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Determining reliability signature by minimal cut method for complex-parallel network

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Abstract

The process of counting minimal cut sets (MCS) in a network is typically more difficult than finding minimal path sets (MPS). The first aim of this work is to provide a precise and effective method for counting the (MCS) of directed networks. The second aim is to calculate the effectiveness of a complex-parallel network using the (MCM) to evaluate the reliability of a complex-parallel network. The (MCM) has been applied for predicting network reliability, minimal (MS), tails signatures (TS), signatures, and Barlow-Brochan indexes.

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Keywords: *Reliability signature, Complex-parallel network, Barlow–proschan index.*

1. Introduction

Since its inception, the central concern in reliability theory has been the method of determining a network's reliability criteria based on its components' reliability requirements. Network reliability refers to the likelihood of a functional connection existing between a source(sink) nodes [1, 4]. Furthermore, the issue of evaluating network reliability is NP-hard [3], the use of assessing network reliability in different network s [2, 3]. In this paper, we will find the (MCS) for a direct network using the set of nodes for paths by finding the inverse of the nodes using the definition of (MCS) in the algorithms [5], using

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the relationship between the set and its complement. Authors in both the past and present have extensively examined the reliability of intricate networks. The significance of a network's signature, which refers to its identifying trait, is thoroughly examined and researched concerning the network's performance.

2. Proposed algorithm

In this section we will propose an algorithm to find all MCS by nodes of MPS for a directed network, using the definition MCS in [8, 10].

Step 1: Delete the source nodes s from each path $w_1 = p - \{s\}$

Step 2: Delete the sink nodes t from each path $w_2 = p - \{t\}$

Step 3: Delete the source nodes s from each path $w_3 = (P_i^a \cup P_j^b) - \{t\}$

Step 4: Find complement for w_1, w_2, w_3 and $\{s\}, \{t\}$.

Step 5: Find MCs for w_1, w_2 and w_3.

3. Algorithm for finding Signature of Complex-parallel Network

Signature analysis allows reliability engineers to enhance network efficiency by assessing the failure probability of each component (edge) based on its value [4, 6, 7].

Stage 1: Suggest that the ith element with the shortest useful life in the complex-parallel network is used to calculate the signature Si as

$$S_i = \frac{1}{\binom{n}{n-i+1}} \sum_{\substack{H \subseteq [n] \\ |H| = n-i+1}} \varphi(H) - \frac{1}{\binom{n}{n-1}} \sum_{\substack{H \subseteq [n] \\ |H| = n-1}} \varphi(H) \tag{1}$$

The quadratic of reliability can be expressed in an alternative form. The polynomial will be denoted as

$$H(R) = \sum_{j=1}^{n} \alpha_j \binom{n}{j} R^j Q^{n-j} \tag{2}$$

Where $a_j = \sum_{i=n-j+1} S_i$ for $j = 1,2, \dots, n$

Stage 2: If we assume that (TS) is computed for the complex-parallel network under consideration, is $(n + 1)$- Array of $\bar{S}(\bar{S}_0, \bar{S}_1, \dots, \bar{S}_n)$ by

$$\bar{S}_i = \sum_{k=i+1} S_i = \frac{1}{\binom{n}{n-i}} \sum_{|H|=n-i} \varphi(H) \tag{3}$$

Stage 3: Calculate the reliability function of the polynomial using a Taylor expansion with its center at $R = 1$ by

$$P(R) = R^n H\left(\frac{1}{R}\right) \tag{4}$$

Stage 4: Ascertain the (TS) of the intricate network utilizing Eq. 3 as

$$\bar{S}_i = \frac{(n-i)!}{n!} D^i P(1), i = 0,1,2, \dots, n \tag{5}$$

Stage 5: Ascertain the signature of a complex-parallel network utilizing Eq. 5 as

$$S_i = \bar{S}_{i-1} - \bar{S}_i, i = 1,2, \dots, n \tag{6}$$

Barlow–Proschan of the Complex-Parallel Network

Calculates the (BPI) of the $i.i.d.$ elements that its reliability function provides

$$I_{BP}^i = \int_0^1 (\partial_i H)(\mathrm{r}) d\mathrm{r}, i = 1,2, \dots, n \tag{7}$$

Illustrative example: Consider Figure 1 to find MCS, for the system network, we need to find MPS, it was found by an algorithm [8, 13], we get MPS:

$P_1 = \{e_{sv_1}, e_{v_1 t}\}, P_2 = \{e_{sv_1}, e_{v_1 v_2}, e_{v_2 t}\}, P_3 = \{e_{sv_2}, e_{v_2 t}\}, P_4 = \{e_{sv_3}, e_{v_3 t}\},$

