

Design and analysis of a novel self-excited 5-D hyperchaotic system with maximum (n-2) positive lyapunov exponents

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Abstract

In this work, analytic and designed a new self -existed 5-D with thirteen terms nine of the term exhibit linearity, while the remaining one demonstrates nonlinearity. It has the highest Lyapunov Exponent with three positive ones to leverage the sequence in data security, encryption, and other related aspects, it's advantageous to utilize a complex system where the key remains unbreakable It contains fifteen keys sensitive ICs and parameters. The system is a highly intricate dynamical system characterized by two equilibrium points, as demonstrated through examinations such as observation of phase plane trajectories, the investigation into eigenvalue structure, and assessment of dissipative stability. These analyses provide evidence supporting the existence of chaotic and hyperchaotic attractors within the system. The results of the simulations are derived from MATLAB 2021b, and Mathematica 11. A detailed and comprehensive evaluation of the recommended techniques has been carried out, offering insights into their effectiveness and suitability for theoretical computation and numerical simulation.

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1. Introduction

Hyperchaotic systems have become increasingly prominent in scientific research due to their wide-ranging potential applications, sparking significant interest among researchers. A 1+ve LE may be dangerous in several real-world applications defined as chaotic systems, including encryption [1], environmental modeling, neural networks [2], and electronic circuits, Consequently, the

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development of (3D, 4D, and 5D) systems with L_i^E more than 1+ve which defined as hyperchaotic systems. The systems are commonly classified as possessing either hidden or the concept of self-excited attractors. Yet, the coexistence of systems featuring both hidden and self-excited attractors has largely been overlooked in earlier studies. This lack of understanding prompted the unveiling of a new, relatively uncomplicated 5-D system that has the largest L_i^E with FDL_i^E that demonstrates the simultaneous presence of self-excited attractors and a variety of behaviors has two equilibria points (EP). Table 1 compares our work with various hyperchaotic and chaotic 5-D behaviors based on L_i^E , FDL_i^E , no. of terms and classified hidden or self-excited. This research makes a significant contribution to the field through the main topic [3, 10].

1. It has (n-2) +ve L_i^E to guarantee that the system's dynamics expand in three distinct phase space directions.
2. we found a superior $L_1^E = 14.291$ to guarantee a quick exponential growth in phase space, understanding various 5-D aspects is outlined in Table 1. Additionally, the proposed system is deemed more intricate compared to another system
3. A superior $FDL_i^E = 4.9866$ value to give a high attractor complexity.
4. The determination of two EPs is R_0 and R_1 with both nonhyperbolic and unstable.

Table 1
Compare the new 5-D model with another model.

No.	No of terms	Attractor, and system behaviors	Source	Max L_i	FDL_i^E	No.of +ve L_i^E	Year
2	13	Hyperchaotic, hidden	[3]	0.6892	4.0216	2	2018
3	13	Hyperchaotic, hidden	[4]	0.29379	4.0216	3	2019
4	15	Hyperchaotic, hidden	[5]	0.9001	3.0011	3	2019
6	11	Hyperchaotic, hidden	[6]	0:0937	3.1899	2	2021
7	11	Hyperchaotic - self-excited	[7]	12.659	4.189	3	2022
8	13	chaotic - hidden	[8]	3.034	4,002	1	2022
9	10	Hyperchaotic- multi	[9]	0.3049	4.3801	3	2023
10	12	Hyperchaotic, hidden	[10]	0.9999	-	3	2023
11	13	Hyperchaotic – self-excited	New system	14. 291	4. 9866	3	2024

2. The Dynamic Features of a Self-Excited 5-D System

Consider the new 5-D differential equations described as:

$$\left\{ \begin{array}{l} \frac{dx}{dt} = -\alpha x + \beta z^2 - y \\ \frac{dy}{dt} = \sigma \sin z - \eta y - xz + u \\ \frac{dz}{dt} = \xi y - \varrho z + \delta xy \\ \frac{dw}{dt} = \psi w + \phi z \\ \frac{du}{dt} = -\gamma z \end{array} \right. \quad (1)$$

