

Neutrosophic invariant rings related with some applications

Nibras H. Ali *

Majid Mohammed Abed [†]

Department of Mathematics

College of Science

University of Anbar

Ramadi

Iraq

Abstract

This article gives some new results of a notion namely neutrosophic invariant ring (NIR) and their relationship with an important module named multiplication module. Other properties have been presented in this paper. We proved (NC)-module over neutrosophic invariant ring is neutrosophic multiplication module. Also, neutrosophic f-generated is neutrosophic multiplication module over invariant ring. Finally, some remarks, examples and properties have introduced in this article.

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1. Introduction

Here we present a concept Neutrosophic set (NS). Also, as a generalization of vector space, we introduce structure algebra named module. One of the important properties, is any commutative group is a $\mathbb{Z}$ -module. As a mathematics system, we consider a regular concept is very important in abstract algebra. It is known that every ring need two binary operations. It is also known that there are several types of the rings,

This research was conducted at University of Anbar

* E-mail: Nib22u2006@uoanbar.edu.iq (Corresponding Author)

[†] E-mail: majid_math@uoanbar.edu.iq

including the regular ring, which is the subject of our study in this paper. Invariant ring in [1] refer to each right (left) ideal. More details about invariant ring and their relation with multiplication module in [1]. Also, Injective Module with Divisible module [2]. The author in [3], studied pseudo valuation module and injectivity property. Some Neutrosophic Algebra structures and (NN)-algebraic structures in [4]. On simple submodules and C-rings [5]. Several results of generalizations of Fuzzy k-ideals in a KU-algebra with semigroup in [6]. The author in [7] presented basic Neutrosophic and their application of fuzzy and Neutrosophic module. Smarandache semigroups in [8]. In [9], some new results of neutrosophic submodules. More details of n-refined Neutrosophic modules in [10]. Also, in [11] we find some properties of Some results of topological rings. More information about Injective modules, projective modules and their relationships to some rings [12]. Semi homomorphisms and algebraic relations between strong refined Neutrosophic modules and strong Neutrosophic modules in [13].

2. Preliminaries

Now we give the main tools which it has a direct relationship with neutrosophic invariant ring (shortly $(VR_n(I))$ and multiplication neutrosophic module $M_n(I)$.

Definition 2.1: Let S be a collection of points belongs to S named S . A neutrosophic sets $H \in S$ is a truth membership mapping T_H , Indeterminacy membership mapping I_H and falsity membership mapping F_H . Note that $T_H(a)$, $I_H(a)$ and $F_H(a)$ are real standard of the interval $[0,1]$ which is.

$$H = \{ \langle S, (T_H(a), I_H(a), F_H(a)) \rangle : a \in \Omega, T_H(a), I_H(a), F_H(a) \in [0,1] \},$$

where Ω named to universe.

Definition 2.2: We define fuzzy set on X , we here x is universal set as $A = \{ \langle X, \mu_a(x) : x \in X \rangle \text{ such that } \mu_a : X \rightarrow [0,1] \}$.

Remark 2.3: $\mu_a = \{ \langle x, \infty_a(\infty), A_a(x) \rangle, \infty \in X \}$ means Neutrosophic mapping $\infty_a : X \rightarrow [0,1]$

Definition 2.4: We say $R_n(I)$ is a neutrosophic ring, where Which means it generated by R and I with clear operation.

Example 2.5: $(Z_n(I))$ is called neutrosophic integer ring such that $(Z_n(I)) = \{a + bI : a, b \in \mathbb{Z}\}$.

Definition 2.6: A characteristic of the ring $R_n(I)$ is $aIbI = oI$ such that $aI \in R_n(I)$.

Example 2.7: $\mathbb{C}_n(I)$ represent to neutrosophic ring of complex numbers, has $Ch\mathbb{C}_n(I) = o + oI$.

3. Neutrosophic Invariant rings (NIR)

This section contains several results of neutrosophic invariant ring. Also, some proportion and examples have been presented.

Definition 3.1: We define invariant neutrosophic ring $(VR_n(I))$ by means each of its right (left) neutrosophic ideal is also neutrosophic ideal $A_n(I)$.

Remark 3.2: Every neutrosophic ideal $A_n(I)$ of $R_n(I)$ generated by idempotent $(a^2)I = aI$ is neutrosophic multiplication module.

Example 3.3: $R_n(I)$ is a neutrosophic invariant, so every neutrosophic cyclic $R_n(I)$ is a neutrosophic multiplication module.

Example 3.4: Neutrosophic invariant ideal $VA_n(I)$ of neutrosophic invariant ring $(VN_n(I))$ generated by a set and for each $N(I) \leq (M_n(I))$, we have $K_n(I) = A_n(I) (M_n(I))$; $A_n(I)$ is an ideal of $N_n(I)$. (1)

Remark 3.5: Every neutrosophic $N_n(I)$ -module which have only two trivial neutrosophic sub-module are $\{O \cup I\}$ and $\{M_n(I)\}$ is neutrosophic multiplication module.

Theorem 3.6: Let $R_n(I)$ be a (NR). If $(VR_n(I))$ is a (NIR), then each (NC)-modules $(M_n(I))$ over $(VR_n(I))$ are neutrosophic multiplication.

$$N_n(I) = A_n(I)M_n(I). \quad (2)$$

Proof: Suppose that $M_n(I)$ is neutrosophic cyclic $(VR_n(I))$ -module; $(M_n(I)) = (mI) (VR_n(I))$. Let $N_n(I) \leq (M_n(I)) = (mI) (VN_n(I))$ and

$$N_n(I) = B_n(I). \quad (3)$$

But $N_n(I)$ subset of $(mI) (VN_n(I))$. So $N_n(I) \subseteq (mI) N_n(I) : mI \subseteq N_n(I)$. Hence,

$$N_n(I) = (mI)B_n(I). \quad (4)$$

Also, $B_n(I)$ is a neutrosophic of $(VN_n(I))$ because $(VN_n(I))$ is a neutrosophic invariant ring. Then

$$\begin{aligned} R_n(I) &= (mI)B_n(I) = (mI)((VN_n(I))B_n(I)) = ((mI)(VN_n(I)))B_n(I) \\ &= (M_n(I))B_n(I). \end{aligned} \quad (5)$$

Thus $(M_n(I))$ is neutrosophic multiplication module.

Corollary 3.7: *If free principal ideal $N_n(I)$ of $N_n(I)$ is a ideal, then $(VR \cup I)$ is a (NIR).*

Proof: Suppose that $N_n(I)$ is a ideal of $(VN_n(I))$. But $(VN_n(I))$ is a multiplication module. So $\exists B_n(I)$ is a ideal of

$$(VN_n(I)) \ni N_n(I) = (VR_n(I))B_n(I). \quad (6)$$

Hence

$$\begin{aligned} N_n(I) &= (VN_n(I))B_n(I) \\ &= (VN_n(I))(VN_n(I))B_n(I) \\ &= (VN_n(I))N_n(I). \end{aligned} \quad (7)$$

Then $N_n(I)$ is a ideal of $(VN_n(I))$. Thus $(VN_n(I))$ is a (NIR).

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