

Construction of cubic magic with cryptography application

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Abstract

There are numerous potential applications for the formidable instrument known as Cubic Magic. Combining mathematics, engineering, and art allows the creation of cubes with specific properties. This technique may be applied in many other contexts. This article will discuss how it works and its uses in different industries. While the concept has been around for millennia, it has only lately gained popularity due to its diverse uses. Our work is to build Cubic Magic using two techniques and use them with cryptography applications and the Matlab program presented.

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1. Introduction

Both mathematicians and non-specialists have shown an interest in the interesting mathematical concept known as magic squares. They are regarded as

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stunning mathematical illustrations and have a variety of uses [1]and[7]. The origins of magic squares can be found in a Chinese myth known as He-Tu and Lo-Shu, which goes back to 2200 B.C. The Lo-Shu, which was said to have been transported on the shell of a tortoise from the Lo River, is the earliest magic square that has ever been discovered. In China, magic squares have been used in several academic disciplines, Astrology, divination, the interpretation of natural phenomena, philosophy, and social psychology are only a few examples. For more see [15]and[18]

Numerous varieties of magic squares are found, each with a distinct set of characteristics and guidelines, such as anti-magic squares, hetero squares, semi-magic squares, etc. Three-dimensional figures that make up magic cubes can develop the peculiar and intriguing qualities of magic squares. [3],[10]and[14]. In a letter from P. Fermat dated 1640, the first magic cube made an appearance. Magical cubes and squares have long been a source of mathematical entertainment and perplexing mathematical questions. These topics fall under the category of recreational mathematics and have a sizable fan base, many of whom are amateur mathematicians. Books like[4 -6]as well as more current works like[8] and [17], provide details on many kinds of magic cubes, construction techniques, and historical relevance. A lot of applications were found that relate to combinatorics in different areas of mathematics such as [11-12] and [13].

2. Preliminaries

Some definitions that relate to our work were introduced as,

Magic Cube [2] Is a square arrangement of n^2 numbers, usually distinct integers, is referred to as a magic square of order n if the total of the n numbers in all the rows, columns, and diagonals equals a fixed value known as the magic number. An $n \times n$ matrix can be used to represent a typical magic square. The numbers 1 to n^2 are utilized in a typical magic square, and there are such squares for all orders $n=1$ except for $n=2$. The following formula can be used to get the magic constant for ordinal normal magic squares of order n [5].

$$M_n = \frac{n(n^2 + 1)}{2}$$

It can be seen from the example 3×3 normal magic in Figure 1 that the sum of each of the square's three vertical, three horizontal, and two corner diagonal columns is 15, giving all eight columns that total.

8	1	6
3	5	7
4	9	2

Figure 1
Magic square of order 3

A semi-magic square is an n by n matrix whose elements are positive integers and whose row sum and column sum are equal to the magic constant but whose main diagonal sum or both of them do not equal the magic constant, as seen in Figure 2.

1	5	9
6	7	2
8	3	4

Figure 2
Semi magic square

Construction of Magic Square

Designing cubes and magic squares is an attractive math behavior that emphasizes the beauty and complex nature of number arrangement. To be certain that the sum of each row, column, and diagonal coincides with the same constant, you need to arrange the numbers in this fashion. Using magic squares, a rationally ordered grid of numbers is created with each piece blending into the next to form an interesting pattern.

Making cubes and magic squares is a required math action that focuses on the beauty and complicated nature of number arrangement. To get the sum of each row, column, and diagonal fit with the same constant. Using magic squares, an objectively ordered grid of numbers is created with each component dissolving into the next to generate an intriguing pattern.

In this part, we will discuss two different constructions of the magic square and cube, which are as follows:

A. Magic square Construction

Magic squares cannot be built using a single method or limited algorithm that suits all types of magic squares. For example, without further modification or work, a method designed for even-order squares cannot be used to odd-order squares, unless via experiment and error or by building computer software using programming languages. There have been many different ways to create magic squares over time. It is useful to explain the three most popular methods for creating magic squares, the single even order, the double even order technique, and those odd order as well as to split magic squares into three classes based on their matrix dimensions [8, 9]

B. Magic Cube Construction [16]

The term "magic cube" refers to a three-dimensional arrangement of integers from 1 to n^3 , where n is the cube's order, where the total of the integers in each row, column, pillar, and four space diagonals equals a fixed amount known as the cube's magic constant. The magic constant can be computed using the formula that follows:

$$M_n = \frac{n(n^3+1)}{2}$$

From Figure 3, the smallest standard magic cube of 3x3x3 dimension with a sequence of numbers from 1 to 27 that are arranged in three horizontal planes, there are nine rows, nine columns, nine pillars, and four space diagonals. The sum of the numbers in any row, column, and pillar equals 42 (for instance, row: 4 + 20 + 18, column: 26+ 12 +14, and pillar: 1 + 17 + 24). Additionally, each space diagonal's sum equals 42 (1 + 14 + 27, 18 + 14 + 10, 26+ 14+ 2, 4+14 + 24). However, it is not required for the sum along every face diagonal to equal the magic sum (for instance, 27 + 21 + 24 $\neq$ 42).

