

New structure of algebras using permutations in symmetric groups

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Abstract

The permutation BG-algebras were first introduced as a novel kind of algebra. In this work, their basic qualities were investigated to better understand how they relate to one another and how they might be combined to construct various sorts of superstructures. We investigated permutation BG-algebras, permutation group-derived, permutation BG-subalgebra, and permutation BG-normal. Furthermore, we explored certain novel concepts in permutation theory for the initial time. We additionally looked into BG-algebra isomorphism

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theorems, BG-algebra homomorphism, quotient permutation BG-algebras, and equivalence relations.

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1. Introduction

BCI – algebras and BCK – algebras are two forms of abstract algebras proposed by K. Is'eki and S. Tanaka ([1,2]). It is widely acknowledged that the BCK – algebra class is a legitimate subclass of the BCI – algebra class. Chang and Hee [3] proposed the concept of BG – algebras, an extension of B – algebras. The permutation sets are provided, which are then explored for the characteristics and applications. Following that, the ideas of permutation B – algebra [4] and permutation BH – algebra [5] are presented. In this paper, we introduced permutation BG – algebras as a new class of BG – algebras and discussed some of its key properties. Permutation group–derived, permutation BG – subalgebra, and permutation BG – normal are all novel ideas in permutation BG – algebras. We also looked into BG – algebra isomorphism theorems, BG – algebra homomorphism, quotient permutation BG – algebras, and equivalence relations.

2. Preliminaries

In this section, we shall review the main concepts and findings that are required for this research.

Definition 2.1 [3]: For any set $W \neq \emptyset$ with a map $*$: $W \times W \rightarrow W$ and for some $0 \in W$. This triple $(W, *, 0)$ is called a B –algebra if satisfy: (1) $e * f = 0$, (2) $e * 0 = e$, (3) $(e * f) * d = e * (d * (0 * f))$, $\forall e, f, d \in W$.

Definition 2.2 [3]: Assume that $K(\tau)$ is the class of all algebras with type $\tau = (2, 0)$. A BG – algebra is a system $(W, *, 0)$ and such that: (1) $e * e = 0$, $\forall e \in W$, (2) $e * 0 = e$, $\forall e \in W$, (3) $(e * f) * (0 * f) = e$, $\forall e, f \in W$. This term 0 is called unit.

Definition 2.3 [4]: If $\mu = \prod_{p=1}^{c(\mu)} \lambda_p \in \mathcal{S}_n$ is a permutation in a symmetric group $\mathcal{S}_n$ with $\{\lambda_p\}_{p=1}^{c(\mu)}$ are pairwise disjoint cycles and $\lambda_p = (t_1^p, t_2^p, \dots, t_{\alpha_p}^p)$, $1 \leq p \leq c(\mu)$, for some $1 \leq \alpha_p, c(\mu) \leq n$. Then μ –sets of $\{\lambda_p\}_{p=1}^{c(\mu)}$ are given as $\lambda_p^\mu = \{t_1^p, t_2^p, \dots, t_{\alpha_p}^p\} | 1 \leq p \leq c(\mu)$.

Definition 2.4 [4]: Let $W = \{\lambda_i^\beta\}_{i=1}^{c(\beta)} \cup \{1\}$ be a collection of β –sets $\{\lambda_i^\beta\}_{i=1}^{c(\beta)}$ with $\{1\}$, where $\beta \in \mathcal{S}_n$. We say $(W, \#, \{1\})$ is a permutation

B – algebra ($PB - A$), if $\#: W \times W \rightarrow W$ such that: (1) $\lambda_i^\beta \# \lambda_i^\beta = \{1\}$, (2) $\lambda_i^\beta \# \{1\} = \lambda_i^\beta$, (3) $(\lambda_i^\beta \# \lambda_j^\beta) \# \lambda_k^\beta = \lambda_i^\beta \# (\lambda_k^\beta \# (\{1\} \# \lambda_j^\beta))$, $\forall \lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in W$.