Thirteen terms make up system (1) four terms of their nonlinearity, with x , y , z , u , and w serving as five variables. α , η , $\delta > 0$, $\xi > 0$, ϱ , β , and σ , are parameters, with ψ , ϕ , and γ are control, under stratified sensitive conditions the hyperchaotic behavior is an illustration in the form of a phase portrait is evident in Figure 1 when utilizing the parameters, $\alpha = 11$, $\eta = 1$, $\delta = 5$, $\xi = 10$, $\varrho = 2.5$, $\beta = 1.38$, $\sigma = 15$, $\psi = 14.3$, $\phi = 4$, and $\gamma = 3$ (2) with ICs are (3.6, 1, 2.5, 5, 8) respectively. The system boundedness, there are bounds dependent on system solutions $x(t)$, $y(t)$, $z(t)$, $w(t)$, and $u(t)$ [11, 12, and 13].

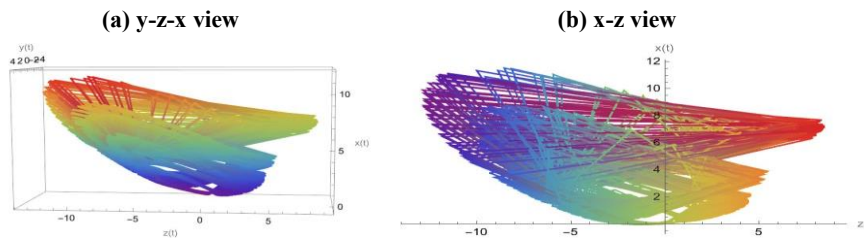


Figure 1

An illustration in the form of a phase portrait showcases the system's behavior

2.1 Equilibria points, and Divergence

From System (1) produces Eqs. (3-7) equal to zero that can be analyzed and solved to evaluate EPs:

$$-\alpha x + \beta z^2 - y = 0 \quad (3)$$

$$\sigma \sin z - \eta y - xz + u = 0 \quad (4)$$

$$\xi y - \varrho z + \delta xy = 0 \quad (5)$$

$$\psi w + \phi z = 0 \quad (6)$$

$$-\gamma z = 0 \quad (7)$$

By Eq. (7,6) we acquire $z = w = 0$, for Eq. (5) either $y = 0$ or $x = \frac{-\xi}{\delta}$, if $y = 0$ then $x = u = 0$ from Eq. (3,4). If $x = \frac{-\xi}{\delta}$ we acquire $y = \frac{\alpha\xi}{\delta}$ and $u = \frac{\alpha\eta\xi}{\delta}$. The analysis led to two cases from the system (1)

- If $\xi = 0$, $\alpha, \eta, \delta \neq 0$ and the system have a unique trivial EP. $R_0 = (0, 0, 0, 0, 0)$
- If all parameters are not equal to zero the determination of two EPs, one of them is trivial R_0 and the other is nontrivial $R_1 = \left(\frac{-\xi}{\delta}, \frac{\alpha\xi}{\delta}, 0, 0, \frac{\alpha\eta\xi}{\delta}\right)$.

Theorem 1: R_0 and R_1 under the parameters (2) is nonhyperbolic and unstable.

Proof: One way to compute the Jacobian matrix for system (1) is derived as:

$$J = \begin{bmatrix} -\alpha & -1 & 2z\beta & 0 & 0 \\ z & -\eta & -x + \sigma\cos[z] & 0 & 1 \\ y\delta & x\delta + \xi & -\varrho & 0 & 0 \\ 0 & 0 & \phi & \psi & 0 \\ 0 & 0 & -\gamma & 0 & 0 \end{bmatrix}$$

and the characteristic associated formula is

$$\lambda^5 + A\lambda^4 + E + B\lambda^3 + C\lambda^2 + D\lambda = 0 \quad (8)$$

As per the Hurwitz conditions [11], [12], at R_0 by add conditions parameters (2) to Eq.(8) then $\Delta_3 < 0$,

$$-1320 + 6260\lambda + 883.5\lambda^2 + 133\lambda^3 + 10.5\lambda^4 + \lambda^5 = 0 \quad (9)$$

with roots are $\lambda^1 = 0.1958, \lambda^2 = -11, \lambda^3 = 4, \lambda^{4,5} = -1.848 \pm 12.2396i$, now at R_1 under the same condition (2) then $\Delta_1 < 0$,

$$1320 - 6400\lambda - 2413.5\lambda^2 + 254\lambda^3 - 11.5\lambda^4 + \lambda^5 = 0 \quad (10)$$

with roots are $\lambda^1 = 6.7134, \lambda^2 = 0.2249, \lambda^3 = -4, \lambda^{4,5} = 0.2809 + 14.7822i$ in both cases concluded that the R_0 and R_1 nonhyperbolic unstable EPs.