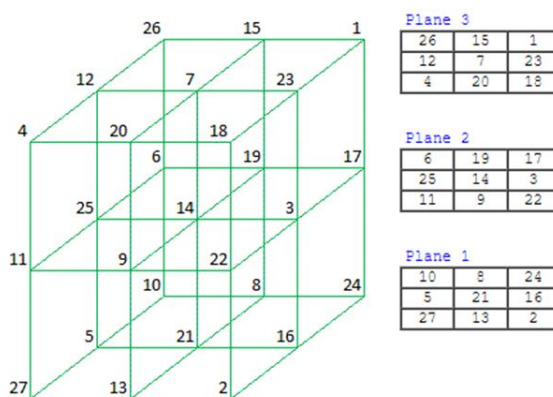


Figure 3
A magic cube of order 3

3. Design of Cube Magic

This section deals with the construction of a magic cube. In this section, the construction of a magic cube with two different views was given.

I. The first method of construction

This work depends on the distribution of numbers from 1 to n^3 in the faces of a cube that is, if the faces of cubes were taken and filled the number in them using one of the exact methods that are used to build magic squares and consequently fill them such that all faces are magic. The result is the sum of two opposite square faces that have the same constant, that is, (5)

1. Put the magic square faces of the cube as given in Figure 4 such that each one is a magic square.

8	1	6
3	5	7
4	9	2

Face 1

17	10	15
12	14	16
13	18	11

Face 2

26	19	24
21	23	25
22	27	20

Face 3

35	28	33
30	32	34
31	36	29

Face 4

44	37	42
39	41	43
40	45	38

Face 5

53	46	51
48	50	52
49	54	47

Face 6

Figure 4
Construction of Magic Cube

- The sum of two squares which are (1,6),(2,5), and (3,4) have the same magic constant 165.
- Thus the magic cube is obtained.

That is it can be built any magic cube by the first distribution of numbers 1 to n^2 into a magic square and combined them to form a magic cube.

II. The second method of construction

Construct the smallest standard magic cube of order 3 without a cavity inside the cube, with a sequence of numbers from 1 to 27 without the value 14 that are arranged in the faces of a cube that is, if the faces of cubes were taken, assign variables to each cell of the cube, starting from 1 in the top left corner and increasing by 1 for each cell as shown in the following Figure 5.

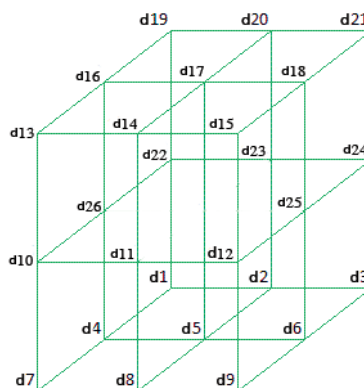


Figure 5
A cube with their faces

d_1	d_2	d_3
d_4	d_5	d_6
d_7	d_8	d_9

 f_1

d_7	d_8	d_9
d_{10}	d_{11}	d_{12}
d_{13}	d_{14}	d_{15}

 f_2

d_{13}	d_{14}	d_{15}
d_{16}	d_{17}	d_{18}
d_{19}	d_{20}	d_{21}

 f_3

d_{15}	d_{18}	d_{21}
d_{12}	d_{25}	d_{24}
d_9	d_6	d_3

 f_4

d_{13}	d_{16}	d_{19}
d_{10}	d_{26}	d_{22}
d_7	d_4	d_1

 f_5

d_{19}	d_{20}	d_{21}
d_{22}	d_{23}	d_{24}
d_1	d_2	d_3

 f_6

Write down the equations for each row, and column, of the cube. The sum of the numbers in any row, or column is 42, that is 24 linear equations were obtained.