Definition 2.5 [5]: We say $(X, \#, \{1\})$ is a *permutation BH – algebra* ($PBH - A$), where $\beta \in S_n$ and $X = \{\lambda_i^\beta\}_{i=1}^{c(\beta)} \cup \{1\}$, if $\#$ satisfies: (1) $\lambda_i^\beta \# \lambda_i^\beta = \{1\}$, (2) $\lambda_i^\beta \# \lambda_j^\beta = \{1\}$ and $\lambda_j^\beta \# \lambda_i^\beta = \{1\} \Rightarrow \lambda_i^\beta = \lambda_j^\beta$, (3) $\lambda_i^\beta \# \{1\} = \lambda_i^\beta$, $\forall \lambda_i^\beta, \lambda_j^\beta \in X$.

3. Permutation BG –algebras [6]

In this part, We will look at some of the basic properties of permutation BG-algebra and discuss some novel ramifications for the subject.

Definition 3.1: Assume that $X = \{\lambda_i^\beta\}_{i=1}^{c(\beta)}$ is a collection of β –sets, where $\beta \in S_n$. We say X is a *permutation BG – algebra* ($PBG - A$) if $\#: X \times X \rightarrow X$ satisfies: (1) $\lambda_i^\beta \# \lambda_i^\beta = \{1\}$, $\forall \lambda_i^\beta \in X$, (2) $\lambda_i^\beta \# \{1\} = \lambda_i^\beta$, $\forall \lambda_i^\beta \in X$, (3) $(\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_k^\beta) = \lambda_i^\beta \# \lambda_k^\beta$, $\forall \lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in X$. We say that $\{1\}$ is a constant element in X .

Example 3.2: Suppose that (S_9, o) is a symmetric group, let $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 1 & 5 & 9 & 3 & 7 & 6 & 2 & 4 \end{pmatrix}$ be a permutation in S_9 . Since $\beta = (1\ 8\ 2)$

$(3\ 5)(4\ 9)(6\ 7)$. So $X = \{\lambda_i^\beta\}_{i=1}^4 \cup \{1\} = \{\{1,2,8\}, \{3,5\}, \{4,9\}, \{6,7\}, \{1\}\}$. Define $\#: X \times X \rightarrow X$ by Table 1. Then $(X, \#, \{1\})$ is a $(PBG - A)$.

Table 1
 $(X, \#, T)$ is a $(PBG - A)$.

#	$\{1\}$	$\{3,5\}$	$\{4,9\}$	$\{1,2,8\}$
$\{1\}$	$\{1\}$	$\{3,5\}$	$\{4,9\}$	$\{1,2,8\}$
$\{3,5\}$	$\{3,5\}$	$\{1\}$	$\{1,2,8\}$	$\{4,9\}$
$\{4,9\}$	$\{4,9\}$	$\{1,2,8\}$	$\{1\}$	$\{3,5\}$
$\{1,2,8\}$	$\{1,2,8\}$	$\{4,9\}$	$\{3,5\}$	$\{1\}$

Remark 3.3: Note that in above example if $\lambda_i^\beta = \{3,5\}$, $\lambda_j^\beta = \{4,9\}$, and $\lambda_k^\beta = \{1,2,8\}$, we can satisfy the three conditions in Definition (3.1) as following: (1) $\lambda_i^\beta \# \lambda_i^\beta = \lambda_j^\beta \# \lambda_j^\beta = \lambda_k^\beta \# \lambda_k^\beta = \{1\}$, (2) $\lambda_i^\beta \# \{1\} = \lambda_i^\beta$, $\lambda_j^\beta \# \{1\} = \lambda_j^\beta$, & $\lambda_k^\beta \# \{1\} = \lambda_k^\beta$, (3) $(\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_k^\beta) = (\{3,5\} \# \{4,9\}) \# (\{1\} \# \{4,9\}) = \{1,2,8\} \# \{4,9\} = \{3,5\} = \lambda_i^\beta$.