Remark 2: The system exhibits the capacity to spontaneously produce an attractor, leading to self-excited oscillations. Theorem:

- If $< \alpha + \eta + \varrho$, then new 5-D exhibits dissipative.
- If $= \alpha + \eta + \varrho$, then new 5-D exhibits conservative.
- If $> \alpha + \eta + \varrho$, then the new 5-D is unbounded.

Proof: Considering that system (2) divergence is described as:

$$\nabla \cdot S = \frac{\partial z}{\partial z} + \frac{\partial w}{\partial w} + \frac{\partial u}{\partial u} + \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} = -\alpha - \eta - \varrho + \psi$$

prove that system (1) might be dissipative, unbounded, or conservative based on the relationship shown above.

2.2 Fractional Dimension, Lyapunov Exponent

Apart from mirroring the dynamic characteristics of a chaotic system, the Lyapunov exponent (L_i^E) serves as a vital indicator for discerning whether a system exhibits hyperchaotic or chaotic behavior.

Theorem 3: System (1) has "n-2" +ve L_i^E and it is dissipative "hyperchaotic dissipative attractors" where parameter (2) is satisfied.

Proof: We will use Wolf [1] to calculate L_i^E , as below, evaluated by the MATLAB 2021b with $T=1000$ with step units $T=0.05$, then

$$L_1^E = 14.291, L_2^E = 0.151, L_3^E = 0.0292, L_4^E = 0, L_5^E = -14.66672$$

Since $\sum_{i=1}^5 L_i^E = -0.2$, consequently, the system (2) is dissipative and remarkably displays three +ve $L_i^E, i = 1, 2, \dots, 5$, demonstrating the fulfillment of the condition of having "n-2" +ve L_i^E is essential for classifying a system as possessing hyperchaotic dissipative attractors. for $L_1^E > L_2^E > L_3^E > L_4^E > L_5^E$, The fractional dimension FDL_i^E [8] is established by relying on the L_i^E , system (1) calculated as follow: $FDL_i^E = 4 + \frac{14.4712}{14.66672} = 4.9866$, we obtained a non-integer value FDL_i^E which represents indicating extremely intricate dynamics behavior. **Figure 2** showcase the L_i^E calculation for the newly proposed 5-D system.

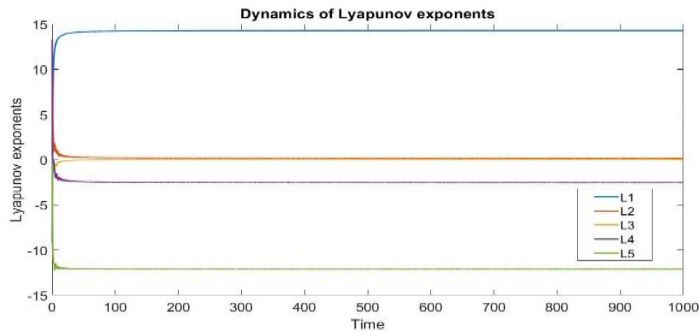


Figure 2
Showcase the L_i^E for system (2)

3. Discussion and Conclusion

This paper designed a new 5-D hyperchaotic system with thirteen terms nine of the term exhibit linearity, while the remaining one demonstrates nonlinearity characterized by two unstable hyperbolic equilibrium points, meeting the criteria for classification as a self-excited attractor. We compared a new 5-D hyperchaotic system that has unstable equilibria with other systems, we found this system has a superior fractional dimension $FDL_i^E = 4.9866$ and maximum $L_1^E = 14.291$ with $(n-2) + L_i^E$ therefore this enables us to use the system for data security. These diverse dynamical behaviors are confirmed through the analysis of phase portrait plots and Lyapunov Exponent plots. The proposed system exhibits complex behavior, characterized by the simultaneous presence of transient chaotic behavior and an attractor.

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