$P_5 = \{e_{sv_4}, e_{v_4 v_3}, e_{v_3 t}\}, P_6 = \{e_{sv_4}, e_{v_4 t}\}$ we can conclusion all minimal path sets of Figure 1 by: $P_1^{S_{AB}} = \{x_1 x_6\}$, $P_2^{S_{AB}} = \{x_2 x_7\}$, $P_3^{S_{AB}} = \{x_1 x_5 x_7\}$, $P_4^{S_{AB}} = \{x_4 x_{10}\}, P_5^{S_{AB}} = \{x_3 x_9\}, P_6^{S_{AB}} = \{x_4 x_8 x_9\}$

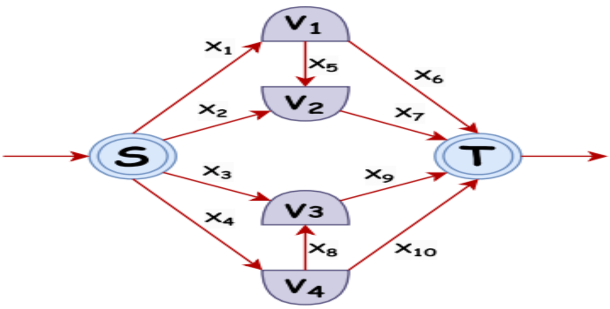

Figure 1
A Complex-parallel network.

From Table 1,2and 3 we getting all MCS of Figure 1, as:

$$C_1^{S_{AB}} = \{x_1 x_2 x_3 x_4\},$$

$$C_2^{S_{AB}} = \{x_1 x_3 x_4 x_7\}, C_3^{S_{AB}} = \{x_3 x_4 x_6 x_7\}, C_4^{S_{AB}} = \{x_2 x_3 x_4 x_5 x_6\},$$

$$C_5^{S_{AB}} = \{x_1 x_2 x_4 x_9\}, C_6^{S_{AB}} = \{x_1 x_4 x_7 x_9\}, C_7^{S_{AB}} = \{x_4 x_6 x_7 x_9\},$$

$$C_8^{S_{AB}} = \{x_2 x_4 x_5 x_6 x_9\}, C_9^{S_{AB}} = \{x_1 x_2 x_9 x_{10}\}, C_{10}^{S_{AB}} = \{x_1 x_7 x_9 x_{10}\},$$

$$C_{11}^{S_{AB}} = \{x_6 x_7 x_9 x_{10}\}, C_{12}^{S_{AB}} = \{x_1 x_2 x_3 x_8 x_{10}\}, C_{13}^{S_{AB}} = \{x_1 x_3 x_7 x_8 x_{10}\},$$

$$C_{14}^{S_{AB}} = \{x_3 x_6 x_7 x_8 x_{10}\}, C_{15}^{S_{AB}} = \{x_2 x_5 x_6 x_9 x_{10}\}, C_{16}^{S_{AB}} = \{x_2 x_3 x_5 x_6 x_8 x_{10}\}.$$

Table 1
Delete a source nodes s from each path $w_1 = p - \{s\}$ to find MCS.

$w_1 = P_i - \{t\}$	$\bar{w}_1$	MC_{ni}
$\{s\}$	$\{v_1, v_2, v_3, v_4, t\}$	$\{sv_1, sv_2, sv_3, sv_4\}$
$\{s, v_1\}$	$\{v_2, v_3, v_4, t\}$	$\{sv_2, sv_3, sv_4, v_1v_2, v_1t\}$
$\{s, v_2\}$	$\{v_1, v_3, v_4, t\}$	$\{sv_1, sv_3, sv_4, v_2t\}$
$\{s, v_3\}$	$\{v_1, v_2, v_4, t\}$	$\{sv_1, sv_2, sv_4, v_3t\}$
$\{s, v_4\}$	$\{v_1, v_2, v_3, t\}$	$\{sv_1, sv_2, sv_3, v_4v_3, v_4t\}$
$\{s, v_1, v_2\}$	$\{v_3, v_4, t\}$	$\{sv_3, sv_4, v_1t, v_2t\}$
$\{s, v_4, v_3\}$	$\{v_1, v_2, t\}$	$\{sv_1, sv_2, v_4t, v_3t\}$

Table 2
Delete a sink nodes t from each path $w_2 = p - \{t\}$ to find MCS.