Theorem 3.4: Let $(X, \#, \{1\})$ be a $(PBG - A)$. Then: (i) The right cancellation law holds in X , i.e. $\lambda_i^\beta \# \lambda_j^\beta = \lambda_k^\beta \# \lambda_j^\beta \Rightarrow \lambda_i^\beta = \lambda_k^\beta$, (ii) $\{1\} \# (\{1\} \# \lambda_i^\beta) = \lambda_i^\beta \forall \lambda_i^\beta \in X$, (iii) If $\lambda_i^\beta \# \lambda_j^\beta = \{1\}$, then $\lambda_i^\beta = \lambda_j^\beta \forall \lambda_i^\beta, \lambda_j^\beta \in X$, (iv) If $\{1\} \# \lambda_i^\beta = \{1\} \# \lambda_j^\beta$, then $\lambda_i^\beta = \lambda_j^\beta \forall \lambda_i^\beta, \lambda_j^\beta \in X$, (v) $(\lambda_i^\beta \# (\{1\} \# \lambda_i^\beta)) \# \lambda_i^\beta = \lambda_i^\beta \forall \lambda_i^\beta \in X$.

Proof: (i) Assume that $\lambda_i^\beta \# \lambda_j^\beta = \lambda_k^\beta \# \lambda_j^\beta$. Then $\lambda_i^\beta = (\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = (\lambda_k^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = \lambda_k^\beta$. From [Definition 3.1], we have [(ii)-(v)] are hold.

Theorem 3.5: Suppose that $(X, o, \{1\})$ is a group. If $\lambda_i^\beta \# \lambda_j^\beta = \lambda_i^\beta o \lambda_j^{\beta^{-1}}$, then $(X, \#, \{1\})$ is a $(PBG - A)$.

Proof: We see that $\lambda_i^\beta \# \lambda_i^\beta = \lambda_i^\beta \# \lambda_i^{\beta^{-1}} = \{1\}$ and $\lambda_i^\beta \# \{1\} = \lambda_i^\beta o \{1\}^{-1} = \lambda_i^\beta o \{1\} = \lambda_i^\beta . (\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = (\lambda_i^\beta o \lambda_j^{\beta^{-1}}) o (\{1\} o \lambda_j^\beta)^{-1} = \lambda_i^\beta o (\lambda_j^{\beta^{-1}} o \lambda_j^\beta) = \lambda_i^\beta o \{1\} = \lambda_i^\beta, \forall \lambda_i^\beta, \lambda_j^\beta \in X$. Hence $(X, \#, \{1\})$ is a $(PBG - A)$.

Definition 3.6: A group (X, o) with identity $\{1\}$ that can determine a $(PBG - A)$ $(X, \#, \{1\})$ is called a *permutation group-derived* $(PG - D)$.

Lemma 3.7: Assume that $(X, \#, \{1\})$ is a $(PBG - A)$ with the identity $\{1\}$ such that $(\lambda_i^\beta \# \lambda_j^\beta) \# \lambda_k^\beta = \lambda_i^\beta \# (\{1\} \# ((\{1\} \# \lambda_j^\beta) \# \lambda_k^\beta)) \forall \lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in X$. Then $(X, \#, \{1\})$ is $(PG - D)$.

Proof: It is easy from [Theorem (3.4)–(ii)], we consider that $(X, \#, \{1\})$ is a $(PG - D)$.

Theorem 3.8: Every $(PB - A)$ is a $(PBG - A)$.

Proof: Let $(X, \#, \{1\})$ be a $(PB - A)$, then axioms (1) and (2) of the $(PBG - A)$ are satisfied $(\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = \lambda_i^\beta \# ((\{1\} \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta)) = \lambda_i^\beta \# \{1\} = \lambda_i^\beta \forall \lambda_i^\beta, \lambda_j^\beta \in X$. Hence every $(PB - A)$ is a $(PBG - A)$.

