathcal{S}v}(cl_{\mathcal{S}v}(\mathcal{V}))\big)$, then we get $lnt_{\mathcal{S}v}(\mathcal{V}) \subseteq lnt_{\mathcal{S}v}\big(cl_{\mathcal{S}v}\big(lnt_{\mathcal{S}v}(cl_{\mathcal{S}v}(\mathcal{V}))\big)\big)$, but $lnt_{\mathcal{S}v}(cl_{\mathcal{S}v}(\mathcal{V})) = lnt_{\mathcal{S}v}(\mathcal{V})$ and $lnt_{\mathcal{S}v}(\mathcal{V}) = \mathcal{V}$, hence $\mathcal{V} \subseteq lnt_{\mathcal{S}v}\big(cl_{\mathcal{S}v}(\mathcal{V})\big)$, therefore $\mathcal{V}$ is a pre $-$ supra $v\ -$ open.

Proposition 2.13: $\mathcal{P}(\mathcal{S}v\mathcal{O}_\mathcal{X}) = \beta(\mathcal{S}v\mathcal{O}_\mathcal{X})$.

Proposition 2.14: $\mathcal{R}(\mathcal{S}v\mathcal{O}_\mathcal{X}) \subset \mathcal{P}(\mathcal{S}v\mathcal{O}_\mathcal{X})$.

Proposition 2.15: $\mathcal{R}(\mathcal{S}v\mathcal{O}_\mathcal{X}) \subset \mathcal{S}v\mathcal{O}_\mathcal{X}$.

Proposition 2.16: Assume that $(\mathcal{X}, \mathcal{S}v\mathcal{O}_\mathcal{X})$ be a supra $v\ -$ space. Then $\mathcal{S}v\mathcal{O}_\mathcal{X} \subset \mathcal{P}(\mathcal{S}v\mathcal{O}_\mathcal{X})$.

Definition 2.17: Suppose that $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ is a supra $\mathcal{v}-$space. Then a subset $\mathcal{V}$ of $\mathcal{X}$ is said to be $\alpha-$supra $\mathcal{v}-$open if $\mathcal{V} \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V})))$.

the set of all $\alpha-$supra $\mathcal{v}-$open is denoted by $\alpha(\mathcal{S}\mathcal{v}O_{\mathcal{X}})$.

complement for an $\alpha-$supra $\mathcal{v}-$open named $\alpha-$supra $\mathcal{v}-$closed.

A set for every $\alpha-$supra $\mathcal{v}-$closed is $\alpha(\mathcal{S}\mathcal{v}C_{\mathcal{X}})$.

Proposition 2.18: Every supra $\mathcal{v}-$open is a $\alpha-$supra $\mathcal{v}-$open.

Theorem 2.19: Let $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ be a supra $\mathcal{v}-$space. Then every $\alpha-$supra $\mathcal{v}-$open is a supra $\mathcal{v}-$open.

Proof: Assume $\mathcal{V}$ be an $\alpha-$supra $\mathcal{v}-$open, so

$$\mathcal{V} \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}))).$$

when $\mathcal{V} = \phi \; or \; \mathcal{V} = \mathcal{X}$, hence $\mathcal{V}$ is supra $\mathcal{v}-$open.

Thus, let $\phi \neq \mathcal{V} \neq \mathcal{X}$.

Now, either $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) = \phi$ or $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) \neq \phi$.

If $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) = \phi$, then $cl_{\mathcal{S}\mathcal{v}}(lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V})) = \phi$, hence

$lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}))) = \phi$, thus $\mathcal{V} \subseteq \phi$ contradiction .

Hence, $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) \neq \phi$, since $\mathcal{V} \neq \phi$, then by Theorem (2.9) we get $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) = \mathcal{V}$. Therefore $\mathcal{V}$ is a supra $\mathcal{v}-$open.

Proposition 2.20: $\alpha(\mathcal{S}\mathcal{v}O_{\mathcal{X}}) = \mathcal{S}\mathcal{v}O_{\mathcal{X}}$.

Proposition 2.21: Every regular $-$ supra $\mathcal{v}-$open is a $\alpha-$supra $\mathcal{v}-$open.

Proposition 2.22: Assume $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ is a supra $\mathcal{v}-$space. then $\mathcal{R}(\mathcal{S}\mathcal{v}O_{\mathcal{X}}) \subset \alpha(\mathcal{S}\mathcal{v}O_{\mathcal{X}})$.

Proposition 2.23: Every $\alpha-$ supra $\mathcal{v}-$open is a pre $-$ supra $\mathcal{v}-$open.