Rows	Columns
$d_1 + d_2 + d_3 = 42, d_4 + d_5 + d_6 = 42$	$d_2 + d_5 + d_8 = 42, d_7 + d_{10} + d_{13} = 42$
$d_7 + d_8 + d_9 = 42, d_{10} + d_{11} + d_{12} = 42$	$d_8 + d_{11} + d_{14} = 42, d_9 + d_{12} + d_{15} = 42$
$d_{13} + d_{14} + d_{15} = 42, d_{16} + d_{17} + d_{18} = 42$	$d_{14} + d_{17} + d_{20} = 42, d_{19} + d_{22} + d_1 = 42$
$d_{19} + d_{20} + d_{21} = 42, d_{22} + d_{23} + d_{24} = 42$	$d_{20} + d_{23} + d_2 = 42, d_{21} + d_{24} + d_3 = 42$
$d_{15} + d_{18} + d_{21} = 42, d_{12} + d_{25} + d_{24} = 42$	$d_{18} + d_{25} + d_6 = 42, d_4 + d_{26} + d_{16} = 42$
$d_9 + d_6 + d_3 = 42, d_7 + d_4 + d_1 = 42$	$d_{10} + d_{26} + d_{22} = 42, d_{13} + d_{16} + d_{19} = 42$

By solving the system of equations by Mathematica Program, the values of each variable that satisfy all equations are found. The solutions are obtained by guessing $d_1, d_2, d_4, d_5, d_{10}, d_{11}, d_{16}$ and d_{17} plugging out these guesses in the aforementioned equations, resulting in 192 third-order magic cubes

$d_3 = 42 - d_1 - d_2$	$d_{18} = 42 - d_{16} - d_{17}$
$d_6 = 42 - d_4 - d_5$	$d_{19} = 42 - d_1 + d_{10} - d_{16} - d_4$
$d_7 = 42 - d_1 - d_4$	$d_{20} = 42 + d_{11} - d_{17} - d_2 - d_5$
$d_8 = 42 - d_2 - d_5$	$d_{21} = -42 + d_1 - d_{10} - d_{11} + d_{16}$ $+d_{17} + d_2 + d_4 + d_5$
$d_9 = -42 + d_1 + d_2 + d_4 + d_5$	$d_{22} = -d_{10} + d_{16} + d_4$
$d_{12} = 42 - d_{10} - d_{11}$	$d_{23} = -d_{11} + d_{17} + d_5$
$d_{13} = d_1 - d_{10} + d_4$	$d_{24} = 42 + d_{10} + d_{11} - d_{16} - d_{17}$ $-d_4 - d_5$
$d_{14} = -d_{11} + d_2 + d_5$	$d_{25} = -42 + d_{16} + d_{17} + d_4 + d_5$
$d_{15} = 42 - d_1 + d_{10} + d_{11} - d_2$ $-d_4 - d_5$	$d_{26} = 42 - d_{16} - d_4$

The solutions for $d_i, i = 1, \dots, 26$ are $d_1 = 6$ $d_2 = 10$ $d_3 = 26$ $d_4 = 11$ $d_5 = 27$ $d_6 = 4$ $d_7 = 25$ $d_8 = 5$ $d_9 = 12$ $d_{10} = 15$ $d_{11} = 19$ $d_{12} = 8$ $d_{13} = 2$ $d_{14} = 18$ $d_{15} = 22$ $d_{16} = 24$ $d_{17} = 1$ $d_{18} = 17$ $d_{19} = 16$ $d_{20} = 23$ $d_{21} = 3$ $d_{22} = 20$ $d_{23} = 9$ $d_{24} = 13$ $d_{25} = 21$ $d_{26} = 7$ respectively, we obtained the faces

6	10	26
11	27	4
25	5	12

 f_1

25	5	12
15	19	8
2	18	22

 f_2

2	18	22
24	1	17
16	23	3

 f_3

16	23	3
20	9	13
6	10	26

 f_4

22	17	3
8	21	13
12	4	26

 f_5

2	24	16
15	7	20
25	11	6

 f_6

Algorithm of generating the magic cube of the second method

Step 1: Input eight variables $d_1, d_2, d_4, d_5, d_{10}, d_{11}, d_{16}$ and d_{17} each ranging from 1 to 27.

Step 2: Use nested loops to iterate through all possible combinations of the eight variables.

Step3: Check if each variable is not equal to 14 and is not equal to any of the other variables.

If all conditions are met, set the value of d_3 as $42 - d_1 - d_2$.

Step 4: For each of the variables d_6 to d_{26} , generate a random number between 1 and 27, ensuring that the number is not equal to 14 and is distinct from all previously generated numbers

Step 5: Create a matrix w with the values d_1 to d_{26} .

Step 6: Create six 3×3 matrices, f_1 through f_6 , from the elements of matrix w

4. Application of Magic cube

The magic cube and squares have several practical uses in a variety of areas, some of which are applied[15] and others which are pure[14]. We will take an application in cryptography, and the other will be in the next paper that relates to the mass of gravity.

4.1 Magic Cube Cryptosystem

Constructing a produced set of magic cubes requires knowledge of only one magic cube and construction of the others based on it. In section 2, the first one is presented, and an algorithm is provided for this purpose. The approach that is suggested is described using the steps below, for instance for $n = 3$.

Step 1: The generator magic square is f_1 for the first face of a cube.