Theorem 3.9: Every $(PBG - A)$ is a $(PBH - A)$.

Proof: It comes according to the Theorem 3.4-(3).

Theorem 3.10: Assume that X is a collection of β -sets contains $\{1\}$. Let " $\#$ "

be a binary operation on X define as $\lambda_i^\beta \# \lambda_j^\beta = \begin{cases} \lambda_i^\beta & \text{if } \lambda_i^\beta = \{1\} \\ \{1\} & \text{if } \lambda_i^\beta = \lambda_j^\beta, \forall \lambda_i^\beta, \lambda_j^\beta \in X \\ \lambda_i^\beta & \text{otherwise} \end{cases}$

X , then $(X, \#, \{1\})$ is a $(PBG - A)$.

Proof: Let $\lambda_i^\beta = \{1\}$, then $(\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = (\{1\} \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = \{1\}, \forall \lambda_i^\beta, \lambda_j^\beta \in X$. Assume $\lambda_i^\beta \neq \{1\}$. If $\lambda_j^\beta = \lambda_i^\beta$, then $(\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = (\lambda_i^\beta \# \lambda_i^\beta) \# (\{1\} \# \lambda_i^\beta) = \{1\} \# (\{1\} \# \lambda_i^\beta) = \lambda_i^\beta$. If $\lambda_j^\beta \neq \lambda_i^\beta$, then $(\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \lambda_j^\beta) = \lambda_i^\beta \# (\{1\} \# \lambda_j^\beta) = \lambda_i^\beta \# \lambda_j^\beta = \lambda_i^\beta$. Thus $(X, \#, \{1\})$ is a $(PBG - A)$.

Theorem 3.11: Let $(X, \#, \{1\})$ be a $(PBG - A)$ and $|X| \geq 3$, obtained in Theorem 3.10 above is not $(PG - D)$.

Proof: Let $\lambda_i^\beta, \lambda_j^\beta, \lambda_k^\beta \in X$ be distinct elements and $\lambda_k^\beta \neq \{1\}$. Then $\lambda_k^\beta \# \lambda_i^\beta = \lambda_k^\beta \# \lambda_j^\beta = \lambda_k^\beta$. If $(X, \#, \{1\})$ is not $(PG - D)$ considered from the group $(X, \#)$ by Theorem 3.10, then $\lambda_k^\beta = \lambda_k^\beta \# \lambda_i^\beta = \lambda_k^\beta \circ \lambda_i^{\beta^{-1}} \& \lambda_k^\beta = \lambda_k^\beta \# \lambda_j^\beta = \lambda_k^\beta \circ \lambda_j^{\beta^{-1}}$, i.e. $\lambda_i^\beta = \lambda_j^\beta$ a contradiction.

Definition 3.12: Assume that $(X, \#, \{1\}_X)$ and $(Y, \#, \{1\}_Y)$ are two $(PBG - As)$. A mapping $\theta: X \rightarrow Y$ is called a permutation BG -homomorphism $(PBG - Hom)$ if $\theta(\lambda_i^\beta \#_X \lambda_j^\beta) = \theta(\lambda_i^\beta) \#_Y \theta(\lambda_j^\beta) \forall \lambda_i^\beta, \lambda_j^\beta \in X$. A $(PBG - Hom)$ $\theta: X \rightarrow Y$ is said to be a permutation BG -isomorphism $(PBG - Isom)$ if θ is a bijection, denoted as $X \cong Y$. If $\theta: X \rightarrow Y$ is a $(PBG - Hom)$, We say $\{\lambda_i^\beta \in X | \theta(\lambda_i^\beta) = \{1\}_Y\}$ is a kernel of the $(PBG - Hom)$ θ , denoted by $Ker\theta$.