Proof: Assume that $\mathcal{V}$ is a $\alpha-$supra $\mathcal{v}-$open. Then $\mathcal{V} \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V})))$. Now, we have $lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}) \subseteq \mathcal{V}$, hence $lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}))) \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))$, thus $\mathcal{V} \subseteq lnt_{\mathcal{S}\mathcal{v}}(cl_{\mathcal{S}\mathcal{v}}(\mathcal{V}))$. Therefore, $\mathcal{V}$ is a pre $-$ supra $\mathcal{v}-$open.

Proposition 2.24: $\alpha(\mathcal{S}\mathcal{v}O_{\mathcal{X}}) \subset \mathcal{P}(\mathcal{S}\mathcal{v}O_{\mathcal{X}})$.

Proposition 2.25: Every $\alpha-$ supra $\mathcal{v}-$open is a $\beta-$supra $\mathcal{v}-$open.

Proposition 2.26: Assume $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ is a supra $\mathcal{v}-$space. then $\alpha(\mathcal{S}\mathcal{v}O_{\mathcal{X}}) \subset \beta(\mathcal{S}\mathcal{v}O_{\mathcal{X}})$.

Definition 2.27: Suppose that $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ is a supra $\mathcal{v}-$space. Then asubset $\mathcal{V}$ of $\mathcal{X}$ is called semi $-$ supra $\mathcal{v}-$open if $\mathcal{V} \subseteq cl_{\mathcal{S}\mathcal{v}}(lnt_{\mathcal{S}\mathcal{v}}(\mathcal{V}))$.

- the set of all semi $-$ supra $\mathcal{v}-$open is denoted by $\mathcal{g}(\mathcal{S}\mathcal{v}O_{\mathcal{X}})$.

- the complement of semi $-$ supra v $-$ open is called semi $-$ supra v $-$ closed.
- the set of all semi $-$ supra v $-$ closed is denoted by $g(SvC_X)$.

Proposition 2.28: Every supra v $-$ open is a semi $-$ supra v $-$ open.

Theorem 2.29: *If (X, SvO_X) is a supra v $-$ space, then every semi $-$ supra v $-$ open is a supra v $-$ open.*

Proof: Assume V is semi $-$ supra v $-$ open, so when $V = \phi$ *or* $V = X$, we get V is supra v $-$ open.

Thus, let $\phi \neq V \neq X$.

Now, either $lnt_{Sv}(V) = \phi$ or $lnt_{Sv}(V) \neq \phi$.

If $lnt_{Sv}(V) = \phi$, then $cl_{Sv}(lnt_{Sv}(V)) = \phi$.

Hence $V \subseteq \phi$ contradiction .

Thus, $lnt_{Sv}(V) \neq \phi$, since $V \neq \phi$, then we get $lnt_{Sv}(V) = V$. So V is supra v $-$ open.

Proposition 2.30: $g(SvO_X) = SvO_X$.

Theorem 2.31: Let (X, SvO_X) *be a supra v $-$ space* & $V \subseteq X$. Then V is a α $-$ supra v $-$ open iff V is a semi $-$ supra v $-$ open.

Conclusion

The main results are obtained are:

1. Every supra v $-$ open is a pre $-$ supra v $-$ open.
2. Every β $-$ supra v $-$ open is a pre $-$ supra v $-$ open.
3. Every supra v $-$ open is a α $-$ supra v $-$ open.
4. Every supra v $-$ open is a semi $-$ supra v $-$ open.
5. Every regular $-$ supra v $-$ open is a supra v $-$ open.
6. Assume (X, SvO_X) is a supra v $-$ space & $\phi \neq V \subseteq X$. If $lnt_{Sv}(V) \neq \phi$,then $lnt_{Sv}(V) = V$.
7. Suppose (X, SvO_X) is a *supra v $-$ space* & $V \subset X$. If $cl_{Sv}(V) \neq X$,then $cl_{Sv}(V) = V$.
8. Assume (X, SvO_X) is a supra v $-$ space, then every β $-$ supra v $-$ open is a pre $-$ supra v $-$ open.
9. If (X, SvO_X) is a supra v $-$ space, then every semi $-$ supra v $-$ open is a supra v $-$ open.
10. Assume (X, SvO_X) is a supra v $-$ space. Then $g(SvO_X) = SvO_X$.
11. Let (X, SvO_X) *be a supra v $-$ space* & $V \subseteq X$. Then V is a α $-$ supra v $-$ open iff V is a semi $-$ supra v $-$ open.

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