Step 2: Use the other faces obtained for the magic cube which are $f_2, f_3, f_4, f_5, f_6, f_7, f_8$.

For all n dimensions of the magic cube, one can obtain more people of this magic cube as shown in Figure 6.

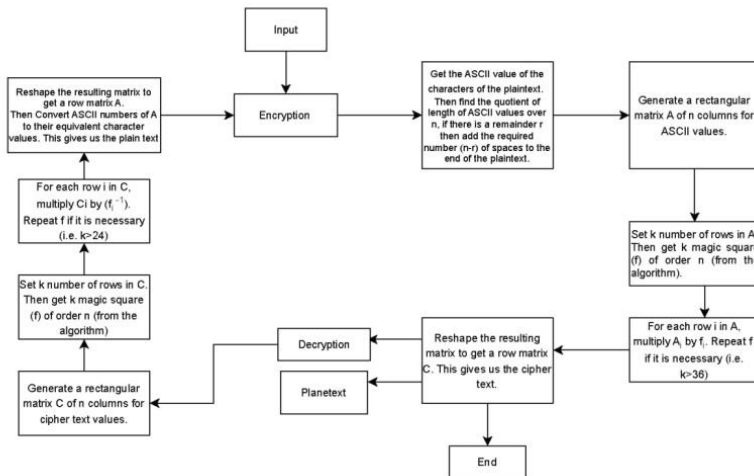


Figure 6
Block diagram for the algorithm of generating semi-magic squares set

Example:

I. Encryption: We will take circle magic of order 3.

1. Specify "I Like Mathematics" in the plaintext's characters. The comparable ASCII values are: 73 32, 76, 105, 107, 101, 32, 77, 116, 104, 101, 109, 116, 105, 99, 115.
2. It must add spaces to the end of plain text to obtain the letter A Since there are 18 ASCII characters
3. Create a matrix A with the plain text code in it, where

$$A = \begin{bmatrix} 73 & 32 & 76 \\ 105 & 107 & 101 \\ 116 & 104 & 101 \\ 109 & 97 & 116 \\ 105 & 99 & 115 \end{bmatrix}$$

One can add a space with the ASCII code 32 if the matrix is not arranged.

4. Then $k=6$.
5. Use $f_1, f_2, f_3, f_4, f_5, f_6$.

$$6. \quad \text{The matrix for the cipher text is } C = \begin{bmatrix} 2690 & 1974 & 2938 \\ 4432 & 4376 & 4338 \\ 3464 & 2884 & 2304 \\ 4542 & 4614 & 4326 \\ 4566 & 4354 & 4604 \\ 4570 & 4478 & 4350 \end{bmatrix}$$

7. Convert C in the one-dimensional matrix.
 $C = [2690 \ 1974 \ 2938 \ 4432 \ 4376 \ 4338 \ 3464 \ 2884 \ 2304 \ 4542 \ 4614 \ 4326 \ 4566 \ 4354 \ \dots]$

II. Decryption

$$1. \quad C = \begin{bmatrix} 2690 & 1974 & 2938 \\ 4432 & 4376 & 4338 \\ 3464 & 2884 & 2304 \\ 4542 & 4614 & 4326 \\ 4566 & 4354 & 4604 \\ 4570 & 4478 & 4350 \end{bmatrix}$$

2. Set $k=6$.
3. Take the inverse of $f_1, f_2, f_3, f_4, f_5, f_6$ according to the definition of the inverse for f .

$$4. \quad A = \begin{bmatrix} 73 & 32 & 76 \\ 105 & 107 & 101 \\ 116 & 104 & 101 \\ 109 & 97 & 116 \\ 105 & 99 & 115 \end{bmatrix}$$

5. Reshape the resulting matrix to get $A = 73 \ 32 \ 76 \ 111 \ 118 \ 101 \ 32 \ 77 \ 97 \ 116 \dots$
6. The plain text "I Like Mathematics" were got after translating the integers in A to their character equivalents.

All the obtained computation is performed using MATLAB language.

5. Conclusion

Many branches of science continue to study magic squares, the building blocks of magic cubes, and their many uses in fields as diverse as computer science, engineering, and physics. Based on existing ways for designing magic squares and creating merging between them, our work established a two-method design for a magic cube. Additionally, new apps were presented. This paper introduces the magic cube and its significant importance in cryptosystems. A method and an algorithm for building the magic cube set were provided, which can be used to cipher any plaintext. In the example, we demonstrated that the output is a string of unintelligible integers. That being the case, only authorized parties possess the inverse of the key, which is symbolized by the inverse of semi-magic squares, and hence are able to decipher the encrypted text. This approach requires a large number of computations, and its complexity grows as n increases. Our computation is executed on a PC with the MATLAB programming language.

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