Definition 3.13: Suppose that $(X, \#, \{1\})$ is a $(PBG - A)$ and $\emptyset \neq S \subseteq X$. We say S is permutation BG -subalgebra $(PBG - SA)$ of X if: $\lambda_i^\beta \# \lambda_j^\beta \in S, \forall \lambda_i^\beta, \lambda_j^\beta \in S$. Note that each $(PBG - SA)$ of a $(PBG - A)$ is a $(PBG - A)$ too.

Proposition 3.14: Suppose that $(X, \#, \{1\})$ is a $(PBG - A)$ and $\emptyset \neq S \subseteq X$. Then the following are equivalent:

(a) S is a $(PBG - SA)$ of X , and (b) $\lambda_i^\beta \# (\{1\} \# \lambda_j^\beta), \{1\} \# \lambda_j^\beta \in S \forall \lambda_i^\beta, \lambda_j^\beta \in S$.

Proof: (a) $\Rightarrow$ (b) Since $S \neq \{1\}$, there exists an element $\lambda_i^\beta \in S$ and so $\{1\} = \lambda_i^\beta \# \lambda_i^\beta \in S$ since S is closed under " $\#$ ", $\{1\} \# \lambda_j^\beta \in S$ and thus $\lambda_i^\beta \# (\{1\} \# \lambda_j^\beta) \in S$. (b) $\Rightarrow$ (a) Since $\lambda_i^\beta \# \lambda_j^\beta = \lambda_i^\beta \# (\{1\} \# (\{1\} \# \lambda_j^\beta))$, $\lambda_i^\beta \# \lambda_j^\beta \in S \forall \lambda_i^\beta, \lambda_j^\beta \in S$.

Definition 3.15: If $\emptyset \neq N \subseteq X$. We say N permutation BG – normal ($PBG - N$) of X if $(\lambda_i^\beta \# \lambda_m^\beta) \# (\lambda_j^\beta \# \lambda_n^\beta) \in N \forall \lambda_i^\beta \# \lambda_j^\beta, \lambda_m^\beta \# \lambda_n^\beta \in N$.

Proposition 3.16: Any ($PBG - N$) subset N of a ($PBG - A$) X is a ($PBG - SA$) of X .

Proof: If $\lambda_i^\beta, \lambda_j^\beta \in N$, then $\lambda_i^\beta \# \{1\}, \lambda_j^\beta \# \{1\} \in N$. Since N is ($PBG - N$), we get $\lambda_i^\beta \# \lambda_j^\beta = (\lambda_i^\beta \# \lambda_j^\beta) \# (\{1\} \# \{1\}) \in N$. Thus N is a permutation subalgebra of X . It should be noted that Proposition 3.16's opposite is not necessary to hold.

Lemma 3.17: Assume that N is a ($PBG - N$) of ($PBG - A$) X and let $\lambda_i^\beta, \lambda_j^\beta \in N$. Then $\lambda_j^\beta \# \lambda_i^\beta \in N$, if $\lambda_i^\beta \# \lambda_j^\beta \in N$.

Proof: Let $\lambda_i^\beta \# \lambda_j^\beta \in N$. Since $\lambda_j^\beta \# \lambda_j^\beta = \{1\} \in N$ and N is ($PBG - N$), $\lambda_j^\beta \# \lambda_i^\beta = (\lambda_j^\beta \# \lambda_i^\beta) \# (\lambda_j^\beta \# \lambda_j^\beta) \in N$.

Definition 3.18: Let $(X, \#, \{1\})$ be a ($PBG - A$) and let N be a ($PBG - N$) of X by $\lambda_i^\beta \sim_N \lambda_j^\beta$ if and only if $\lambda_i^\beta \# \lambda_j^\beta \in N$, where $\lambda_i^\beta, \lambda_j^\beta \in X$. $\sim_N$ is then simply demonstrated to be an equivalence relation on X . Put λ_i^β in the equivalence class represented by $[\lambda_i^\beta]_N$, i.e. $[\lambda_i^\beta]_N = \{\lambda_j^\beta \in X \mid \lambda_i^\beta \sim_N \lambda_j^\beta\}$ and then $\frac{X}{N} = \{[\lambda_i^\beta]_N \mid \lambda_i^\beta \in X\}$ is called the *quotient permutation BG – algebra* ($QPBG - A$) of X by N .