$w_2 = P_i - \{s\}$	$\bar{w}_2$	MC_{ni}
$\{t\}$	$\{s, v_1, v_2, v_3, v_4\}$	$\{v_1t, v_2t, v_3t, v_4t\}$
$\{v_1, t\}$	$\{s, v_2, v_3, v_4\}$	$\{sv_1, v_2t, v_3t, v_4t\}$
$\{v_2, t\}$	$\{s, v_1, v_3, v_4\}$	$\{sv_2, v_1v_2, v_1t, v_3t, v_4t\}$
$\{v_3, t\}$	$\{s, v_1, v_2, v_4\}$	$\{sv_3, v_1t, v_2t, v_4v_3, v_4t\}$
$\{v_4, t\}$	$\{s, v_1, v_2, v_3\}$	$\{sv_4, v_1t, v_2t, v_3t\}$
$\{v_1, v_2, t\}$	$\{s, v_3, v_4\}$	deleted
$\{v_4, v_3, t\}$	$\{s, v_1, v_2\}$	deleted

Table 3
Delete a source nodes s from paths $w_3 = (P_i^a \cup P_j^b) - \{t\}$to find MCS.

$w_3 = (P_i^a \cup P_j^b) - \{t\}$	$\bar{w}_3$	MC_{ni}
$\{s, v_1, v_3\}$	$\{v_2, v_4, t\}$	$\{sv_2, sv_4, v_1v_2, v_1t, v_3t\}$
$\{s, v_1, v_4\}$	$\{v_2, v_3, t\}$	$\{sv_2, sv_3, v_1v_2, v_1t, v_4v_3, v_4t\}$
$\{s, v_2, v_4\}$	$\{v_1, v_3, t\}$	$\{sv_1, sv_3, v_2t, v_4v_3, v_4t\}$
$\{s, v_2, v_3\}$	$\{v_1, v_4, t\}$	$\{sv_1, sv_4, v_2t, v_3t\}$

Now to Evaluating the reliability of Figure 1, using Minimal Cut Method [9, 11] For $i.i.d$ reliability components, becomes:

$$R_s^{AB}(\gamma) = 4\gamma^2 + 2\gamma^3 - 10\gamma^4 - 2\gamma^5 + 11\gamma^6 + 2\gamma^7 - 11\gamma^8 + 6\gamma^9 - \gamma^{10} \quad (8)$$

And by using Algorithm for finding a reliability signature, we get:

$$\bar{S}_i =< 1,1,1,1,\frac{67}{70},\frac{51}{63},\frac{58}{105},\frac{17}{60},\frac{4}{45},0,0 >$$

$$S =< 0,0,0,\frac{63}{70},\frac{31}{210},\frac{9}{35},\frac{113}{420},\frac{7}{36},\frac{4}{45},0 >$$

The Barlow–Proschan index I_{BP}^{i} for $K = (1,2,\ldots,10)$ of the composition of all elements is illustrated, as demonstrated in [10, 12]:

$$I_{BP}^{i} = \left(\frac{23}{168},\frac{263}{2520},\frac{263}{2520},\frac{23}{168},\frac{11}{630},\frac{263}{2520},\frac{23}{168},\frac{11}{630},\frac{23}{168},\frac{263}{2520}\right)$$

Now, using Eq. 13, The network's (MS) is derived in:

$$M = (0,4,2,-10,-2,11,2,-11,6,-1)$$

So, Table 4. The results of the cost analysis and the probability of component failure indicate that the network is in satisfactory condition. At the same time, the majority of the components are in satisfactory condition and will be able to perform their functions effectively.

Table 4
Analysis of various elements' (MS), (TS), signature, and (BPI).

No.	0	1	2	3	4	5	6	7	8	9	10
MS	–	0	4	2	-10	-2	11	2	-11	6	-1
TS	1	1	1	1	$\frac{67}{70}$	$\frac{51}{63}$	$\frac{58}{105}$	$\frac{17}{60}$	$\frac{4}{45}$	0	0
S	–	0	0	0	$\frac{63}{70}$	$\frac{31}{210}$	$\frac{9}{35}$	$\frac{113}{420}$	$\frac{7}{36}$	$\frac{4}{45}$	0
BPI	–	$\frac{23}{168}$	$\frac{263}{252}$	$\frac{263}{2520}$	$\frac{23}{168}$	$\frac{11}{630}$	$\frac{263}{252}$	$\frac{23}{168}$	$\frac{11}{630}$	$\frac{11}{630}$	$\frac{263}{252}$

4. Conclusion

This study presents a novel approach for identifying all minimal cut sets in a complex-parallel network. This approach is user-friendly and efficient, requiring minimal effort for implementation, provided that the pathways of the directed network are known. The investigation revealed that the component's signature exhibits a positive correlation with both the anticipated cost and the anticipated lifespan. The (MS), (TS), (BPI), and signature of the system under consideration are calculated according to the methodology presented in Table 4.

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