Proposition 3.19: The quotient X/N is a ($PBG - A$) if N is a ($PBG - N$) of a ($PBG - A$) X .

Proof: Define " $\#$ " on X/N by $[\lambda_i^\beta]_N \# [\lambda_j^\beta]_N = [\lambda_i^\beta \# \lambda_j^\beta]_N$, $\forall [\lambda_i^\beta]_N, [\lambda_j^\beta]_N \in X/N$, then the operation " $\#$ " is well-defined, since if $\lambda_i^\beta \sim_N \lambda_a^\beta$ and $\lambda_j^\beta \sim_N \lambda_b^\beta$, then $\lambda_i^\beta \# \lambda_a^\beta \in N$ and $\lambda_j^\beta \# \lambda_b^\beta \in N$ implies $(\lambda_i^\beta \# \lambda_j^\beta) \# (\lambda_a^\beta \# \lambda_b^\beta) \in N$ by normality of N . So $\lambda_i^\beta \# \lambda_j^\beta \sim_N \lambda_a^\beta \# \lambda_b^\beta$ and so $[\lambda_i^\beta \# \lambda_j^\beta]_N = [\lambda_a^\beta \# \lambda_b^\beta]_N$. Note

that $[\{1\}]_N = \{\lambda_i^\beta \in X | \lambda_i^\beta \sim_N \{1\}\} = \{\lambda_i^\beta \in X | \lambda_i^\beta \# \{1\} \in N\} = \{\lambda_i^\beta \in X | \lambda_i^\beta \in N\} = N$.

Definition 3.20: The mapping $\gamma: X \rightarrow X/N$ given by $\gamma(\lambda_i^\beta) = [\lambda_i^\beta]_N$ is called the natural permutation *BG – homomorphism (NPBG – Hom)* of X onto X/N .

Proposition 3.21: Let N be a $(PBG - N)$ of a $(PBG - A)$ X . Then the natural mapping $\gamma: X \rightarrow X/N$ given by $\gamma(\lambda_i^\beta) = [\lambda_i^\beta]_N$ is a surjective $(PBG - Hom)$, and $Ker\gamma = N$.

Proposition 3.22: Assume that $\theta: X \rightarrow Y$ is a $(PBG - Hom)$. Then θ is injective if and only if $Ker\theta = \{\{1\}_X\}$.

Proof: Let $\lambda_i^\beta, \lambda_j^\beta \in X$ with $\theta(\lambda_i^\beta) = \theta(\lambda_j^\beta)$. Then $\theta(\lambda_i^\beta) \# \theta(\lambda_j^\beta) = \{1\}_Y$. So $\lambda_i^\beta \# \lambda_j^\beta \in Ker\theta$. Since $Ker\theta = \{\{1\}_X\}$, $\lambda_i^\beta \# \lambda_j^\beta = \{1\}_X$. Thus $\lambda_i^\beta = \lambda_j^\beta$ from Theorem 3.4–(3). Hence θ is injective. Conversely, if θ is injective then $\lambda_i^\beta = \lambda_j^\beta \Rightarrow \theta(\lambda_i^\beta) = \theta(\lambda_j^\beta)$. Then $\lambda_i^\beta \# \lambda_i^\beta = \{1\} = \lambda_i^\beta \# \lambda_j^\beta \Rightarrow \theta(\lambda_i^\beta \# \lambda_j^\beta) = \{1\}$.

4. Conclusion

In this study, certain novel concepts and outcomes are examined and proven using the composition of *BG – algebra* with permutation sets, including $(PBG - As)$, $(PG - D)$, $(PBG - SA)$, and $(PBG - N)$. In future work, we'll use neutrosophic set theory to combine neutrosophic sets and *BG – algebra*, then consider a few ideas and investigate the results.

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